SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

YEAR 11 PRELIMINARY EXAMINATION

SEPTEMBER 2007

Time Allowed: 120 minutes

Direction to Candidates:

- Approximately marks are shown alongside each question
- All necessary working should be shown .Marks may not be awarded for careless or badly arranged work
- Begin answering each question on a new page

Name: _____ Teacher:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL

QUESTION 1 (10 marks)

a) Simplify $5\sqrt{2} - \sqrt{32}$ 1

b) Solve for x, 2

$$|x + 1| = 3$$

c) State the domain of $y = \sqrt{x - 1}$ 1

d) Solve for x: 2

$$\frac{2x}{3} - 1 = \frac{x+1}{4}$$

e) (i) Sketch the graph of $y = |x + 1|$ 1

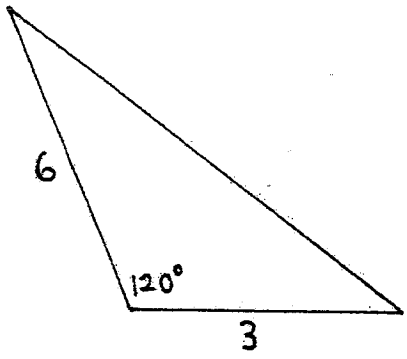
(ii) State its range 1

f) Find $\lim_{x \to 2} \frac{x^2-4}{x-2}$ 2

QUESTION 2 (11 marks)

a) If $sin\theta = \frac{3}{7}$ and $0^\circ \leq \theta \leq 90^\circ$, find $cos\theta$ in surd form 2

b) Find the area of the triangle below leaving your answer in surd form. 2



c) Sketch the region given by: $(x-2)^2 + (y+3)^2 > 9$ 3

d) Simplify $\frac{4a+2b}{8a+4b}$ 2

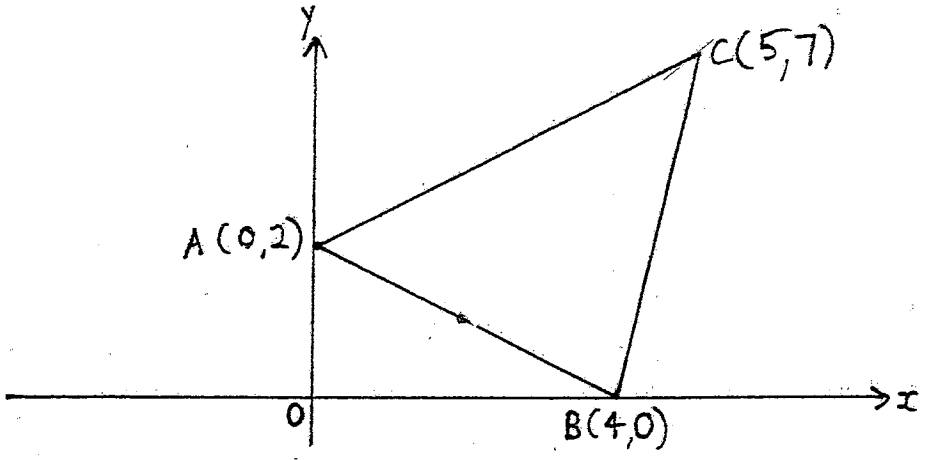
e) $f(x) = \begin{cases} x^3+1 & \text{if } x > 2 \\ 2x & \text{if } -1 \leq x \leq 2 \\ 5 & \text{if } x < -1 \end{cases}$ 2

Find $f(-2) - f(3) + +f(2)$

QUESTION 3 (11 marks)

a) Find the perpendicular distance from the point $(3,2)$ to the line $3x - 4y + 7 = 0$ 2

b)



i) Find the gradient of AB 1

ii) Find the coordinates of D, the midpoint of AB 1

iii) Find the equation of the line passing through D and perpendicular to AB 2

iv) Show that C lies on this line 1

v) Find the lengths of AB and CD in surd form. 2

vi) Find the area of the quadrilateral $ACBO$ 2

QUESTION 4 (11 marks)

a) Solve $|2x - 1| < 3$ 2

b) Write as a single fraction $\frac{1}{x-3} + \frac{1}{x+3}$ 2

c) Solve $2\sin\theta = -1$ for $0^\circ \leq \theta \leq 360^\circ$ 2

d) Simplify $\cos\theta + \cos\theta tan^2\theta$ 3

e) Prove that $sec^2\theta = \frac{1}{(1-sin\theta)(1+sin\theta)}$ 2

QUESTION 5 (11 marks)

