Student ID

Teacher

St Vincent's College

MATHEMATICS

HSC
Task 2
15 March 2013

Weighting 25%

Total marks	/100

General Instructions

- Reading Time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
- Draw diagrams using pencil, but go over final diagram in pen.
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- In Questions 11-16, show relevant mathematical reasoning and/or calculations
- Write your ID number on the top of this page and on each sheet of paper used

Section I
10 marks

- Attempt Questions 1 - 10
- Each question is worth 1 mark
- Allow about 15 minutes for this section
- Record your answers on the Multiple Choice answer sheet provided

Section II
90 marks

- Attempt Questions 11 - 16
- Each question is worth 15 marks
- Allow about 2 hours and 45 minutes for this section
- Write your response on the writing paper provided
- Start a new sheet of paper for each new Question.

Section I

10 marks
Attempt Questions 1–10 (1 mark each)
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

1 Convert the angle 22° into radians, correct to 3 significant figures:

(A) 0.122
(B) 0.384
(C) 1.26
(D) 0.768

2 Which of the following is a primitive function of : $4x - \frac{x^2}{2}$?

(A) $4 - 2x$
(B) $2x^2 - \frac{3x^3}{2x} + c$
(C) $2x^2 - \frac{x^3}{6} + 7$
(D) $x\left(4 - \frac{x}{2}\right)$

3 Which of the following functions is increasing for all x ?

(A) $f(x) = x^2$
(B) $f(x) = x^3$
(C) $f(x) = -x^2$
(D) $f(x) = -x^3$

4 What is the equation of the tangent to the parabola $y = 4 - x^2$ at the point (2, 0) ?

(A) $2x + y - 4 = 0$
(B) $2x + y + 4 = 0$
(C) $4x + y - 8 = 0$
(D) $4x + y + 8 = 0$

5 What are the solutions to the equation $2\cos x + 1 = 0$ for $0 \leq x \leq 2\pi$

(A) $-\frac{5\pi}{6}$ and $\frac{5\pi}{6}$
(B) $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$
(C) $-\frac{2\pi}{3}$ and $\frac{2\pi}{3}$
(D) $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$

6 Which of the following is a property of <u>all</u> rectangles, but <u>not all</u> rhombuses.

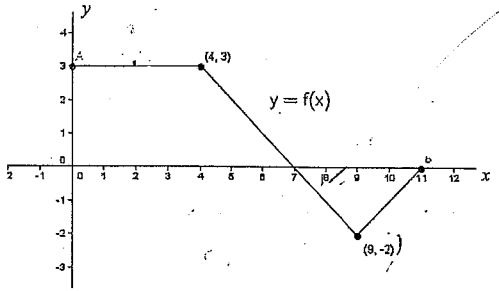
(A) opposite angles are equal

(B) diagonals are perpendicular

(C) diagonals bisect each other

(D) diagonals are equal

7 The following graph defines the function $y = f(x)$ over the domain $0 \leq x \leq 11$:



Using the details of this graph, evaluate the integral $\int_0^{11} f(x)dx$.

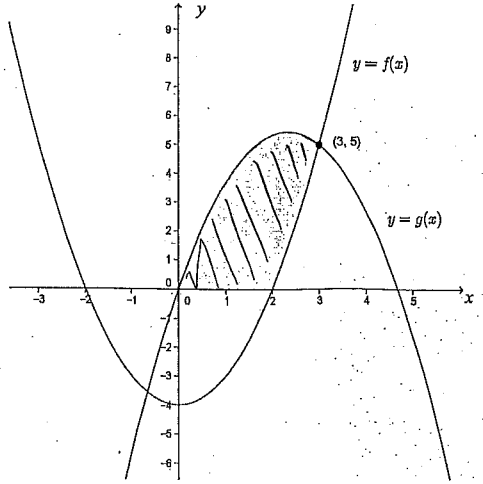
(A) 12.5

(B) 20.5

(C) 23

(D) 60.5

8 Which of the following does **not** represent the shaded area shown?



(A) $\int_0^3 (g(x) - f(x))dx$

(B) $\int_0^2 g(x)dx + \int_2^3 (g(x) - f(x))dx$

(C) $\int_0^3 g(x)dx - \int_2^3 f(x)dx$

(D) $\int_0^2 g(x)dx + \int_2^3 g(x)dx - \int_2^3 f(x)dx$

9 A sector of a circle, has an angle of 2 radians at the centre and an area of $10\ cm^2$. What is the exact radius of the circle?

