

1. Simplify these surds:

(a)  $\sqrt{28}$ ,

(b)  $\sqrt{72}$ ,

(c)  $\sqrt{50} - \sqrt{18}$ ,

(d)  $3\sqrt{2} + 5\sqrt{8}$ ,

(e)  $3\sqrt{15} \times 5\sqrt{6}$ ,

(f)  $18\sqrt{150} \div 3\sqrt{3}$ ,

(g)  $3\sqrt{x} + \sqrt{9x}$ ,

(h)  $5a\sqrt{a} - \sqrt{a^3}$ .

2. Expand and simplify:

(a)  $(5 + 2\sqrt{2})(3 - 4\sqrt{10})$ ,

(b)  $(2 + \sqrt{3})^2$ ,

(c)  $(3 - 2\sqrt{5})^2$ ,

(d)  $(4 - 7\sqrt{2})(4 + 7\sqrt{2})$

(e)  $(1 + \sqrt{m})^2$ ,

(f)  $(3\sqrt{p} - 2)(3\sqrt{p} + 2)$ .

3. Rationalize the denominator of:

(a)  $\frac{5}{\sqrt{5}}$ ,

(b)  $\frac{3\sqrt{2}}{5\sqrt{10}}$ ,

(c)  $\frac{1}{\sqrt{6} - \sqrt{5}}$ ,

(d)  $\frac{5 + \sqrt{2}}{3 - 2\sqrt{2}}$ .

4. Simplify these expressions, leaving your answer as a single fraction without any surds in the denominator:

(a)  $2 + 5\sqrt{3} + \frac{1}{2 + 5\sqrt{3}}$ ,

(b)  $\frac{\sqrt{x} + \sqrt{2+x}}{\sqrt{x} - \sqrt{2+x}} + \frac{\sqrt{x} - \sqrt{2+x}}{\sqrt{x} + \sqrt{2+x}}$ .

5. Given  $x = 2\sqrt{3} - 1$ , find  $\frac{x}{x + \sqrt{2}}$  expressing your answer as a fraction with a rational denominator.

QUESTION 1

(a)  $\sqrt{28} = 2\sqrt{7}$

(b)  $\sqrt{72} = 6\sqrt{2}$

(c)  $\sqrt{50} - \sqrt{18} = 5\sqrt{2} - 3\sqrt{2}$

(d)  $3\sqrt{2} + 5\sqrt{8} = 3\sqrt{2} + 10\sqrt{2}$

(e)  $3\sqrt{15} \times 5\sqrt{6} = 15\sqrt{90}$

$= 15 \times 3\sqrt{10}$

$= 45\sqrt{10}$

(f)  $18\sqrt{150} \div 3\sqrt{3} = 6\sqrt{50}$

$= 6 \times 5\sqrt{2}$

$= 30\sqrt{2}$

$= 42\sqrt{2}$

(h)  $5a\sqrt{a} - \sqrt{a^3} = 5a\sqrt{a} - a\sqrt{a}$

$= 4a\sqrt{a}$

(g)  $3\sqrt{x} + \sqrt{9x} = 3\sqrt{x} + 3\sqrt{x}$

$= 6\sqrt{x}$

QUESTION 2

(a)  $(5+2\sqrt{2})(3-4\sqrt{10}) = 15 - 20\sqrt{10} + 6\sqrt{2} - 8\sqrt{20}$

$= 15 - 20\sqrt{10} + 6\sqrt{2} - 16\sqrt{5}$

(b)  $(2+\sqrt{3})^2 = 4 + 4\sqrt{3} + 3$

$= 7 + 4\sqrt{3}$

(c)  $(3-2\sqrt{5})^2 = 9 - 12\sqrt{5} + 20$

$= 29 - 12\sqrt{5}$

(d)  $(4-7\sqrt{2})(4+7\sqrt{2}) = 16 - 98$

$= -82$

(e)  $(1+\sqrt{m})^2 = 1 + 2\sqrt{m} + m$

(f)  $(3\sqrt{p}-2)(3\sqrt{p}+2) = 9p-4$

QUESTION 3

(a)  $\frac{5}{\sqrt{5}} = \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{5\sqrt{5}}{5}$

$= \sqrt{5}$

(b)  $\frac{3\sqrt{2}}{5\sqrt{10}} = \frac{3\sqrt{2}}{5\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$

$= \frac{3\sqrt{20}}{50}$

$= \frac{6\sqrt{5}}{50}$

$= \frac{3\sqrt{5}}{25}$

(c)  $\frac{1}{\sqrt{6}-\sqrt{5}} = \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

$= \frac{\sqrt{6}+\sqrt{5}}{6+5}$

$= \frac{\sqrt{6}+\sqrt{5}}{11}$

(d)  $\frac{5+\sqrt{2}}{3-2\sqrt{2}} = \frac{5+\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$

$= \frac{15+10\sqrt{2}+3\sqrt{2}+4}{9-8}$

$= 19 + 13\sqrt{2}$

QUESTION 4

(a)  $2+5\sqrt{3} + \frac{1}{2+5\sqrt{3}} = 2+5\sqrt{3} + \frac{1}{2+5\sqrt{3}} \times \frac{2-5\sqrt{3}}{2-5\sqrt{3}}$

$= 2+5\sqrt{3} + \frac{2-5\sqrt{3}}{4-75}$

$= \frac{2+5\sqrt{3}}{1} + \frac{2-5\sqrt{3}}{-71}$

$= \frac{71(2+5\sqrt{3}) - (2-5\sqrt{3})}{71}$

$= \frac{142+355\sqrt{3}-2+5\sqrt{3}}{71}$

$= \frac{140+360\sqrt{3}}{71}$

(b)  $\frac{\sqrt{x}+\sqrt{2+x}}{\sqrt{x}-\sqrt{2+x}} + \frac{\sqrt{x}-\sqrt{2+x}}{\sqrt{x}+\sqrt{2+x}}$

$= \frac{(\sqrt{x}+\sqrt{2+x})^2 + (\sqrt{x}-\sqrt{2+x})^2}{x-(2+x)}$

$= \frac{x+2\sqrt{x(2+x)}+2+x+x-2\sqrt{x(2+x)}+2+x}{-2}$

$= \frac{4x+4}{-2}$

$= -2x-2$

QUESTION 5

$\frac{x}{x+\sqrt{2}} = \frac{2\sqrt{3}-1}{2\sqrt{3}-1+\sqrt{2}}$

$= \frac{2\sqrt{3}-1}{(2\sqrt{3}-1)+\sqrt{2}} \times \frac{(2\sqrt{3}-1)-\sqrt{2}}{(2\sqrt{3}-1)-\sqrt{2}}$

$= \frac{(2\sqrt{3}-1)^2 - \sqrt{2}(2\sqrt{3}-1)}{(2\sqrt{3}-1)^2 - 2}$

$= \frac{13-4\sqrt{3}+2\sqrt{6}+\sqrt{2}}{13-4\sqrt{3}-2}$

$= \frac{13-4\sqrt{3}-2\sqrt{6}+\sqrt{2}}{11-4\sqrt{3}} \times \frac{11+4\sqrt{3}}{11+4\sqrt{3}}$

$= \frac{143-44\sqrt{3}-22\sqrt{6}+11\sqrt{2}+52\sqrt{3}-48-8\sqrt{18}+4\sqrt{6}}{121-48}$

$= \frac{95+8\sqrt{3}-18\sqrt{6}+11\sqrt{2}-24\sqrt{2}}{73}$

$= \frac{95+8\sqrt{3}-18\sqrt{6}-13\sqrt{2}}{73}$