Name:_____

SYDNEY GIRLS HIGH SCHOOL

2 UNIT MATHEMATICS – EXTENSION 1

Topic Test – Basic Arithmetic & Algebra – 2007

Time Allowed: 75 minutes

Instructions:

- ⇨ There are FOUR questions, of equal value.
- ⇨ All questions are to be attempted
- ⇨ All answers and working out are to be handed in on the lined paper provided
- ⇨ Full marks may not be awarded for careless, badly set out or incomplete working.

QUESTION 1: *(20 marks)*

a Find the value of $\frac{5.16}{\sqrt{7.12-2.78}}$ to 3 significant figures **2**

b. Find the value of $16^{-1.6}$ correct to 2 significant figures **2**

c Alison buys a Maths textbook which has been marked down by 20% If she pays \$52 for the book, what was the original price? **2**

d The price of petrol has risen by 12%. If it now costs \$55 to fill the tank, how much did it cost before the price rise, to the nearest cent? **2**

e. Convert $0.\dot{7}\dot{8}$ to a fraction in its simplest form **2**

A particular shadecloth cuts out 20% of the light and lets through the remaining 80%.

i Show that two layers of the shadecloth let through 64% of the light **1**

ii. How many layers of the shadecloth are required to cut out at least 95% of the light? **2**

g. Express $\sqrt{50}-\sqrt{18}$ in the form $k\sqrt{2}$ **2**

If $x=\frac{3-2\sqrt{2}}{3+2\sqrt{2}}$, evaluate in exact form:

i. $x+\frac{1}{x}$ **2**

ii. $x^2+\frac{1}{x^2}$ **3**

QUESTION 2: *(20 marks)*

a Expand and simplify $3\sqrt{2}\left(2\sqrt{7}-\sqrt{2}\right)$ **2**

b Find the values of a and b if $\frac{\sqrt{2}}{3\sqrt{2}-1}=a+b\sqrt{2}$ **2**

c. Factorise $54x^3+16$ **2**

d Factorise $4m-4n-m^2+n^2$ **3**

e. Simplify $\frac{(x+y)^{-1}+(x-y)^{-1}}{(x+y)^{-1}-(x-y)^{-1}}$ **3**

f. Find the exact value of $\frac{A^4}{B^3}$ if $A=\frac{4}{27}$ and $B=\frac{8}{81}$ **3**

g Evaluate $|-5|-|-7|$ **1**

h. Factorise $15m^2n - 3mn + 21mn^2$ 1

i. Factorise $x^2 - 5x - 6$ 1

j. Factorise $24x - 16 - 9x^2$ 2

QUESTION 3: *(20 marks)*

a. Factorise $4a^2 - 5ab - 6b^2$ 2

b. Simplify $\dfrac{1-x^2}{x^2-2x+1}$ 3

c. Write with positive indices $\left(a^2b^{-2}\right)^{-2}$ 3

d. Simplify $\dfrac{2}{x^2-16} - \dfrac{7}{x^2-x-12}$ 4

e. Simplify $\dfrac{2x^2-8}{y^3-27} \times \dfrac{y^2-9}{x^2+5x+6}$ 4

f. Using Pascal's triangle or otherwise, write the expansion for $(2x-y)^4$ 4

QUESTION 4 ON NEXT PAGE

QUESTION 4: *(20 marks)*

a. Solve $\dfrac{x}{2} - 2 = \dfrac{x}{4} + \dfrac{1}{6}$ 2

b. Solve $5x^2 + 3x - 2 = 0$ 2

c. Solve $x^2 - 4x - 2 = 0$. Leave your answer in simplified surd form 2

d. Solve $|2x+5| = 3x+10$ 3

e. Solve $\begin{aligned} x^2 - y^2 &= 16 \\ x - y &= 3 \end{aligned}$ 3

f. Solve $\dfrac{4}{x-1} \geq 1$ 4

g. Solve $|x+1| < 2|x-2|$ 4

LEVEL: Prelim Ext 1 YEAR: _____

EXAM TYPE: Basic Arithmetic and Algebra

Question 1:

a) $\frac{5.16}{\sqrt{7.12-2.78}} \doteq 2.48$ (2)

b) $16^{-1.6} \doteq 0.012$ (2)

c) let original price = x

$80\% \times x = 52$

$0.8x = 52$

$x = 65$

∴ original price was $65 (2)

d) let original price = x

$112\% \times x = 55$

$1.12x = 55$

$x = \$49.11$ (nearest cent)

∴ original price was $49.11 (2)

e) let $x = 0.7888\ldots$ ①

$10x = 7.888\ldots$ ②

$100x = 78.88\ldots$ ③

③ − ② $90x = 71$

$x = \frac{71}{90}$ (2)

f) i) let amt of light = x

Layer 1 allows 80% of x

Layer 2 allows 80% of Layer 1

∴ Layer 2 allows $80\% \times 80\%$ of x

i.e. Layer 2 = $0.8 \times 0.8 \times x$

$= 0.64x$

$= 64\% x$

∴ Layer 2 lets through 64% of the light (1)

ii) No more than 5% can pass.

