SYDNEY GRAMMAR SCHOOL



2012 Annual Examination

# FORM V MATHEMATICS 2 UNIT

Wednesday 29th August 2012

#### General Instructions

- Writing time 2 hours
- · Write using black or blue pen.
- Board-approved calculators and templates may be used.

#### Total — 100 Marks

All questions may be attempted.

#### Section I - 10 Marks

- Ouestions 1-10 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

#### Section II - 90 Marks

- Questions 11–16 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

#### Collection

- Write your name, class and master on each booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single wellordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place your multiple choice answer sheet inside the answer booklet for Question Eleven.
- Write your name and master on this question paper and submit it with your answers.

5P: SG

5Q: BDD

5R: RCF

#### Checklist

- SGS booklets 6 per boy
- · Multiple choice answer sheet

• Candidature — 49 boys

Examiner

**RCF** 

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#### SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

#### QUESTION ONE

Which of the following is the derivative of  $2x^3 + 2x - 1$ ?

- (A)  $6x^2$
- (B)  $6x^2 + 2$
- (C)  $6x^3 + 2x$
- (D)  $6x^2 1$

#### QUESTION TWO

What is the gradient of the interval joining point A(-2,4) and B(-5,0)?

- (A)  $-\frac{4}{3}$
- (B)  $\frac{4}{7}$
- (C)  $\frac{4}{3}$
- (D)  $\frac{3}{4}$

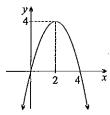
#### QUESTION THREE

Which of the following lines does NOT pass through (1, -3)?

- (A) y+3=2(x-1)
- (B) x = -3
- (C) x + 2y = -5
- (D) y = -3x

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#### QUESTION FOUR



What is the correct equation for the parabola sketched above?

(A) 
$$y=4-(x-2)^2$$

(B) 
$$y = x^2 + 4$$

(C) 
$$y = -(x+2)^2 - 4$$

(D) 
$$y = x^2 - 2x - 4$$

#### QUESTION FIVE

Which of the following expressions is equivalent to  $\log \left(\frac{3x^2}{4}\right)$ ?

(A) 
$$\log 3 + \log x^2 \div \log 4$$

(B) 
$$2(\log 3x - \log 4)$$

(C) 
$$2\log x + \log 3 - \log 4$$

(D) 
$$3\log x^2 - \log 4$$

#### QUESTION SIX

If the discriminant of a quadratic equation is a square number, which of the following types of roots will the equation have?

- (A) Real, rational and distinct roots
- (B) Equal real roots
- (C) No real roots
- (D) Real, irrational and distinct roots

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#### QUESTION SEVEN

Given the sequence 5, 7, 9, 11, ..., which of the following statements is INCORRECT?

(A) 
$$S_5 = 45$$

(B) The common difference is 2.

(C) 
$$S_n = \frac{n}{2}(8+2n)$$

(D) The *n*th term is  $5 \times 2^{n-1}$ 

#### QUESTION EIGHT

Given a geometric progression with first term 5 and common ratio 2, the sum of the first ten terms is:

(A) 
$$\frac{2(5^{10}-1)}{4}$$

(C) 
$$5(1-2^{10})$$

(D) 
$$5(2^{10}-1)$$

#### QUESTION NINE

The solutions of the equation  $2x^2 - 6x + 3 = 0$  are:

(A) 
$$x = \frac{3+\sqrt{3}}{2}$$
 and  $x = \frac{3-\sqrt{3}}{2}$ 

(B) 
$$x = 1$$
 and  $x = \frac{3}{2}$ 

(C) 
$$x = \frac{-6 + \sqrt{12}}{4}$$
 and  $x = \frac{-6 - \sqrt{12}}{4}$ 

(D) There are no real solutions

(C)  $2\cos^2\theta - 1$ (D)  $2\cos^2\theta + 3$ 

End of Section I

GGS Annual 2012				
			QUESTION ELEVEN (15 marks) Use a separate writing booklet.	Mark
			(a) Simplify $2x^3 \times (-3x^5)$ .	1
b) Expand $(2x-1)^2$ .	1			
c) Evaluate, leaving your answers in exact form:				
(i) $2\sqrt{72} - 3\sqrt{8}$	1			
(ii) $\sin 60^{\circ} + \sin 90^{\circ}$	1			
(iii) $\frac{10(3^4-1)}{3-1}$	1			
(iv) log <sub>2</sub> 32	1			
d) Differentiate the following:				
(i) $3x^5$	1			
(ii) $\sqrt{x}$	1			
(iii) $\frac{1}{3x^2}$	2			
e) Solve:				
(i) $(2x-3)(3x+5)=0$	1			
(ii) $\tan \theta = \sqrt{3}$ , for $0^{\circ} \le \theta \le 360^{\circ}$	1			
(iii) $\log_3 x = 4$				
(iv) $2^{z+3} = \sqrt{32}$	2			

