Name	
Class	

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 10 TERM 3 EXAMINATION
AUGUST 2004

ADVANCED MATHEMATICS

Time Allowed: 70 minutes

Directions to Candidates

- Attempt all questions.
- **All necessary working should be shown.**
- Unless otherwise specified, answers must be given in their simplest form.
- Approved calculators may be used in all sections.
- Use a ruler when drawing straight lines.
- Marks may be deducted for careless or poorly arranged work.

Section 1	Section 2	Section 3	Section 4	Section 5	TOTAL
/13	/13	/13	/13	/13	/65

Questions and working space.	Answers Only
1. Simplify $\dfrac{(2x^2)^3}{2x}$	
2. Estimate the surface area of a tennis ball· (A) 50 cm^2 (B) 75 cm^2 (C) 140 cm^2 (D) 450 cm^2	
3. By factorising or otherwise find which is the factor of $6x^2 + x - 15$? (A) $2x - 3$ (B) $2x + 3$ (C) $2x - 5$ (D) $2x + 5$	
4. Expand and simplify $(5\sqrt{2} + 3)^2$	
5. Factorise $x - xy - 4 + 4y$	
6. Evaluate $\dfrac{\sqrt{16} + \sqrt{9}}{\sqrt{16 + 9}}$	

Marks column (left side): 1, 1, 1, 2, 2, 1

7. Simplify $\dfrac{x+2}{x^2-5x+6} - \dfrac{1}{x-3}$

2

8. A computer purchased for $2400 depreciates at a rate of 21% p.a.
   i) Find its value after 10 years

   i)

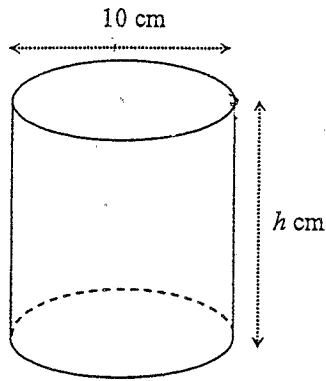   ii) By how much did it depreciate in the second year?

   ii

3

Section 2

2

1. Solve the equation $x^2 - 3x - 9 = 0$ leaving the roots in simplest surd form.

1

2. Write another score to make the range equal to the mode.

   16 19 19 19 24 26 30

4

3. Solve   (a) $x - \dfrac{x+2}{4} = 1$

   (b) $4 - 2x \geq x + 6$

   (a)

   (b)

4

4. a) Calculate the total area of the flat
   surfaces of this closed cylinder
   (leave your answer in terms of π).

   10 cm

   h cm

   a)

   b) Write down an expression for the
   area of the curved surface

   b)

3

   c) What should the value of h be so that the curved surface area is the same
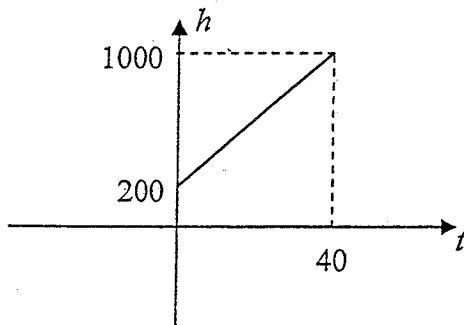   as the flat surface area?
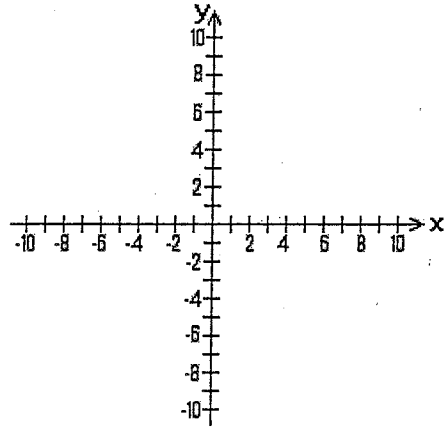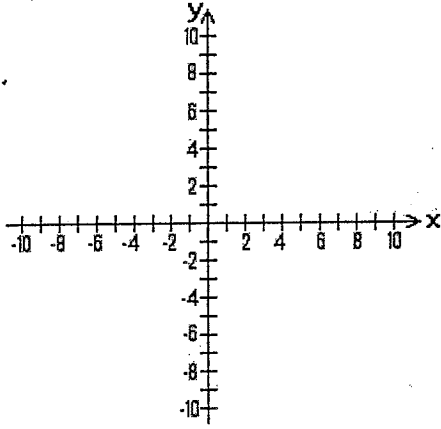
   c)