a) Differentiate

i) $-3x^4$ 1

ii) $\frac{2x-1}{x+4}$ 2

iii) $(3x^2-5)^6$ 2

iv) $(2x+3)(x^2+x+1)$ 2

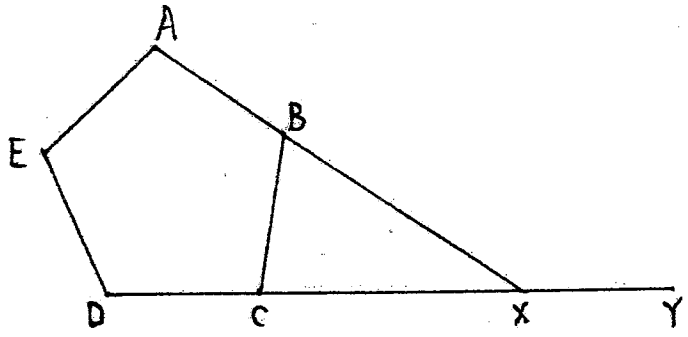
b) Find the x co-ordinate of the point on the curve $y = x^2 + 2$ where the tangent has the gradient of -2 1

c) Find the equation of the tangent to $y = 2x^2 - 2x + 1$ at he point $x = 1$ 3

QUESTION 6 (11 marks)

a) Factorise $8 - 27x^3$ 2

b)



In the diagram, $ABCDE$ is a regular pentagon and AB and DC are produced to meet at X. The point Y lies on DCX produced.

(i) Find $\angle ABC$ 2

(ii) Find $\angle BXY$ giving reasons 3

c) (i) Find the discriminant of $x^2 + (k-1)x + 1$ in simplest form 2

(ii) Find the range of values of k for which the quadratic expression above is positive definite. 2

QUESTION 7 (10 marks)

a) (i) Express the equation of the parabola $8y = x^2 - 8x - 24$ in the form $(x-h)^2 = 4a(y-k)$ 2

(ii) Write down the coordinates of the vertex and equation of the directrix for this parabola 2

b) If α and β are the roots of the quadratic equation $x^2 + 3x - 5 = 0$, find

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\alpha^2 + \beta^2$ 2

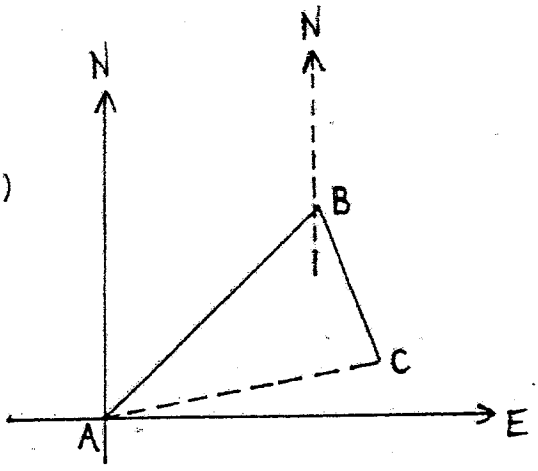
(iv) $\alpha^3\beta + \alpha\beta^3$ 2

QUESTION 8 (11 marks)

a) If $f(x) = \frac{x-1}{x+2}$, show that $f(1-x) = \frac{x}{x-3}$ 2

b) Solve $9^x + 3.3^x - 18 = 0$ by first reducing this equation to a quadratic 3

c)



Copy this diagram onto your answer sheet and mark the following information on it.

(i) An ultralight plane is flown from an airport *A* on a bearing of $030^\circ T$ for $150km$ to a position *B*. From position *B* the ultralight is then flown $100km$ on a new course bearing $135^\circ T$ to position *C*. Use the above diagram to find how far (to the nearest km) *C* is from *A*. 3

(ii) Use the Sine Rule to help find the bearing of *C* from *A*. 3

(nearest degree)

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Solution to 2007 2 Unit Final Prelim. Exam