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QUESTION TWELVE (15 marks) Use a separate writing booklet.

Marks

- (a) Consider the arithmetic sequence 15, 18, 21, ....
  - (i) State the values of a and d.

(iii) Find the sum of the first one hundred terms.

(ii) Find the one hundredth term.

2

- (b) Consider the parabola with equation  $y = x^2 6x + 5$ .
  - (i) State the y-intercept.

(ii) Find the x-intercepts.

(iii) State the equation of the axis of symmetry.

1

(iv) Hence find the co-ordinates of the vertex. (v) State the minimum value of  $x^2 - 6x + 5$ .

- 1 1
- 2 (vi) Sketch the graph of  $y = x^2 - 6x + 5$ , clearly marking all of the above features.
- (vii) Hence solve the inequation  $x^2 6x + 5 \ge 0$ .
- (c) Find the limiting sum of the geometric series  $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} \dots$ 2

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QUESTION THIRTEEN (15 marks) Use a separate writing booklet.

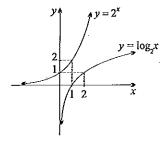
Marks

(a) Differentiate the following, without using the product or quotient rules:

(i) 
$$f(x) = x^2(2x^4 - 5x)$$

(ii) 
$$f(x) = \frac{x+1}{\sqrt{x}}$$

(b)



The diagram above shows the graphs of  $y = 2^x$  and  $y = \log_2 x$ .

(i) Sketch the graph of  $y = 2^{-x}$ , showing clearly any intercepts.

- 1
- (ii) On a separate set of axes, sketch the graph of  $y = \log_2(x-1)$ , showing clearly any intercepts and asymptotes.
- [1] (iii) State the domain for  $y = \log_2(x-1)$ .
- (c) Find the equation of the tangent to the parabola  $y = x^2 6x + 5$  at the point (4, -3). Give your answer in general form.
- (d) A cricket ball is hit skywards so that at any time t seconds after leaving the bat, the height of the ball is given by  $h = 35t - 5t^2$  metres.
  - (i) Find the time at which the ball reaches its maximum height.

- (ii) Hence determine the maximum height attained.
- (e) Find the natural domain of  $f(x) = \sqrt{9-x}$ .

SGS Annual 2012 ...... Form V Mathematics 2 Unit ..... Page 9 QUESTION FOURTEEN (15 marks) Use a separate writing booklet. Marks (a) Use the chain rule to differentiate: (i)  $y = (3x^2 + 5)^4$ 2 (ii)  $y = \sqrt{3-2x}$ 2 (b) If  $\alpha$  and  $\beta$  are the roots of the equation  $2x^2 - 4x + 3 = 0$ , find: (i)  $\alpha + \beta$ (ii) αβ (iii)  $\frac{2}{\alpha} + \frac{2}{\beta}$ (iv)  $\alpha^2 + \beta^2$ (c) Consider the quadratic function  $f(x) = x^2 + 4x - 3$ . (i) Complete the square. (ii) Hence, or otherwise, find the minimum value of the function. (d) Prove the identity  $\frac{\sin \beta}{\cos \beta} + \frac{\cos \beta}{\sin \beta} = \sec \beta \csc \beta$ . 2

QUESTION FIFTEEN (15 marks) Use a separate writing booklet.	Marks
(a) Differentiate $f(x) = x^2 - 5x$ from first principles.	3
(b) An arithmetic sequence has first term 8 and sixth term 28.	
(i) Find the common difference.	1
(ii) Find the first term larger than one hundred.	2
(iii) Find the value of $n$ for which $S_n = 680$ .	3
(c) Use the quotient rule to differentiate $f(x) = \frac{5x+3}{3x-4}$ .	3
(d) Use the product rule to differentiate $f(x) = 2x^4(3x+1)^3$ , expressing the derivative in fully factorised form.	3

