ST. URSULA'S COLLEGE, KINGSGROVE

MATHEMATICS

PRELIMINARY YEAR

FINAL EXAMINATION

2009

Time Allowed: 2 hours (plus 5 minutes reading time)

Instructions:

- *Commence each question on a separate sheet of paper*
- *Write your student number at the top of every page*
- *You must show all necessary working in order to gain maximum marks*
- *Marks may be deducted for careless or badly arranged work*
- *Attempt all questions*

Question 1 (*Start a new sheet of paper*) **Marks**

(a) Evaluate, correct to three significant figures

$$\sqrt{\frac{(5.2)^2 \times 3.6}{(2.3)^3}}$$ 2

(b) Simplify $\sqrt{8}+3\sqrt{50}$ 2

(c) Simplify $\frac{a-3}{2}-\frac{3a+1}{3}$ 2

(d) Factorise each of the following expressions.

(i) $3a+3b+a^2+ab$ 1

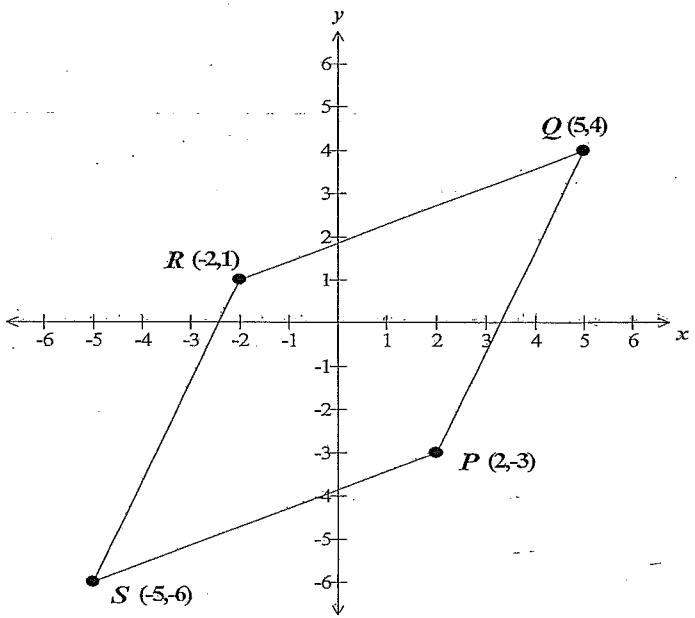
(ii) x^2y^2-4 1

(e) Solve $x^2-x-6=0$ 2

(f) Solve $|x+2|>5$. Graph your solution on a number line. 2

Question 2 (*Start a new sheet of paper*) **Marks**

The diagram shows the points $P(2,-3)$, $Q(5,4)$, $R(-2,1)$ and $S(-5,-6)$.
The four points form the parallelogram $PQRS$.



(a) Write down the coordinates of the midpoint of PR. **1**

(b) Find the exact length of PQ. **2**

(c) Show that the equation of the line passing through Q and R is $3x - 7y + 13 = 0$. **2**

(d) Show whether or not the point $(12,7)$ lies on the line $3x - 7y + 13 = 0$. **1**

(e) Calculate the perpendicular distance from P to QR. **2**

(f) Write down the equation of the line that is parallel to the y-axis and passes through point Q. **1**

(g) Write down the equation of the line that has a gradient of zero and passes through point R. **1**

(h) Prove that PR is perpendicular to QS. **2**

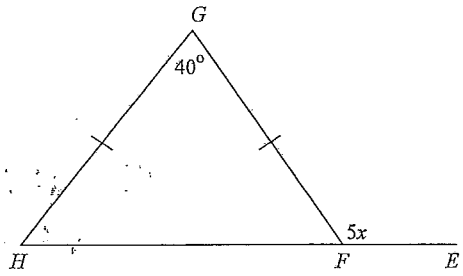
Question 3 (*Start a new sheet of paper*) **Marks**

(a) The cost of an Ipod is \$306. This price includes a 15% discount on the original price. Calculate the original price of the Ipod. **2**

(b) Given that $\sec(3x + 12) = \operatorname{cosec}(2x + 8)$, solve for x. **1**

(c) Evaluate $\lim_{x \to 5} \dfrac{x^2 - 9}{x + 3}$ **1**

(d) In the diagram below, triangle FGH is isosceles. Find the value of x, giving reasons. **2**



(e) Sketch each of the graphs below, showing all important features.