(A) $\sqrt{10\pi}\ cm$

(B) $\sqrt{10}\ cm$

(C) $10\pi\ cm$

(D) $10\ cm$

10 Given that, $\int_1^k \frac{1}{x^2}dx = a$, find k in terms of a.

(A) $k = \frac{1}{1-a}$

(B) $k = \frac{1}{1+a}$

(C) $k = \sqrt{\frac{a-3}{3}}$

(D) $k = \sqrt{\frac{a+3}{3}}$

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question on the writing paper provided. Start a new page for each new question.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks)

a) Find each of the following indefinite integrals:
   i) $\int 5x^2 - 1 \ dx$ 2
   ii) $\int \sqrt{x} \ dx$ 2
   iii) $\int \frac{x^4-1}{x^3} \ dx$ 3

b) Evaluate each of the following definite integrals:
   i) $\int_0^2 \frac{t^3}{2} \ dt$ 3
   ii) $\int_{-3}^3 (2x+1)^3 \ dx$ 3

c) Find the values of p, given that $\int_0^p 2x - 3 \ dx = 4$ 2

Question 12 (15 Marks) START A NEW PAGE

a) Write down the size of an angle of 0.5 radians in degrees and minutes to the nearest minute. 1

b) Give the simplified exact value of the following:
   i) $sin\frac{\pi}{3}$ 1
   ii) $cos\frac{3\pi}{4}$ 2
   iii) $sec\frac{\pi}{6}cot\frac{\pi}{3}$ 2

c) Solve the following equations (give exact answers if possible, or correct to 2 decimal places if not)
   i) $sin\theta = 0.9$, for $0 \leq \theta \leq 2\pi$ 2
   ii) $\tan(x) = 1$, for $-\pi \leq x \leq \pi$ 2
   iii) $2cos^2x = 2$, for $0 \leq x \leq 2\pi$ 3

d) Show that $1 - cos^2\left(\frac{\pi}{2} - x\right) \equiv cos^2x$ 2

Question 13 (15 Marks) START A NEW PAGE

a) A circle with radius of 20cm has a sector with an angle of θ=40° at the centre.
   i) Express the size of angle θ in radians (to 3 significant figures). 1
   ii) Find the perimeter of the sector (to 3 significant figures). 2
   iii) Find the area of the sector (to 3 significant figures). 2

b) A circle of radius 5 m has a chord of 2 m.
   i) Find the size of the angle subtended by the chord at the centre (to 2 decimal places). 3
   ii) Find the area of the minor segment formed by the chord (to 2 decimal places). 2

c) The hour hand of the Big Ben clock is 2.7m long, while its minute hand is 4.3m long
   i) Find the distance between the tips of the two hands at 4 o'clock (to 2 decimal places). 3
   ii) What distance does the tip of the minute hand travel along the curved path it takes in the next 20 minutes (to 2 decimal places) 2

Question 14 (15 Marks) START A NEW PAGE

a) Find the area between the curve $y = 9 - x^2$ and the x-axis. 3

b) For the curve $y = x^3 - 2x^2 - 3x$:
   i) Find its x-intercepts. 2
   ii) Find the area bounded by the x-axis and the curve. 3

c) The curves $y = x^2$ and $y = \sqrt{8x}$ cross at the origin.
   i) Find the other point at which they cross. 2
   ii) Find the area between these two curves. 2

d) Find the volume of the solid of revolution formed when the area between the lines $y = x + 1$, $y = 4$ and the y-axis is rotated around the y-axis. 3