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SGS Annual 2012 Form V Mathematics 2 Unit Page 11	
QUESTION SIXTEEN (15 marks) Use a separate writing booklet.	εk
(a) Solve $3 \times 2^{x} = 5775$ . Give your answer correct to two decimal places.	2
(b) A yacht sails $30 \mathrm{km}$ from harbour $A$ to island $B$ on a bearing of $053^\circ$ , then it sails $110 \mathrm{km}$ from island $B$ to marina $C$ on a bearing of $285^\circ$ .	
(i) Draw a clearly labelled diagram to represent this voyage.	1
(ii) Find the distance of harbour $A$ from marina $C$ , correct to the nearest kilometre.	2
(iii) Find $\angle ACB$ , and hence find the bearing of harbour A from marina C, correct to the nearest degree.	2
(c) Consider the quadratic function $f(x) = kx^2 - 8x + 4k$ .	
(i) Find the values of k for which the function has a repeated zero.	2
(ii) Find the values of k for which the function has no real zeroes.	2
$\begin{array}{c} E \\ \hline \\ A \\ \hline \end{array}$	
In the diagram above, the spiral begins with right-angled triangle $ABC$ in which $AB = 1$ m and $\angle CAB = \alpha$ , where $\alpha < 30^{\circ}$ . A similar right-angled triangle $ACD$ is drawn on the hypotenuse $AC$ . This pattern is then continued to form triangles $ADE$ , $AEF$ and so on.	
(i) Show that $CD = \sec \alpha \tan \alpha$ .	2
(ii) Find an expression for side DE.	2 1
(iii) The spiral pattern is continued until the tenth triangle $AKL$ is formed. Find an expression for side $KL$ of this triangle.	1
End of Section II	

END OF EXAMINATION

Tear-off pages follow ...

## MFH FORM 2U AMMAL 2012

Section 1 Multi-Choice One Mark Each

9 
$$2x^2-6x+3=0$$
  
 $a=2$   $b=(-6)$   $c=3$ 

$$x = \frac{6 + \sqrt{6^2 - 4 \times 2 \times 3}}{2 \times 2}$$

A) Vertex 
$$(24)$$
  $y-4=-(x-2)^a$   $= 3\pm \sqrt{3}$  (A) Concave Down  $a=(-1)$ 

(B) GP 
$$a=5$$
  $r=2$ 

$$S_{10} = \frac{a(r^{10})}{r-1}$$

$$= \frac{5(2^{10}-1)}{2^{-1}}$$

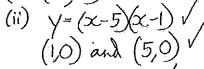
$$= \frac{5(2^{10}-1)}{2^{-1}}$$
(D)

