9C Banked Tracks

Exercise 9C

1. A car travels round a circular bend in a road of radius 45 metres at a speed of 48 km/h. There is no sideways frictional force between the road surface and the tyres. Show that the circular bend is banked at $21°57'$ to the horizontal. (Assume $g = 9{\cdot}8\,\text{m/s}^2$.)

2. A railway line has been constructed around a circular bend of radius 400 m. The distance between the rails is 1·5 m and the outside rail is 8 cm above the inside rail. Show that the optimum speed of a train on this bend (that is, the speed at which the wheels exert no sideways force on the rails) is about 52 km/h. (Assume $g = 9{\cdot}8\,\text{m/s}^2$.)

3. A car of mass 1·2 tonnes is rounding a circular bend of radius 150 m. The bend is banked at 10° to the horizontal. Assume that $g = 9{\cdot}8\,\text{m/s}^2$.

(a) Show that the car must travel at about 58 km/h so that there is no tendency to skid sideways.

(b) Suppose that the car travels into the bend at 72 km/h. Show that the sideways frictional force exerted by the tyres on the road is approximately 1109 Newtons.

4. A railway track around a circular curve of radius 200 m is designed for an optimum speed of 50 km/h. Assume that $g = 9{\cdot}8\,\text{m/s}^2$.

(a) If the gauge of the track is 1·52 m, show that the difference in height between the outer and inner rails is about 15 cm.

(b) Show that the sideways thrust on the rails is about 110 577 Newtons if a train of mass 120 tonnes travels around the curve at 70 km/h.

5. The sleepers of a railway line at a point on a circular bend of radius 100 metres are sloped such that a train travelling at 48 km/h exerts no lateral force on the rails. Show that a locomotive of mass 100 tonnes at rest on this bend would exert a lateral force of about $1{\cdot}75 \times 10^5$ Newtons on the rails.

6. A car travels at v m/s around a curved track of radius R metres.

(a) Show that the inclination θ of the track to the horizontal satisfies $\tan\theta = \dfrac{v^2}{Rg}$ if there is no tendency for the car to slip sideways.

(b) A second car of mass M kg is travelling around the same curved track at V m/s. Prove that the sideways frictional force exerted by the surface of the track on the tyres of this car is $\dfrac{Mg(V^2 - v^2)}{\sqrt{v^4 + R^2g^2}}$ Newtons.

suppose it travels at 80 km/h. What is the sideways frictional force exerted by the tyre on the road.

(b) ② $\sin\theta$ − ① $\cos\theta$

$$F\sin^2\theta + F\cos^2\theta$$

$$= mg\sin\theta - \frac{mv^2}{r}\cos\theta$$

$$= 1400\left(9.8\times\sin 15 - \frac{22.2^2}{120}\cos 15\right)$$

$F \doteqdot \cancel{2040N}\ -2014N$

∴ frictional force is down the plane

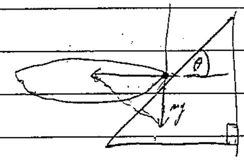
NB: If $v < 63.9$ km/h then $F > 0$

$v > 63.9$ km/h then $F < 0$

$\boxed{\text{Ex. 9C}}$ SOLUTIONS TO EX 9C – CAMBRIDGE 4U – SGS

1. $r = 45$

$v = \frac{40}{3}\ ms^{-1}$



Vert

$N\cos\theta = mg$ ②

Rad

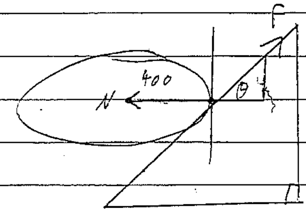
$N\sin\theta = \frac{mv^2}{r}$ ①

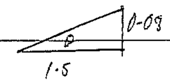
$\frac{①}{②}$: $\frac{\sin\theta}{\cos\theta} = \frac{v^2}{rg}$

$$\tan\theta = \frac{\left(\frac{40}{3}\right)^2}{45\times 9.8}$$

$\theta \doteqdot 21°57'$ #

2.





When $F = 0$,

$\theta = 3°3'$

$v^2 = gr\tan\theta$

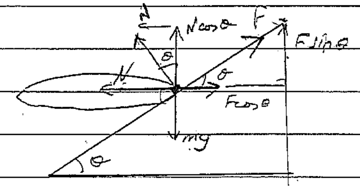
$v^2 = 209.0667$

$v = 14.459\ ms^{-1}$

$= 52.053$ km/h

$= 52$ km/h

3. (a)



$m = 1200$

$r = 150$

$\theta = 10°$

$g = 9.8$

Vert: $F\sin\theta + N\cos\theta = mg$

Horiz: $-F\cos\theta + N\sin\theta = \frac{mv^2}{r}$

$v^2 = rg\tan\theta$

$v^2 = 259.200$

$v = 16.099\ ms^{-1}$

$\doteqdot 58$ km/h

(b) $v_1 = 20\ ms^{-1}$

$F\sin\theta + N\cos\theta = mg$ $\qquad N\sin\theta - F\cos\theta = \frac{mv^2}{r}$

$N\cos\theta = mg - F\sin\theta$ $\qquad N\sin\theta = \frac{mv^2}{r} + F\cos\theta$

$$\therefore \frac{N\sin\theta}{N\cos\theta} = \frac{\frac{mv^2}{r} + F\cos\theta}{mg - F\sin\theta}$$

$\cancel{\tan\theta(mg}$

$mg\tan\theta - \tan\theta(F)\sin\theta = \frac{mv^2}{r} + F\cos\theta$

$F\cos\theta + F\tan\theta\sin\theta = mg\tan\theta - \frac{mv^2}{r}$

$F\cos^2\theta + F\sin^2\theta = mg\sin\theta - \cos\theta\left(\frac{mv^2}{r}\right)$

$F = -1{,}109.28\ldots$

$= -1{,}109N$ ~~down the ram~~ up the ramp

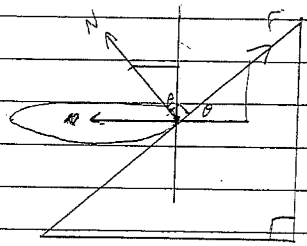
F.