(i) $y = |x - 1|$ **2**

(ii) $y = 2^x - 1$ **2**

(iii) $y = x^2 - 4$, for $x \geq 0$ **2**

Question 4 (*Start a new sheet of paper*) — Marks

(a) Solve $|3a-1|=|a+5|$. — 2

(b) For which value of x is the function $y=\dfrac{x-5}{3x-2}$ discontinuous ? — 1

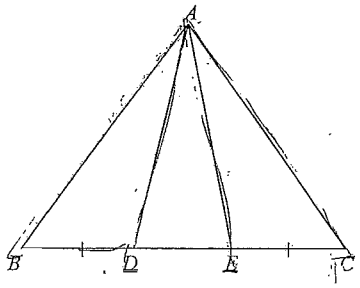
(c) Solve the pair of simultaneous equations

$$x+3y=10$$
$$4x-2y=12$$

— 2

(d) Solve $\sqrt{3}-2\sin\theta=0$, for $0^o\le\theta\le360^o$. — 2

(e) In the diagram, ABC is an equilateral triangle.
The intervals AD and AE meet BC at points D and E respectively, such that $BD = CE$.
Prove that triangles ABD and ACE are congruent. — 3



(f) Differentiate each of the following.

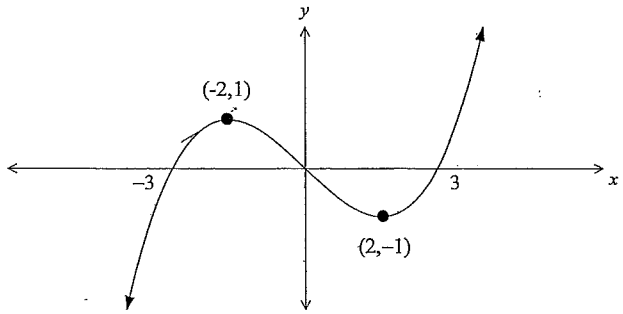
(i) $5x^3+6x-7$ — 1

(ii) $\dfrac{7x^2+6x}{x}$ — 1

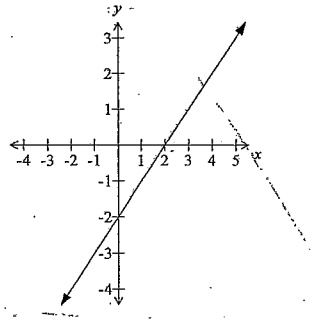
Question 5 (*Start a new sheet of paper*) — Marks

(a) Differentiate $f(x)=x^2-2x+5$ from first principles. — 3

(b) The graph of $y=f(x)$ is drawn below.
Use this graph to sketch the gradient function graph $y=f'(x)$. — 2



(c) The graph of the gradient function $y=f'(x)$ is drawn below.
Use this graph to sketch the function graph $y=f(x)$, given that $f(2)=0$. — 2



(d) Consider the function $f(x)=\sqrt{4-x^2}$.

(i) State the domain and range of $f(x)$. — 2

(ii) State whether the function is odd or even or neither.
You must justify your answer using algebra. — 2

(iii) What is the maximum function value of the graph $y=2f(x)$? — 1

Question 6 (*Start a new sheet of paper*)

Marks

(a) If $f(x) = 2x$, find $f'(3)$. 1

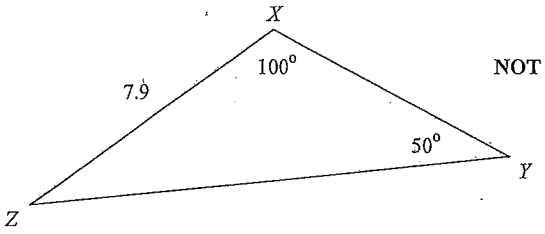
(b) Shade the region in the number plane containing all points that satisfy the set of inequalities below. 3

$$y \geq x^2 \text{ and } y < 4 \text{ and } x \leq 0$$

(c) In the diagram, XYZ is a triangle where $XZ = 7.9$ metres, angle $XYZ = 50^\circ$ angle $ZXY = 100^\circ$.