Question 15 (15 Marks) START A NEW PAGE

a) A curve $y = x(x-2)^2$ is defined over the domain $0 \leq x \leq 3$

i) Find its stationary points. 3

ii) Sketch the graph. 2

iii) What is the maximum y-value of any point on the curve? 1

b) A rectangular sheet of tin is 50 cm long by 30 cm wide.
Four equal squares, of side length $x\ cm$, are cut from its corners to allow the sheet to be formed into a lidless box of height $x\ cm$.

i) Show that the volume of the box is given by $V = 4x^3 - 160x^2 + 1500x$ 2

ii) Find the maximum volume of the box. 3

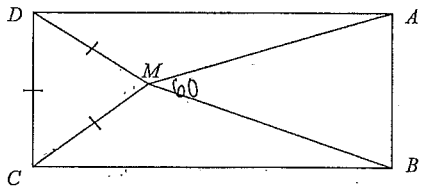
c) A biologist finds the skin of a snake, left behind by a snake after it had moulted. She measures the width of the skin at 20 cm interval along the length of the snake. The first width is measured at the tip of its tail and the last at the tip of its head. The widths, in cm, where recorded in her notebook as: 5, 10, 12, 16, 15, 13, 8.

i) How long was the snake? 1

ii) Use the trapezoidal rule to calculate an approximate area of the skin. 3

Question 16 (15 Marks) START A NEW PAGE

a) In the diagram shown, $ABCD$ is a rectangle and M is positioned so that $\triangle MCD$ is equilateral.



i) Prove that $\triangle MAD \equiv \triangle MBC$ 3

ii) Hence show that $\triangle MAB$ is isosceles. 2

b) The normal to the curve $y = \sqrt{x}$ is drawn at the point (1,1)

i) Show that the equation of the normal is $2x + y - 3 = 0$ 2

ii) Find the area of the region between the curve, the normal and the y-axis. 3

c) For the integral: $\int_0^1 \sqrt{1-x^2}\,dx$

i) Explain what area on the Cartesian plane that this integral represents. 1

ii) Use two applications of Simpson's rule to evaluate this integral. 3

iii) Explain why (i) and (ii) imply that $\pi \approx (1+\sqrt{3}+\sqrt{7}+\sqrt{15})/3$. 1

[End of Exam]

ISC MATHEMATICS: TASK 2 2013 Solutions

1 ① B ② C ③ B ④ C ⑤ D ⑥ D ⑦ A ⑧ A ⑨ B ⑩ A

Working:

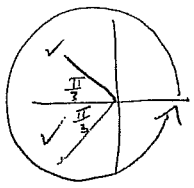
1: $22° \times \frac{\pi}{180°} = 0.38397..$
≈ 0.384 radians (to 3 sig.figs) [B]

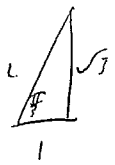
2: $\int (4x - \frac{x^2}{2})\,dx = \frac{4x^2}{2} - \frac{x^3}{3\times 2} + C$
$= 2x^2 - \frac{x^3}{6} + C$ for any C
$= 2x^2 - \frac{x^3}{6} + 7$ is a primitive. [C]

3: Graphically: What is always ↗ . ↗ ✓, ↘ ✓, ↘ ✗, ↘ ✗
Algebraically: Need $f'(x) > 0$
$\frac{d}{dx}(x^3) = 3x^2 \times \frac{d}{dx}(x^3) 3x^2 > 0$ ✓ since $x^2 > 0$.
$\frac{d}{dx}(-x^3) = -3x^2 < 0$ ✗, $\frac{d}{dx}(-x^2) = -2x$ ✗.
$\therefore y = x^3$ [B]

4: $y = 4 - x^2$
$y' = -2x$
@ $x = 2$, $m = -2\times 2 = -4$
∴ tangent $y - y_1 = m(x - x_1)$ @ $(2,0)$
$y - 0 = -4(x-2)$
$y = -4x + 8$
$4x + y - 8 = 0$ [C]