∴ force is down the ramp.

4. (a) $r = 200$

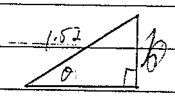
$v = \frac{125}{9}$

$g = 9.8$



Vert: $F\sin\theta + N\cos\theta = mg$

Rad: $N\cos\theta - F\sin\theta = \frac{mv^2}{r}$



$F = 0,$

$v^2 = rg\tan\theta$

$\theta = 5°37'$

$h = 1.52\sin\theta$

$h = 0.1488m$

$\doteqdot 15cm$ #

(b) $\frac{N\sin\theta}{N\cos\theta} = \frac{\frac{mv^2}{r} + F\cos\theta}{mg - F\sin\theta}$

$F\cos\theta + F\sin\theta\tan\theta = mg\tan\theta - \frac{mv^2}{r}$

$m = 120000$

$v = \frac{125}{9}$

$F = mg\sin\theta - \cos\theta\left(\frac{mv^2}{r}\right)$

$F = -110576.8614$

$\doteqdot -110577N$ up slope

∴ sideways thrust is 110577N inwards.

5. $r = 100$

$v = \frac{40}{3}$

$m = 100000$

$g = 9.8$

Vert: $F\sin\theta + N\cos\theta = mg$

Rad: $N\sin\theta - F\cos\theta = \frac{mv^2}{r}$

$v^2 = rg\tan\theta$

$\tan\theta = \frac{v^2}{rg}$ $\quad v_1 = 0,$

$\theta = 10°16'$

$$\frac{N\sin\theta}{N\cos\theta} = \frac{\frac{mv^2}{r} + F\cos\theta}{mg - F\sin\theta}$$

$F = mg\sin\theta - \cos\theta\left(\frac{mv^2}{r}\right)$

$= mg\sin\theta - 0$

$= 174922.88N$

$\approx 1.75\times10^5 N$

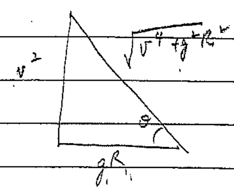
6. (a) Vert: $N\cos\theta = mg$ Rad: $N\sin\theta = \frac{mv^2}{r}$

$\frac{\sin\theta}{\cos\theta} = \frac{\frac{mv^2}{r}}{mg}$

$\tan\theta = \frac{v^2}{rg}$

$r = R$

$\therefore \tan\theta = \frac{v^2}{Rg}$



$\cos\theta = \frac{gR}{\sqrt{v^4+g^2R^2}}$

$\sin\theta = \frac{v^2}{\sqrt{\ldots}}$

(b) ~~m~~ M

Vert: $N\cos\theta + F\sin\theta = Mg$

Rad: $N\sin\theta - F\cos\theta = \frac{MV^2}{r}$

$\tan\theta = \frac{\frac{MV^2}{r} + F\cos\theta}{Mg - F\sin\theta}$

$\frac{v^2}{Rg}Mg - \frac{v^2}{Rg}F\sin\theta = \frac{MV^2}{R} + F\cos\theta$

$F\left(\cos\theta + \frac{v^2}{Rg}\sin\theta\right) = \frac{Mv^2}{R} - \frac{MV^2}{R}$

$$F = \frac{-\frac{M}{R}\left(V^2 - v^2\right)}{\cos\theta + \frac{v^2}{Rg}\sin\theta}$$

$$\cancel{f = \frac{-\frac{M}{R}\left(V^2 - v^2\right)}{\frac{gR}{\sqrt{v^4+g^2R^2}} + \frac{v^2}{Rg}\left(\frac{v^2}{\sqrt{v^4+g^2R^2}}\right)}}$$

$$f = \frac{-\frac{M}{R}\left(V^2 - v^2\right)}{\frac{gR}{\sqrt{v^4+g^2R^2}} + \frac{v^2}{Rg}\left(\frac{v^2}{\sqrt{v^4+g^2R^2}}\right)}$$

$$= \frac{-\frac{M}{\cancel{R}}\left(V^2 - v^2\right)}{\frac{\sqrt{g^2R^2+v^4}}{Rg\,\cancel{\sqrt{v^4+g^2R^2}}}}$$

$$= \frac{-Mg\left(V^2 - v^2\right)}{\sqrt{v^4+g^2R^2}}$$

Since sideways frictional force is scalar, magnitude is taken

$$F = \left| \frac{-Mg\left(V^2 - v^2\right)}{\sqrt{v^4+g^2R^2}} \right|$$

$$= \frac{Mg\left(V^2 - v^2\right)}{\sqrt{v^4+g^2R^2}} \quad \#$$