Let no. shadecloths = n

$0.8^n \leq 0.05$

$n \leq 14$ (by trial and error)

∴ 14 shadecloths are needed (2)

g) $\sqrt{50} - \sqrt{18} = 5\sqrt{2} - 3\sqrt{2}$

$= 2\sqrt{2}$ (2)

h) $x + \frac{1}{x} = \frac{3-2\sqrt{2}}{3+2\sqrt{2}} + \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

$= \frac{(3-2\sqrt{2})^2 + (3+2\sqrt{2})^2}{(3+2\sqrt{2})(3-2\sqrt{2})}$

$= \frac{9-12\sqrt{2}+8+9+12\sqrt{2}+8}{9-8}$

$= \frac{34}{1}$

$= 34$ (2)

ii) $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$= 34^2 - 2$

$= 1156 - 2$

$= 1154$ (3)

Question 2:

a) $3\sqrt{2}(2\sqrt{7} - \sqrt{2}) = 6\sqrt{14} - 3\sqrt{4}$

$= 6\sqrt{14} - 6$ (2)

b) $\frac{\sqrt{2}}{3\sqrt{2}-1} = \frac{\sqrt{2}}{3\sqrt{2}-1} \times \frac{3\sqrt{2}+1}{3\sqrt{2}+1}$

$= \frac{\sqrt{2}(3\sqrt{2}+1)}{9 \times 2 - 1}$

$= \frac{6+\sqrt{2}}{17}$

$= \frac{6}{17} + \frac{1}{17}\sqrt{2}$

$a = \frac{6}{17}$ $b = \frac{1}{17}$ (2)

c) $54x^3 + 16 = 2(27x^3 + 8)$

$= 2(3x+2)(9x^2 - 6x + 4)$ (2)

d) $4m - 4n - m^2 + n^2$

$= 4(m-n) - (m+n)(m-n)$

$= (m-n)[4 - (m+n)]$ (3)

$= (m-n)(4-m-n)$

e) $\frac{1}{x+y} + \frac{1}{x-y}$

$= \frac{x-y+x+y}{(x+y)(x-y)}$

$= \frac{2x}{(x+y)(x-y)}$

$\frac{1}{x+y} - \frac{1}{x-y}$

$= \frac{(x-y)-(x+y)}{(x+y)(x-y)}$

$= \frac{-2y}{(x+y)(x-y)}$

$\frac{(x+y)^{-1} + (x-y)^{-1}}{(x+y)^{-1} - (x-y)^{-1}}$

$= \frac{2x}{x^2-y^2} \div \left(\frac{-2y}{x^2-y^2}\right)$

$= \frac{2x}{x^2-y^2} \times \frac{x^2-y^2}{-2y}$

$= -\frac{x}{y}$ (3)

f) $\frac{A^4}{B^3} = A^4 B^{-3}$

$= \left(\frac{4}{27}\right)^4 \left(\frac{8}{81}\right)^{-3}$

$= \left(\frac{2^2}{3^3}\right)^4 \left(\frac{2^3}{3^4}\right)^{-3}$

$= \frac{2^8}{3^{12}} \times \left(\frac{3^4}{2^3}\right)^3$

$= \frac{2^8}{3^{12}} \times \frac{3^{12}}{2^{-9}}$

$= \frac{1}{2}$ (3)

g) $|-5| - |-1-7| = 5 - 7$

$= -2$ (1)

h) $15m^2n - 3mn + 21mn^2$

$= 3mn(5m - 1 + 7n)$ (1)

i) $x^2 - 5x - 6 = (x-6)(x+1)$ (1)

j) $-16 + 24x - 9x^2$

$= -16 + 12x + 12x - 9x^2$

$= -4(4-3x) + 3x(4-3x)$

$= (4-3x)(3x-4)$

$= -(4-3x)(4-3x)$

$= -(4-3x)^2$ (2)

b) $5x^2+3x-2=0$

$5x^2+5x-2x-2=0$

$5x(x+1)-2(x+1)=0$

$(5x-2)(x+1)=0$

$5x-2=0 \qquad x+1=0$

$5x=2 \qquad x=-1$

$x=\frac{2}{5}$ (2)

c) $x^2-4x-2=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{4\pm\sqrt{(-4)^2-4(1)(-2)}}{2}$

$=\frac{4\pm\sqrt{16+8}}{2}$

$=\frac{4\pm\sqrt{24}}{2}$

$=\frac{4\pm2\sqrt{6}}{2}$

$=2\pm\sqrt{6}$ (2)

d) $|2x+5|=3x+10$

$2x+5=3x+10$ OR $-(2x+5)=3x+10$

$x=5 \qquad -2x-5=3x+10$

$5x=-15$

$x=-3$ (2)