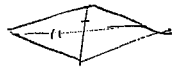
5: $2\cos x + 1 = 0$
$\cos x = -\frac{1}{2}$
[basic $x = \cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$]
$x = \pi - \frac{\pi}{3}$, $\pi + \frac{\pi}{3}$
$x = \frac{2\pi}{3}$, $\frac{4\pi}{3}$ [D]





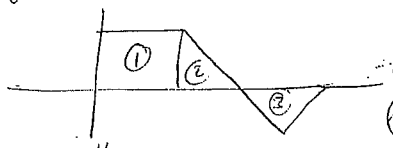
§1 ctd.

Q6. (A) opp $\angle$s equal: true for both rectangle & rhombus (✓)
(B) diagonals perpendicular: false for rectangles, true for rhombus (✗)
(C) diagonals bisect: true for rectangle, true for rhombus (✓)
(D) diagonals equal: true for rectangle, false for some rhombus (✗)



∴ [D]

Q7 $\int_0^{11} f(x)\,dx$ = sum of the "signed areas"

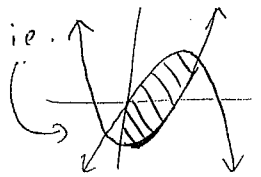


A① $= 4\times 3 = 12$
A② $= \frac{1}{2}\times 3\times 3 = 4.5$
A③ $= -\frac{1}{2}\times 4\times 2 = -4$

$\int_0^{11} f(x)\,dx = 12 + 4.5 - 4 = 12.5$ [A]

Q8 [A] $\int_0^3 (g(x) - f(x))\,dx \neq$ shaded area ∴ [A]

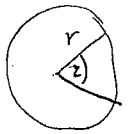
$\int_0^3 (g(x) - f(x))\,dx$ = area between curve



i.e.

the other integrals all give the shaded area in the question..

Q9



$A = 10\pi$
$A = \frac{1}{2}\theta r^2$
$10 = \frac{1}{2}\times 2\times r^2$
$r^2 = 10$
$r = \sqrt{10}$ cm [B]

Q10 $\int_1^k (x^{-2})\,dx = a$
$\left[\frac{x^{-1}}{-1}\right]_1^k = a$
$\left[-\frac{1}{x}\right]_1^k = a$

$\left\{\left(-\frac{1}{k}\right) - \left(-\frac{1}{1}\right)\right\} = a$
$-\frac{1}{k} + 1 = a$
$-\frac{1}{k} = a - 1$
$\frac{1}{k} = 1 - a$
$k = \frac{1}{1-a}$ ∴ [A]

§2 Question 11

a) i) $\int (5x^2 - 1)\,dx = \frac{5x^3}{3} - x + C$ ✓✓

ii) $\int \sqrt{x^3}\,dx = \int x^{\frac{3}{2}}\,dx$ ✓

$= \frac{2x^{\frac{5}{2}}}{5} + C$ ✓

$= \frac{2\sqrt{x^5}}{5} + C$

$= \frac{2x^2\sqrt{x}}{5} + C$ } either.

iii) $\int \frac{x^4 - 1}{x^2}\,dx = \int (x - x^{-3})\,dx$ ✓

$= \frac{x^2}{2} - \frac{x^{-2}}{-2} + C$ ✓

$= \frac{x^2}{2} + \frac{1}{2x^2} + C$ ✓

b) i) $\int_0^2 \frac{t^3}{2}\,dt = \left[\frac{t^4}{8}\right]_0^2$ ✓

$= \left\{\left(\frac{2^4}{8}\right) - \left(\frac{0^4}{8}\right)\right\}$ ✓

$= \frac{16}{8} = 2$ ✓

ii) $\int_{-3}^{3} (2x+1)^3\,dx = \left[\frac{(2x+1)^4}{4\times 2}\right]_{-3}^{3}$ ✓ ✓

$= \left\{\frac{(2(3)+1)^4}{8} - \frac{(2(-3)+1)^4}{8}\right\}$

$= \frac{7^4 - (-5)^4}{8}$

$= \frac{1776}{8}$

$= 222$ ✓

Note: −1 marks for (a) if +C has been consistently left out.