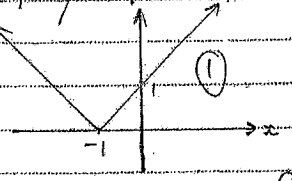
Question 1

a) $5\sqrt{2} - \sqrt{32}$
$5\sqrt{2} - 4\sqrt{2}$
$\sqrt{2}$ ①

b) $|x+1| = 3$
$x+1 = 3$ or $x+1 = -3$ ①
$x = 2$ or $x = -4$ ①

c) $x \geq 1$ ①

d) $\frac{2x}{3} - 1 = \frac{x+1}{4}$
$8x - 12 = 3(x+1)$
$8x - 12 = 3x + 3$ ①
$5x = 15$
$x = 3$ ①

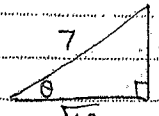
e) i) 

①
ii) Range $y \geq 0$ ①

f) $\lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)}$
$= 4$ ①

Question 2

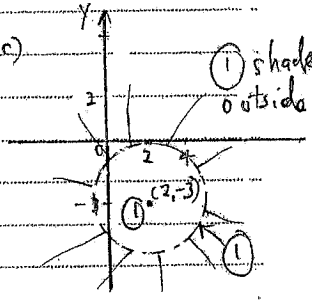
a) $\sin\theta = \frac{3}{7}$



①

$\cos\theta = \frac{\sqrt{40}}{7}$ ①

b) $A = \frac{1}{2}ab\sin C$
$= \frac{1}{2} \times 3 \times 6 \times \sin 120$ ①
$= 9 \times \frac{\sqrt{3}}{2}$
$= \frac{9\sqrt{3}}{2}$ units^2 ①

c) 

① shade outside
① ①

d) $4a + 2b$
$8a + 4b$
$\frac{1}{2}(2a+b)$ ①
$2(2a+b)$
$= \frac{1}{2}$ ①

e) $f(-2) - f(3) + f(2)$
$5 - 28 + 4$ ①
$= -19$ ①

Question 3

a) $d = \frac{|3 \times 3 + -4 \times 2 + 7|}{\sqrt{3^2 + (-4)^2}}$
$= 0$ ①

b) i) $m_{AB} = \frac{-1}{2}$ ①
ii) $D(2,1)$ ①

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iii) $m = 2$ (2,1)
$y - 1 = 2(x-2)$ ①
$y - 1 = 2x - 4$
$y = 2x - 3$ or $2x - y - 3 = 0$ ①

iv) $d_{AB} = \sqrt{(4-0)^2 + (0-2)^2}$
$= \sqrt{20}$ or $2\sqrt{5}$ ①

$d_{CD} = \sqrt{(5-2)^2 + (7-1)^2}$
$= \sqrt{9+36}$
$= \sqrt{45}$ or
$= 3\sqrt{5}$ ①

iv) $7 = 2,5 - 3$
$7 = 10 - 3$ ✓ ①

vi) Area $= \frac{1}{2} \times 4 \times 2 + \frac{1}{2} \times 2\sqrt{5} \times 3\sqrt{5}$ ①
$= 4 + 15$
$= 19$ units^2 ①

Question 4

a) $|2x-1| < 3$ ①
$2x - 1 < 3$ and $2x - 1 > -3$
$2x < 4$ and $2x > -2$
$x < 2$ and $x > -1$
$\therefore -1 < x < 2$ ①

b) $\frac{1}{x-3} + \frac{1}{x+3}$
$\frac{x+3 + x-3}{(x-3)(x+3)}$ ①
$\frac{2x}{(x-3)(x+3)}$ or $\frac{2x}{x^2-9}$

c) $2\sin\theta = -1$
$\sin\theta = \frac{-1}{2}$ ①
Working angle is 30
$\therefore \theta = 210°, 330°$ ①

d) $\cos\theta + \cos\theta\tan^2\theta$
$\cos\theta(1 + \tan^2\theta)$ ①
$\cos\theta \times \sec^2\theta$ ①
$= \sec\theta$ ①

e) $\sec^2\theta = \frac{1}{(1-\sin\theta)(1+\sin\theta)}$
$= \frac{1}{1-\sin^2\theta}$ ①
$= \frac{1}{\cos^2\theta}$ ①
$= \sec^2\theta$

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Question 5

a) (i) $-3x^4$

$\frac{d}{dx} = -12x^3$ (1)

(ii) $\frac{2x-1}{x+4}$ (1)

$\frac{d}{dx} = \frac{(x+4)\times 2 - (2x-1)}{(x+4)^2}$

$\frac{d}{dx} = \frac{9}{(x+4)^2}$ (1)

(iii) $(3x^2-5)^6$ (1)