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Question 1
 a) 2x^{3} \times (-3x^{5}) = -6x^{8}
b) (2x-1)^{2} = 4x^{2} - 4x + 1
c) 0 272 - 3\sqrt{8} = 1212 - 612
(ii) \sin 60^{\circ} + \sin 90^{\circ} = 13 + 1 or (\frac{13+2}{2})
    (ii) 10(3^{+}1) = 5(81-1)
     (M) log_32=5 (since 2=32)
00 \ y = 3x^{5}
2x^{5} \ y = x^{5}
2x^{5} \ y = 3x^{5}
 e) i) (22-3)(3x+5)= O
         x=\frac{3}{3} or x=(\frac{5}{3}) (in) 2^{x+3}=\sqrt{32}
```

$$x=\frac{3}{3}$$
 or  $x=\frac{3}{3}$  / (in)  $x=\sqrt{3}$  (ii)  $x=\sqrt{3}$   $x=\sqrt{3}$  (ii)  $x=\sqrt{3}$   $x=\sqrt{3}$ 

### QUESTION MELVE

b) 
$$y=x^2-6x+5$$
. (b)  $(0,5)$ 



(iv) 
$$Y=3^{2}-6\times3+5=9-18+5$$
  $V(3,-4)/-4$ 

(ii) 
$$x^{a} - 6x + 5 \times 0$$
  
 $x > 5$  or  $x < | \sqrt{ }$ 

a) (i) 
$$f(x) = x^{2}(2x^{2}-5x)$$

$$= 2x^{6}-5x^{3}$$

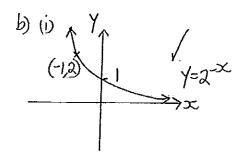
$$f(x) = |2x^{5}-15x^{2}|$$

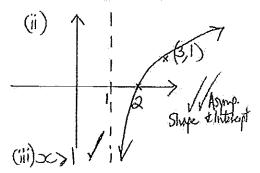
(i) 
$$f(x) = \frac{x+1}{\sqrt{x}}$$

$$= \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$= x^{k} + \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x}}$$

$$f(x) = kx^{-k} - kx^{-k}$$





c) 
$$y=x^2-6x+5$$
  
 $2x-6$   
 $2x-6$   
 $2x-6$   
 $2x-6$   
 $2x-6$   
 $2x-6$ 

e) 
$$f(x) = \sqrt{9-x}$$
 $9-x>0$ 
 $9>x$ 
 $x \le 9$ 

AL MOILCAND a) (i)  $y = (3x^{2}+5)^{4}$ (i) y= (3-20c) Lot u= 3x+5 Y= u+ / = 6x dy= 4n3 / 11=3-2x y=1/2 dm=(-2) dy= 61/2/  $\frac{df}{dx} = \frac{dy}{dx} \times \frac{dw}{dx}$   $= 4(3x+5) \times 6x$ 哉= 以(3-2x)3×(-2) = 24x (3x2+5)2 b) 200- 400x3=0 o) f(x) = x2+4x-3 (ii) Minumum is (-7) d) LHS = Sing + Coops

QUESTION 15 a) f(x) = x2-5x f(x) = Lim [(x+h)-5(x+h)]-(x2-5x)] = Lim [x+dxh+h2-5x-5h-x+5x] h>0 [x+dxh+h2-5x-5h-x+5x] = Lim [h+ 20ch-5h] 1- Lim [ 1/0x-5+h) = Lim (2x-5+h) b) AP a=8 t=28=a+52 (i) : 5d=20 d=4 (ii) tn= a+(n-1) & = 8+4(n-1) Ve require to >100 4n>96 Twenty fifth term / tas=104 (ii) Sn= 2 (2a+(n-1)d) We require Sn= 680 680=6n+2n2 = 2 (16+4n-4) nº+3n-340-0 = 12(12+4n)/ (n+20)(n-17)=0= Gn+2n2 n=17 since n>0/ 10 Siz=680

c) 
$$f(x) = \frac{55x+3}{3x-4}$$
  $u=5x+3$   $v=3x-4$ 

$$f(x) = \frac{5}{3x-4}$$
  $du=5$ 

$$\frac{du}{dx} = 5$$

$$\frac{du}{dx} = 3$$

$$\frac{5}{3x-4}$$
  $du=5$ 

$$\frac{du}{dx} = 3$$

$$\frac{5}{3x-4}$$
  $du=5$ 

$$\frac{du}{dx} = 3$$

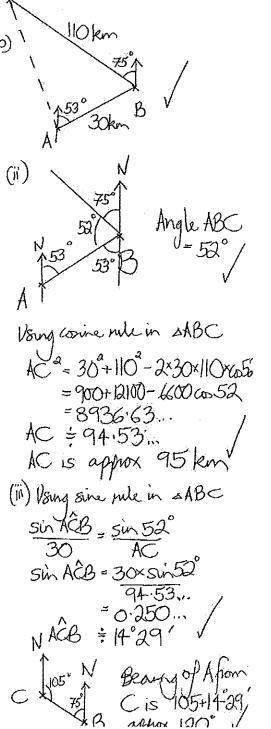
$$\frac{3}{3x-4}$$

$$\frac{du}{dx} = 3$$

f(3)= 2x3(3x4)3 (21x+4)

GUESTION 16

a) 
$$3\times 2^{\times} = 5775$$
 $(\div 3)$ 
 $2^{\times} = 1925$ 
 $\log_{0} 2^{\times} = \log_{0} 1925$ 
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 $(\div 3)$ 
 $2^{\times} = \log_{0} 1925$ 
 $2^{\times$ 



GP a=tax r=secx

KL=tp=ar9 =tanksecx