Q11 ctd

c) $\int_0^p 2x - 3\,dx = 4$

$\left[x^2 - 3x\right]_0^p = 4$

$(p^2 - 3p) - (0^2 - 3(0)) = 4$

$p^2 - 3p = 4$ ✓

$p^2 - 3p - 4 = 0$

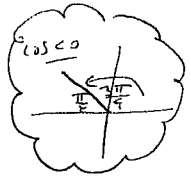
$(p-4)(p+1) = 0$

$\therefore p = 4, -1.$ ✓

Question 12

a) $0.5\,\text{rad} \times \frac{180^\circ}{\pi\,\text{rad}} = \left(\frac{90}{\pi}\right)^\circ = 28.6478..^\circ \approx 28^\circ 38' 52'' \approx 28^\circ 39'$ ✓

b) i) $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ ✓



ii) $\cos\frac{3\pi}{4} = -\cos\left(\pi - \frac{\pi}{4}\right)$

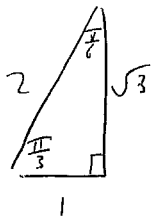
$= -\cos\frac{\pi}{4}$

$= -\frac{1}{\sqrt{2}}$ ✓ ✓

iii) $\sec\frac{\pi}{6}\cot\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{6}} \times \frac{1}{\tan\frac{\pi}{3}}$

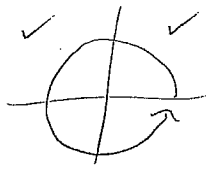
$= \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$ ✓ for one of these correct

$= \frac{2}{3}$ ✓



Q12] ctd.

(c) i) $\sin\theta = 0.9$ $\quad 0 \le \theta \le 2\pi$

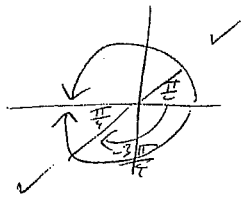


[basic $\theta = \sin^{-1} 0.9 \simeq 1.1197 \simeq 1.12$]

$\theta \simeq 1.12, \pi - 1.12$

$\theta \simeq 1.12, 2.02$ (to 2 d.p.) ✓ ✓

ii) $\tan x = 1$ $\quad -\pi \le \theta \le \pi$

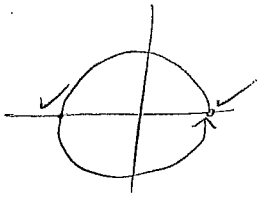


[basic $x = \tan^{-1}(1) = \frac{\pi}{4}$]

$x = \frac{\pi}{4}, -(\pi - \frac{\pi}{4})$

$x = \frac{\pi}{4}, -\frac{3\pi}{4}$ ✓ ✓

iii) $2\cos^2 x = 2$ $\quad 0 \le x \le 2\pi$



$\cos^2 x = 1$

$\cos x = \pm 1$

$x = 0, \pi, 2\pi$

✓ ✓ ✓

(d) show $1 - \cos^2\left(\frac{\pi}{2} - x\right) \equiv \cos^2 x$

L.H.S $= 1 - \cos^2\left(\frac{\pi}{2} - x\right)$

$= 1 - \sin^2 x$ ✓ ✓

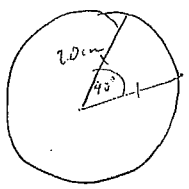
$= \cos^2 x$

✓ $\left[\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta\right]$

"reason required" $[\cos^2\theta + \sin^2\theta = 1]$

Question 13

a)



i) $\theta = 40° \times \frac{\pi}{180°}$

$= \frac{2\pi}{9}$

$\simeq 0.69813..$

$\simeq 0.698$ rad. ✓

ii) $P = r + r + l$ where $l = r\theta$

$= 20 + 20 + \frac{40\pi}{9}$ $\quad = 20 \times \frac{2\pi}{9}$

$= 40 + \frac{40\pi}{9} \simeq 53.96$ $\quad = \frac{40\pi}{9}$ ✓

$= 41.37626..$

$\simeq 41.376$ cm ✓

Note: −1 any rounding error for whole of Q.