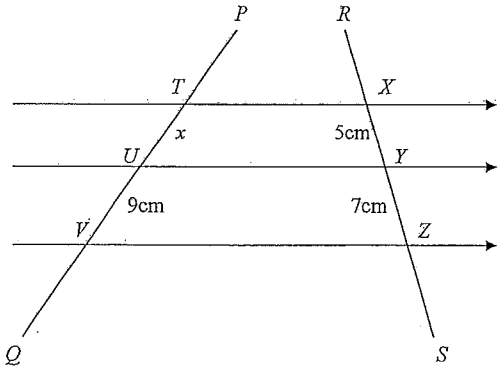
(i) Find the length of YZ, correct to 1 decimal place. 2

(ii) Calculate the area of triangle XYZ, to the nearest square metre. 2



NOT TO SCALE

(d) In the diagram TX, UY and VZ are a set of parallel lines, cut by transversals PQ and RS. Find the value of x. Give reasons to justify your answer. 2

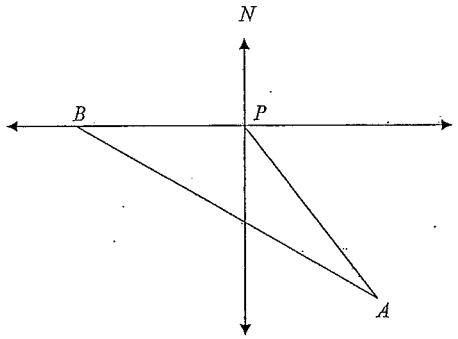


(e) Find values for m for which $m^2 - 2m - 15 \geq 0$. 2

Question 7 (*Start a new sheet of paper*)

Marks

(a) Two ships leave port P at the same time. Ship A sails on a bearing of 145° for a distance of 80km. Ship B sails due west for a distance of 65km.



(i) Copy the diagram onto your answer sheet and then show all of the given information. 1

(ii) Explain why $\angle BPA = 125^\circ$ 1

(iii) Find the distance between the two ships, to the nearest kilometer. 2

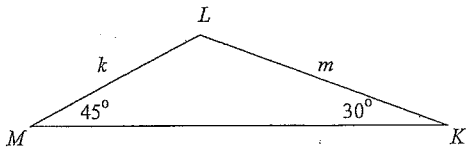
(iv) Find the bearing to the nearest degree of ship A from ship B. 2

(b) Prove the identity $\dfrac{2\sin^2\theta\cos\theta + 2\cos^3\theta}{\sin\theta} = 2\cot\theta$ 2

(c) Simplify $\dfrac{(a^3)^2}{a\sqrt{a}} \div \dfrac{a^4}{\sqrt[3]{a}}$, writing your answer in the form a^k, where k is a fraction. 2

(d) In the diagram, KLM is a triangle where $\angle MKL = 30^\circ$ and $\angle LMK = 45^\circ$.

Find the exact value for the ratio $\dfrac{m}{k}$. 2



(1) (a) $\sqrt{\frac{(5\cdot2)^2 \times 3\%}{(2\cdot3)^3}} = \sqrt{\frac{97\cdot344}{12\cdot167}}$ ← 1 mk