Test $x=-5$

LHS $=|2x-5+5|$

$=|-5|$

$=5$

RHS $=3x(-5)+10$

$=-5$

LHS $\neq$ RHS

$\therefore x=-5$ is not a solution

Test $x=-3$

LHS $=|2x-3+5|$

$=|-1|$

$=1$

RHS $=3x(-3)+10$

$=1$

LHS = RHS

$\therefore x=-3$ is a solution (1)

e) $x^2-y^2=16$ (1)

$x-y=3$ (2)

From (2) $y=x-3$ (3)

Sub (3) into (1)

$x^2-(x-3)^2=16$

$x^2-(x^2-6x+9)=16$

$6x-9=16$

$6x=25$

$x=\frac{25}{6}$

Sub $x=\frac{25}{6}$ into (3)

$y=\frac{25}{6}-3$

$=1\frac{1}{6}$

$\therefore x=4\frac{1}{6}, y=1\frac{1}{6}$ (3)

Question 3:

a) $4a^2-5ab-6b^2$

$=4a^2-8ab+3ab-6b^2$

$=4a(a-2b)+3b(a-2b)$

$=(4a+3b)(a-2b)$ (2)

b) $\frac{1-x^2}{x^2-2x+1}=\frac{(1-x)(1+x)}{(x-1)^2}$

$=\frac{-(x-1)(1+x)}{(x-1)^2}$

$=\frac{-(1+x)}{(x-1)}$ (3)

c) $(a^2b^{-2})^{-2}=\left(\frac{a^2}{b^2}\right)^{-2}$

$=\left(\frac{b^2}{a^2}\right)^2$

$=\frac{b^4}{a^4}$ (3)

d) $\frac{2}{x^2-16}-\frac{7}{x^2-x-12}$

$=\frac{2}{(x+4)(x-4)}-\frac{7}{(x-4)(x+3)}$

$=\frac{2(x+3)-7(x+4)}{(x+4)(x-4)(x+3)}$

$=\frac{2x+6-7x-28}{(x+4)(x-4)(x+3)}$

$=\frac{-5x-22}{(x+4)(x-4)(x+3)}$ (4)

e) $\frac{2(x^2-4)}{(y-3)(y^2+3y+9)}\times\frac{(y-3)(y+3)}{(x+3)(x+2)}$

$=\frac{2(x-2)(x+2)}{(y-3)(y^2+3y+9)}\times\frac{(y-3)(y+3)}{(x+3)(x+2)}$

$=\frac{2(x-2)(y+3)}{(x+3)(y^2+3y+9)}$ (4)

f)

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$(2x-y)^4$

$=(2x)^4-4(2x)^3y+6(2x)^2y^2-4(2x)y^3+y^4$ (4)

$=16x^4-32x^3y+24x^2y^2-8xy^3+y^4$

Question 4:

a) $\frac{x}{2}-2=\frac{x}{4}+\frac{1}{6}$

$\frac{x-4}{2}=\frac{3x+2}{12}$

$2(3x+2)=12(x-4)$

$6x+4=12x-48$

$52=6x$

$x=8\frac{2}{3}$ (2)

f) $\frac{4}{x-1} \geq 1$

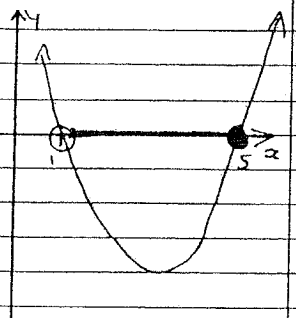
$4(x-1) \geq (x-1)^2$

$0 \geq (x-1)^2 - 4(x-1)$

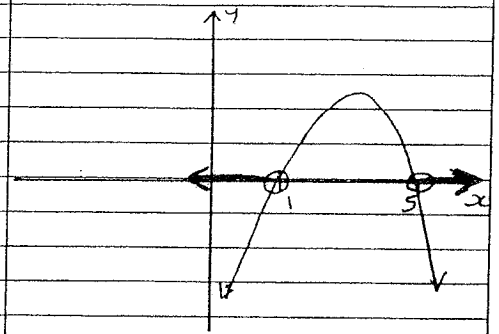
$(x-1)^2 - 4(x-1) \leq 0$

$(x-1)[(x-1)-4] \leq 0$

$(x-1)(x-5) \leq 0$



$1 \leq x \leq 5$ (4)



$x < 1$ or $x > 5$ (4)

g) $|x+1| \leq 2|x-2|$

$(x+1)^2 \leq 4(x-2)^2$

$(x+1)^2 - 4(x-2)^2 \leq 0$

$[(x+1) - 2(x-2)][(x+1) + 2(x-2)] \leq 0$

$(5-x)(3x-3) \leq 0$

$3(5-x)(x-1) \leq 0$

$(5-x)(x-1) \leq 0$