iii) $A = \frac{1}{2} r^2 \theta$

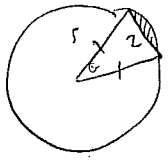
$= \frac{1}{2} \times 20^2 \times \frac{2\pi}{9}$ ✓

$= \frac{400\pi}{9}$

$\simeq 139.6763.$

$\simeq 140$ cm^2 ✓

b)



i) $\cos\theta = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos\theta = \frac{5^2 + 5^2 - 2^2}{2 \times 5 \times 5}$ ✓

$\cos\theta = \frac{46}{50}$ ✓

$\theta = \cos^{-1}\left(\frac{46}{50}\right) = 0.4027..$ rad

$\simeq 0.40$ rad. ✓

[or 23.07°] is ok.

ii) $A = \frac{1}{2} r^2 (\theta - \sin\theta)$ ✓

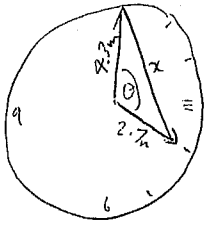
$= \frac{1}{2} \times 5^2 (0.4027.. - \sin 0.4027..)$

$= 0.1349..$

$\simeq 0.13$ m^2 ✓

Q13 (c.t.d)

c)



Let x = distance between tips of hands

i) $\theta = \frac{4}{12} \times 360^\circ = 120^\circ$

$= \frac{4}{12} \times 2\pi$ rad

$= \frac{2\pi}{3}$ rad. ✓

$x^2 = a^2 + b^2 - 2ab\cos\theta$

$= 4.3^2 + 2.7^2 - 2\times4.3\times2.7\times\cos\left(\frac{2\pi}{3}\right)$ ✓

$x^2 = 37.39$

$x = \sqrt{37.39}$

$= 6.1147$

$= 6.11$ m ✓

or 120° just make sure your calc. is set on the matching setting

ii) In 2 minutes minute hand travels $\frac{20}{60}\times360^\circ = 120^\circ = \frac{2\pi}{3}$ rad.

let l = distance travelled by tip of minute hand

$l = r\theta$ ✓

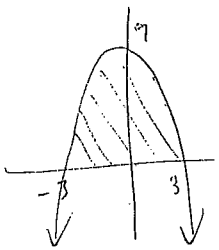
$= 4.3 \times \frac{2\pi}{3}$

$= 9.00589$ ✓

≈ 9.01 m

QUESTION (14)

a)



$y = 9 - x^2$

x-int: $y = 0$

$0 = 9 - x^2$

$x^2 = 9$

$x = \pm3$

Area $= \int_{-3}^{3}(9-x^2)\,dx$ ✓

$= 2\int_0^3 (9-x^2)\,dx$ [since $f(x)$ is even]

$= 2\left[9x - \frac{x^3}{3}\right]_0^3$ ✓

$= 2\left\{\left(9(3) - \frac{3^3}{3}\right) - 0\right\}$

$= 2(27-9)$

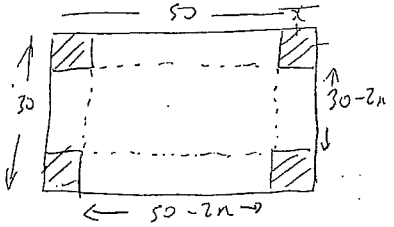
$= 2\times18$

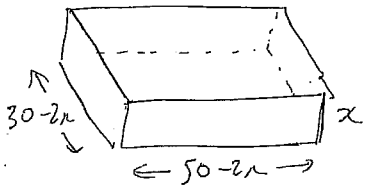
$= 36$ sq. units. ✓

/3

Q15 (c.t.d)