5. Find the equation of the line shown below.

   h

   1000

   200

   40

   t

3

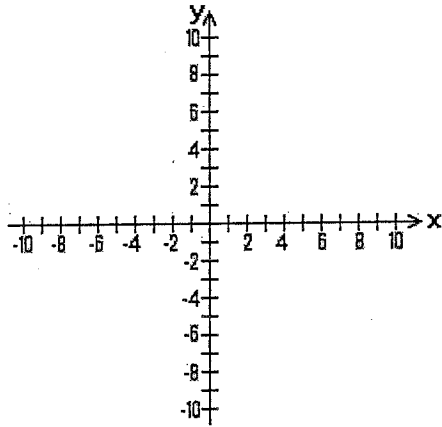1.   Sketch the graphs of the following relations.

(a) $y + x - 3 = 0$

(b) $xy = 3$ (label a point on the graph)

2

5

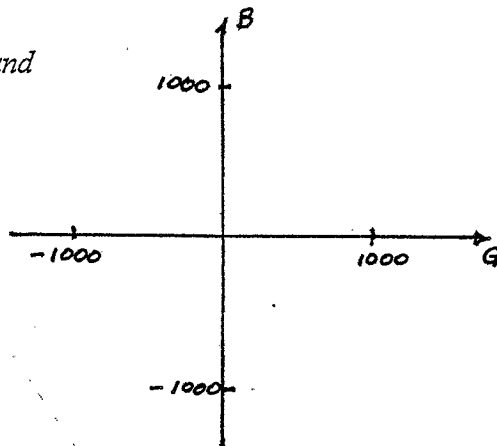2.   (a) Sketch the region $y \geq \dfrac{x}{2} - 2$

(b) *In a school the total number of boys (B) and girls (G) is no more than 1000.*

i)   Write an inequation for this situation in terms of B and G.

....

ii) Sketch the region given by your inequation on the diagram at the right.

3. Under each graph write the letter which corresponds to its equation.

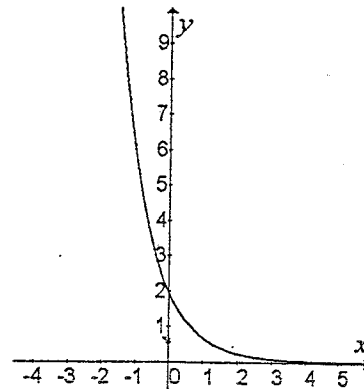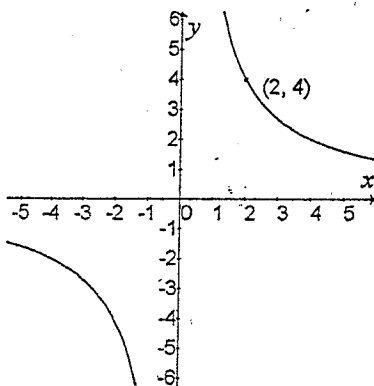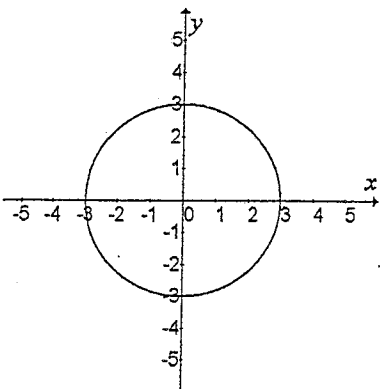A) $y = \frac{2}{x}$ B) $x^2 + y^2 = 9$ C) $y = \frac{4}{x}$ D) $y = 3^{-x+1}$

E) $x^2 + y^2 = 3$ F) $y = 2(3^{-x})$ G) $y = \frac{8}{x}$

4.

Mortgage payments as % of total male average earnings
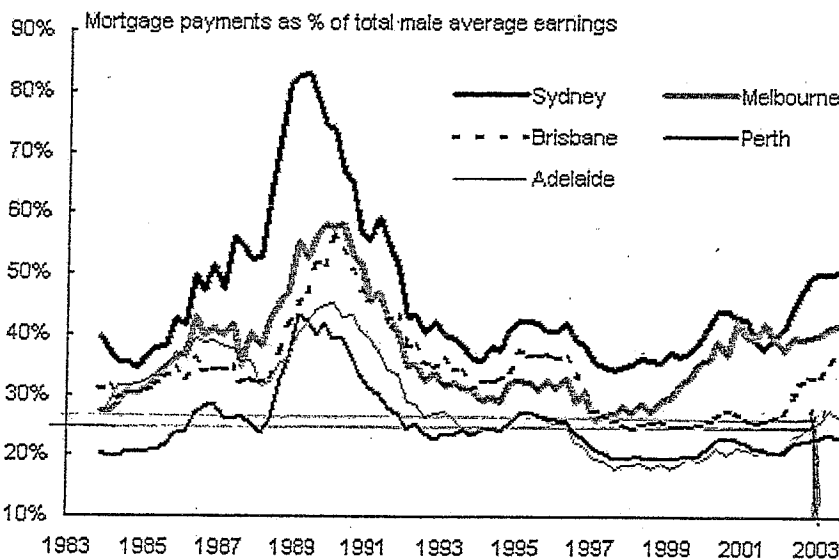
a) In which city is it usually most difficult to pay off a mortgage? _____