$\frac{d}{dx} = 6(3x^2-5)^5 \times 6x$

$\frac{d}{dx} = 36x(3x^2-5)^5$ (1)

(iv) $(2x+3)(x^2+x+1)$ (1)

$\frac{d}{dx} = (2x+3)(2x+1) + (x^2+x+1)\times 2$

$= 4x^2+2x+6x+3+2x^2+2x+2$

$\frac{d}{dx} = 6x^2+10x+5$ (1)

b) $y = x^2+2$

$\frac{dy}{dx} = 2x = -2$

$\therefore x = -1$ (1)

c) $y = 2x^2-2x+1$

$\frac{dy}{dx} = 4x-2$

At $x=1$, $m=2$ (1)

At $x=1$, $y=1$ (1)

$y-y_1 = m(x-x_1)$

$y-1 = 2(x-1)$ or

$y = 2x-1$ (1)

Question 6

a) $8-27x^3 = 2^3-(3x)^3$ (1)

$(2-3x)(4+2\times 3x+9x^2)$

$(2-3x)(4+6x+9x^2)$ (1)

b) (i) Sum of Ext Angles is 360

$\therefore$ Each exterior is $\frac{360}{5} = 72°$ (1)

$\therefore \angle ABC = 180-72$

$= 108°$ (1)

(ii) $\angle BXY = \angle CBX + \angle BCX$ (Exterior angle)

$\angle CBX = \angle BCX = 72°$ from part (i)

$\therefore \angle BXY = 144°$ (1)

c) (i) $x^2+(k-1)x+1$

$\Delta = b^2-4ac$

$= (k-1)^2 - 4\times 1\times 1$ (1)

$= k^2-2k-3$ (1)

(ii) Positive Definite

if $\Delta < 0$

$k^2-2k-3 < 0$ (1)

$(k+1)(k-3) < 0 \Rightarrow -1 < k < 3$

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Question 7

a) (i) $x^2-8x-24 = 8y$

$x^2-8x+16 = 8y+16+24$ (1)

$(x-4)^2 = 8y+40$

$(x-4)^2 = 8(y+5)$ (1)

(ii) Vertex is $(4,-5)$ (1)

Focal Length is 2

Concave Up $\therefore$ directrix is $y = -7$ (1)

b) $x^2+3x-5=0$

(i) $\alpha+\beta = \frac{-b}{a}$

$= -3$ (1)

(ii) $\alpha\beta = \frac{c}{a}$

$= -5$ (1)

(iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (-3)^2 - 2\times -5$

$= 9+10$

$= 19$ (1)

(iv) $\alpha^3\beta + \alpha\beta^3$

$= \alpha\beta(\alpha^2+\beta^2)$ (1)

$= -5\times 19$

$= -95$ (1)

Question 8

a) $f(x) = \frac{x-1}{x+2}$

$f(1-x) = \frac{1-x-1}{1-x+2}$ (1) $= \frac{-x}{3-x} = \frac{x}{x-3}$

b) $9^x + 3.3^x - 18 = 0$

$(3^2)^x + 3.3^x - 18 = 0$

$(3^x)^2 + 3.3^x - 18 = 0$

Let $v = 3^x$

$v^2+3v-18 = 0$ (1)

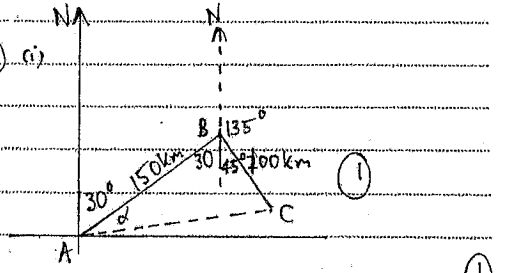
$(v-3)(v+6) = 0$

$v = 3$ or -6 (1)

$\therefore 3^x = 3$ or $3^x = -6$

$\therefore x = 1$ (1) No Soln

c) (i)



(1)

$AC^2 = 150^2+100^2-2\times 150\times 100\times \cos$

$AC^2 = 24735.4$

$AC = 157$km (1)

(ii) Need α

(1) $\frac{\sin\alpha}{100} = \frac{\sin 75}{157}$

$\sin\alpha = \frac{100\times\sin 75}{157}$

$\alpha = 38°$ (1)

$\therefore$ Bearing of C from A is $30+38 = 068°$ or N68°E (1)