(b)





i)

$V = lbh$

$= (50-2x)(30-2x)x$ ✓

$= (1500 - 100x - 60x + 4x^2)x$

$= 1500x - 160x^2 + 4x^3$

$= 4x^3 - 160x^2 + 1500x$ ✓

ii) Max value when $\frac{dV}{dx} = 0$ & $\frac{d^2V}{dx^2} < 0$

$\frac{dV}{dx} = 12x^2 - 320x + 1500$

$\frac{d^2V}{dx^2} = 24x - 320$

when $\frac{dV}{dx} = 0$ $\quad 12x^2 - 320x + 1500 = 0$

$4(3x^2 - 80x + 375) = 0$

$x = \frac{80 \pm \sqrt{80^2 - 4\times375\times3}}{2\times3}$

$= \frac{80 \pm \sqrt{1900}}{6}$

$\approx 6.068,\ 20.59$

check $\frac{dV}{dx}$ $= 6.068\times24 - 320 = -179 < 0$ ∴ MAXIMUM

$= 20.59\times24 - 320 = +179 > 0$ ∴ MIN.

∴ Maximum value when $x = 6.068$

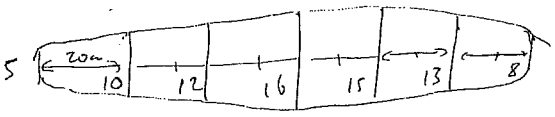
$V = 4(6.068..)^3 - 160(6.068..)^2 + 1500(6.068..)$

$= 4104.410..$

$V \approx 4104$ cm^3

Q15 (ctd)

(c)



i) Length of snake = 6 × 20cm = 120cm ✓

ii) $A \approx \frac{h}{2}[\text{first} + 2\times\text{middles} + \text{last}]$ ✓

$\approx \frac{20}{2}[5 + 2(10+12+16+15+13) + 8]$ ✓

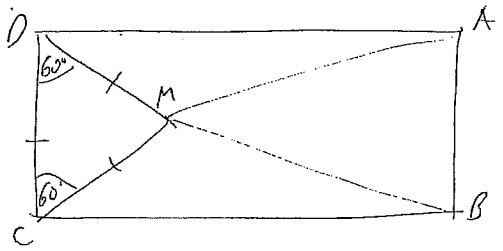
$\approx 10[5 + 2\times 66 + 8]$

$\approx 10 \times 145$

Area $\approx 1450\ cm^2$ ✓

QUESTION 16

(a)



i) In $\triangle MAD$ & $\triangle MBC$

$MD = MC$ (equal sides of equilateral triangle) ✓

$AD = BC$ (opposite sides of rectangle are equal) ✓

$\angle MDA = \angle MCA$ (both complementary to equal angles of equilateral and angles in rectangle are right angles) ✓

$\therefore \triangle MAD \equiv \triangle MBC$ (SAS) ✓

ii) $AM = BM$ (corresponding sides in congruent triangles) ✓

$\therefore \triangle MAB$ is isosceles. (two sides equal) ✓

½

Q16 ctd.

(b) i) $y = \sqrt{x} = x^{\frac{1}{2}}$

$y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

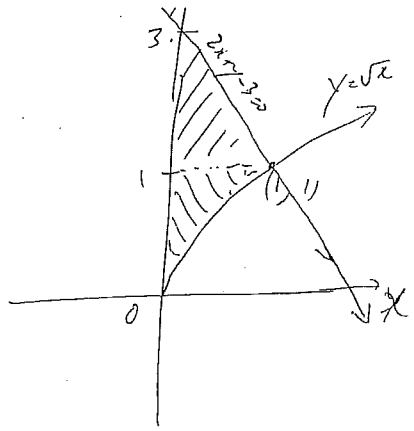
@ $x=1$, $m_T = y' = \frac{1}{2\sqrt{1}} = \frac{1}{2}$ ✓