$= 2\cdot828543356$

$= 2.83$ (to 3 sig figs) ← 1 mk

(b) $\sqrt{8} + 3\sqrt{50}$

$= \sqrt{4}\sqrt{2} + 3\sqrt{25}\sqrt{2}$

$= 2\sqrt{2} + 3 \times 5\sqrt{2}$ ← 1 mk

$= 2\sqrt{2} + 15\sqrt{2}$

$= 17\sqrt{2}$ ← 1 mk

(c) $\frac{a-3}{2} - \frac{3a+1}{3}$

$= \frac{3(a-3) - 2(3a+1)}{6}$ ← 1 mk

$= \frac{3a-9-6a-2}{6}$

$= \frac{-3a-11}{6}$ ← 1 mk

(d) i) $3a + 3b + a^2 + ab$

$= 3(a+b) + a(a+b)$

$= (3+a)(a+b)$ ← 1 mk

ii) $x^2y^2 - 4$

$= (xy-2)(xy+2)$ ← 1 mk

(e) $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$ ← 1 mk

$x - 3 = 0$ or $x + 2 = 0$

$x = 3$ $x = -2$ ← 1 mk

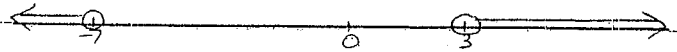
(f) $|x+2| = x+2$ if $x+2 > 0$

$= -(x+2)$ if $x+2 < 0$

$x+2 > 5$ or $-(x+2) > 5$ ← 1 mk

$x > 3$ $x+2 < -5$

$x < -7$



← 1 mk for number line

$\therefore$ $x < -7, x > 3$

Marking Guidelines

(2) (a) $\left(\frac{-2+2}{2}, \frac{1+-3}{2}\right) = (0, -1)$ — 1mk for correct answer

(b) $PQ = \sqrt{(2-5)^2 + (-3-4)^2}$ — 1mk for sub. into correct formula

$= \sqrt{58}$ — 1mk for exact answer

(c) $QR_m = \frac{4-1}{5--2} = \frac{3}{7}$ ← 1mk

Equation QR: $y - 4 = \frac{3}{7}(x-5)$ ← 1mk

$7y - 28 = 3x - 15$

$3x - 7y + 13 = 0$

(d) $3(12) - 7(7) + 13 = 0$ — 1mk for showing correctly

$36 - 49 + 13 = 0$

Point lies on the line

(e) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \frac{|3(2) + (-7)(-3) + 13|}{\sqrt{3^2 + (-7)^2}}$ ← 1mk

$= \frac{40}{\sqrt{58}}$ units ← 1mk

(f) $x = 5$ — 1mk answer

(g) $y = 1$ — 1mk answer

(h) $PR_m = \frac{-3-1}{2--2} = -1$ — 1mk for establishing m_1 and m_2

$QS_m = \frac{4--6}{5--5} = 1$ Since $PR_m \times QS_m = -1$

$\therefore PR \perp QS$ — 1mk for $m_1 \times m_2 = -1$

PRELIMINARY MATHEMATICS
Final Examination 2009

Marking Guidelines

(3) (a) 85% rep. \$306

1% rep \$3·60 ← 1mk

100% rep \$360 ← 1mk

(b) $\sec(3x+12) = \operatorname{cosec}(2x+8)$

$\therefore 3x + 12 + 2x + 8 = 90°$

$5x + 20 = 90°$

$x = 14°$ ← 1mk

(c) $\lim_{x \to 5} \frac{x^2 - 9}{x + 3} = \lim_{x \to 5} \frac{(x+3)(x-3)}{(x+3)}$

$= 2$ ← 1mk

(d) $\angle GFH = (180° - 40°) \div 2$

(base $\angle$'s of isos Δ are equal; Δ sum of $\Delta = 180°$)

$= 70°$

$5x + 70° = 180°$ (straight $\angle$)

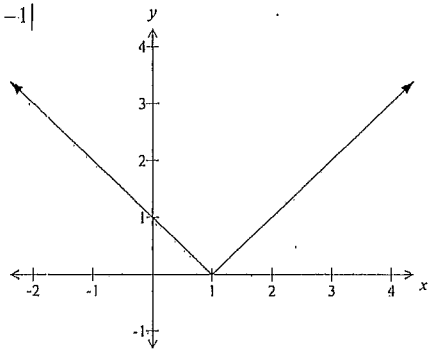
$\therefore x = 22°$

1mk for $x = 22°$

1mk for justificat

Marking Guidelines

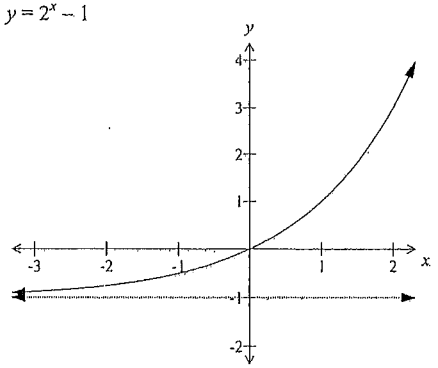
(3) (e) (i) $y = |x-1|$



1mk for 'V' shape

1mk for (1, 0)

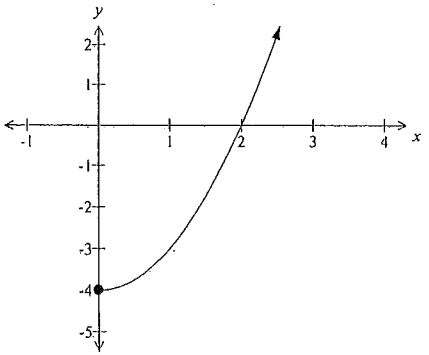
(ii) $y = 2^x - 1$



1mk for correct graph

1mk for asymptote $y = -1$

(iii) $y = x^2 - 4, x \geq 0$



1mk for correct section and (0, -4)

1mk for (2, 0)

PRELIMINARY MATHEMATICS
Final Examination 2009

Marking Guidelines

(4) (a) $|3a-1| = |a+5|$

$3a-1 = a+5$ OR $3a-1 = -a-5$

$2a = 6$ $\quad 4a = -4$

$a = 3$ $\quad a = -1$

1mk for $a = 3$

" " $a = -1$

(b) Discontinuous when $3x-2 = 0$

$\therefore x = \frac{2}{3}$ ← 1mk

(c) $x + 3y = 10 \ldots (1) \times 4$

$4x - 2y = 12 \ldots (2)$

$4x + 12y = 40 \ldots (3)$ −

$4x - 2y = 12 \ldots (4)$

$14y = 28$

$y = 2$ ← 1mk

$x = 4$ ← 1mk

(d) $\sqrt{3} - 2\sin\theta = 0 \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

Related angle $= \sin^{-1}\frac{\sqrt{3}}{2} = 60^\circ$

$\therefore \theta = 60^\circ$ or 120°

1mk for 60°

1mk for 120°

(e) In Δ's ABD and ACE,

(i) AB = AC (equal sides of equil. Δ) ← 1mk (unjustified)

(ii) $\angle ABD = \angle ACE = 60^\circ$ ($\angle$'s of equil. Δ) ← 2mks (justified)

(iii) BD = CE (given) ←

$\therefore \Delta ABD \equiv \Delta ACE$ (SAS) ← 1mk

(f) (i) $15x^2 + 6$

(ii) $\frac{7x^2 + 6x}{x} = 7x + 6 \quad \therefore y' = 7$

1mk each answer

Question 5 (12 marks)

MARKS

a). $f(x+h) = (x+h)^2 - 2(x+h) + 5$
$= x^2 + 2xh + h^2 - 2x - 2h + 5$ — 1mk

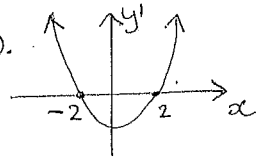
$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{(x^2 + 2xh + h^2 - 2x - 2h + 5) - (x^2 - 2x + 5)}{h}$

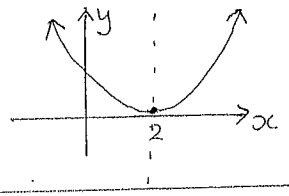
$= \lim_{h\to 0} \frac{2xh + h^2 - 2h}{h}$ — 1mk simplify

$= \lim_{h\to 0} \frac{h(2x + h - 2)}{h}$ — 1mk factorise, cancel; -1 if limit missing

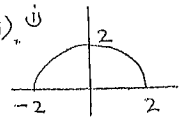
$= 2x - 2$

b). 