$\therefore m_N = -2$

$\therefore$ eq. of normal is $y-1 = -2(x-1)$ ✓

$y - 1 = -2x + 2$

$\underline{2x + y - 3 = 0}$ ½



ii) y-int of $2x+y-3=0$

$x=0$, $y-3=0$

$y=3$

$2x + y - 3 = 0 \Rightarrow$ for $y = \sqrt{x}$

$2x = 3 - y$ $y^2 = x$

$x = \frac{3-y}{2}$ $x = y^2$

$\therefore$ Area $= \int_0^1 y^2\,dy + \int_1^3 \left(\frac{3-y}{2}\right)dy$ ✓✓

$= \left[\frac{y^3}{3}\right]_0^1 + \frac{1}{2}\left[3y - \frac{y^2}{2}\right]_1^3$

$= \left\{\left(\frac{1}{3}\right) - 0\right\} + \frac{1}{2}\left\{\left(3(3) - \frac{(3)^2}{2}\right) - \left(3(1) - \frac{1^2}{2}\right)\right\}$

$= \frac{1}{3} + \frac{1}{2}\left\{\frac{9}{2} - \frac{5}{2}\right\}$

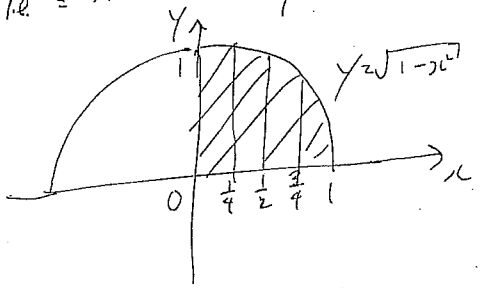
$= \frac{1}{3} + 1$ ✓

$= 1\frac{1}{3}$ sq. units ⅓

(X10)(C10)

(c) i) $\int_0^1 \sqrt{1-x^2}\,dx$ = Area between $y=\sqrt{1-x^2}$, x-axis and $x=0$, $x=1$

i.e. = Area inside quarter circle as shown ✓ either



1

ii)

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
y	1	$\sqrt{1-\left(\frac{1}{4}\right)^2}$ $=\sqrt{\frac{16}{16}-\frac{1}{16}}$ $=\frac{\sqrt{15}}{4}$	$\sqrt{1-\left(\frac{1}{2}\right)^2}$ $=\sqrt{1-\frac{1}{4}}$ $=\frac{\sqrt{3}}{2}$	$\sqrt{1-\left(\frac{3}{4}\right)^2}$ $=\sqrt{1-\frac{9}{16}}$ $=\frac{\sqrt{7}}{4}$	0

$A = \frac{h}{3}\left(y_{first} + 4y_{middle} + y_{last}\right) + \frac{h}{3}$ repeat ✓

$= \frac{1}{4\times 3}\left(1 + 4\times\frac{\sqrt{15}}{4} + \frac{\sqrt{3}}{2}\right) + \frac{1}{4\times 3}\left(\frac{\sqrt{3}}{2} + 4\times\frac{\sqrt{7}}{4} + 0\right)$

$= \frac{1}{12}\left(1 + \sqrt{15} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \sqrt{7}\right)$ ✓

$= \frac{1}{12}\left(1 + \sqrt{15} + \sqrt{3} + \sqrt{7}\right)$

≈ 0.77089

≈ 0.771 sq. unit $\therefore \int_0^1 \sqrt{1-x^2}\,dx \approx 0.771$ ✓ 3

iii) From (i) integral $= \frac{1}{4} \times$ area of circle

$= \frac{1}{4} \times \pi(1^2)$

$= \frac{\pi}{4}$

From (ii) integral $= \frac{1}{12}\left(1+\sqrt{3}+\sqrt{7}+\sqrt{15}\right)$

$\therefore \frac{\pi}{4} = \frac{1}{12}\left(1+\sqrt{3}+\sqrt{7}+\sqrt{15}\right)$ ✓

$\therefore \pi = \left(1+\sqrt{3}+\sqrt{7}+\sqrt{15}\right)/3$ 1