1mk parabola
1mk intercepts 2 and -2

c). 

1mk parabola with axis $x=2$
1mk vertex at $(2,0)$

d). i) 

$d: -2 \le x \le 2$ — 1mk
$r: 0 \le y \le 2$ — 1mk

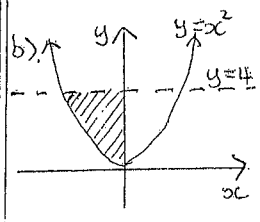
ii) $f(a) = \sqrt{4-a^2}$
$f(-a) = \sqrt{4-(-a)^2} = \sqrt{4-a^2}$ — 1mk
$f(a) = f(-a) \therefore$ even — 1mk

iii). max value $y = \sqrt{4-x^2}$ is 2
$2f(x) = 2 \times 2 = 4$. — 1mk

Question 6 (12 marks)

MARKS

a). $f(x) = 2x$
$f'(x) = 2$ indep of x for all values
$f'(3) = 2$ — 1mk

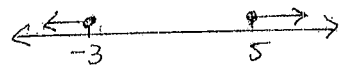
b). 

1 above parabola
1 under dotted $y=4$
1 left of y axis $x \le 0$

c). i) $\frac{YZ}{\sin 100^\circ} = \frac{7.9}{\sin 50^\circ}$ — 1mk
$YZ = \frac{7.9 \sin 100^\circ}{\sin 50^\circ}$
$\doteq 10.2$ — 1mk

ii). $A = \frac{1}{2} ab \sin C$
$= \frac{1}{2} \times 7.9 \times 10.2 \sin 30^\circ$ — 1mk
$\doteq 20\ m^2$ — 1mk

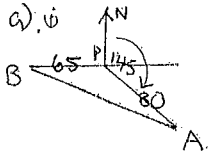
d). $\frac{x}{9} = \frac{5}{7}$
$x = \frac{45}{7} = 6\frac{3}{7}$ — 1mk
(ratio of intercepts made by transversals are equal on $\parallel$ lines) — 1mk

e). $(m-5)(m+3) \ge 0$ — 1mk 5, -3


$m \le -3$, $m \ge 5$ — 1mk

Question 7 (12 marks)

MARKS

a). i) 

1mk diagram

ii) method 1
$180 - 145 = 35^\circ$ (adj supp $\angle$'s)
$\angle BPA = 90 + 35^\circ$
$= 125^\circ$
1mk with working

method 2
$270 - 145 = 125^\circ$
or $360 - 235 = 125^\circ$
($\angle$'s at a pt)

iii). $BA^2 = 65^2 + 80^2 - 2 \times 65 \times 80 \cos 125$ — 1mk rule
$BA = 129$ km — 1mk soln.

iv). $\frac{\sin B}{80} = \frac{\sin 125}{129}$
$\sin B = \frac{80 \sin 125}{129}$
$B \doteq 31^\circ$ — 1mk B
bearing 121° — 1mk bearing

b).
LHS $\frac{2\cos\theta(\sin^2\theta + \cos^2\theta)}{\sin\theta}$ — 1mk factorise
$= \frac{2\cos\theta}{\sin\theta}(1)$ — 1mk simplify
$= 2\cot\theta$
$=$ RHS

c). $\frac{a^6}{a^{3/2}} \times \frac{a^{1/3}}{a^4}$ — 1mk indices
$= \frac{a^{6\frac{1}{3}}}{a^{5\frac{1}{2}}}$
$= a^{5/6}$
$k = \frac{5}{6}$ — 1mk soln.

d).
$\frac{m}{\sin 45^\circ} = \frac{k}{\sin 30^\circ}$ — 1mk
$\frac{m}{k} = \frac{\sin 45^\circ}{\sin 30^\circ}$
$= \frac{1/\sqrt{2}}{1/2}$
$= \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{2\sqrt{2}}{\sqrt{2}}$
$= \sqrt{2}$
1mk, $\frac{2}{\sqrt{2}}$ or $\sqrt{2}$