b) The average male earner in Adelaide in December 2003 earned $700/week. How much (approximately) would he have had left after paying the mortgage?

_____

c) State one aspect of the graph which is usually true over time.

_____

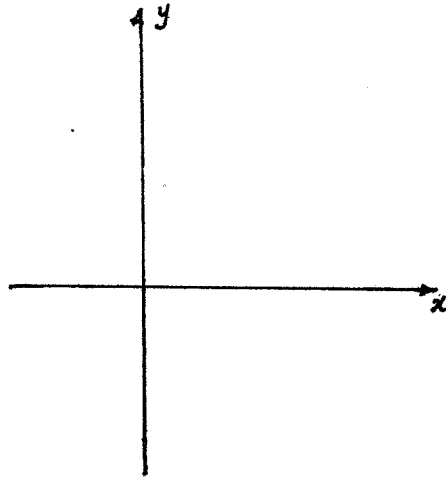1. Sketch the following curves, **showing the x and y intercepts for each one:**

   (a) $y = x(x-2)$

   (b) $y = (x-2)^2$

   (c) $y = -x^2 + 1$

6
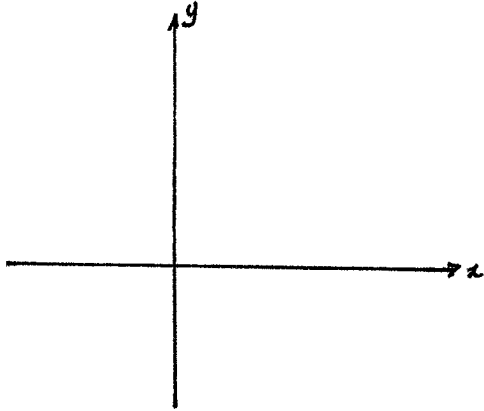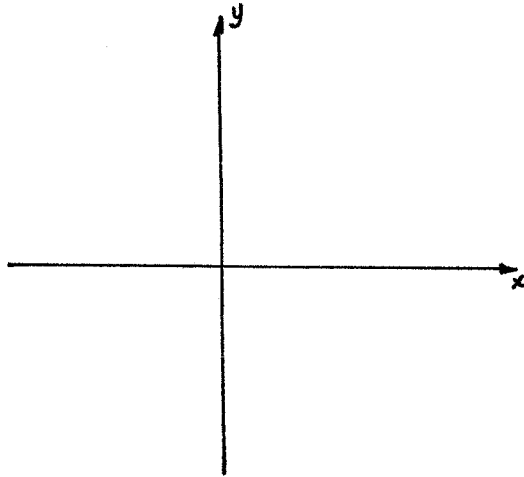
2. a) What is the maximum value of the expression $120 - (x-3)^2$?

   b) For what value of x does this occur?

2

3. For the curve $y = 2x^2 - 8x + 5$

    a)   Find the equation of the axis of symmetry.

                                      (a) _____

2

    b)   Find the coordinates of the vertex.

                                        (b) _____

---

4. Write the equation $y = x^2 - 8x + 25$ in the form $y = (x - A)^2 + B$.

2

_____

---

5. A farmer can sell 50 lambs for $20 each. For every $1 she adds to the price, one lamb will remain unsold.

    If x is the number of unsold lambs, write an expression for the amount of money she receives.

1

_____

Section 5

Write answers in the spaces provided

1	1. Name a measure of spread that is not affected by outliers (extremely high or low scores).	
3	2. For these scores $5\ 2\ 2\ 4\ 7\ 6\ 9\ 10$ use your calculator to find (to 1 dec. place) a) the mean b) the median c) the standard deviation	a) b) c)

3.

This *stacked bar graph* shows the numbers of students enrolled in different faculties in 1993 and in 2003.

University Enrolments by Faculty
Comparison 1993 with 2003

3
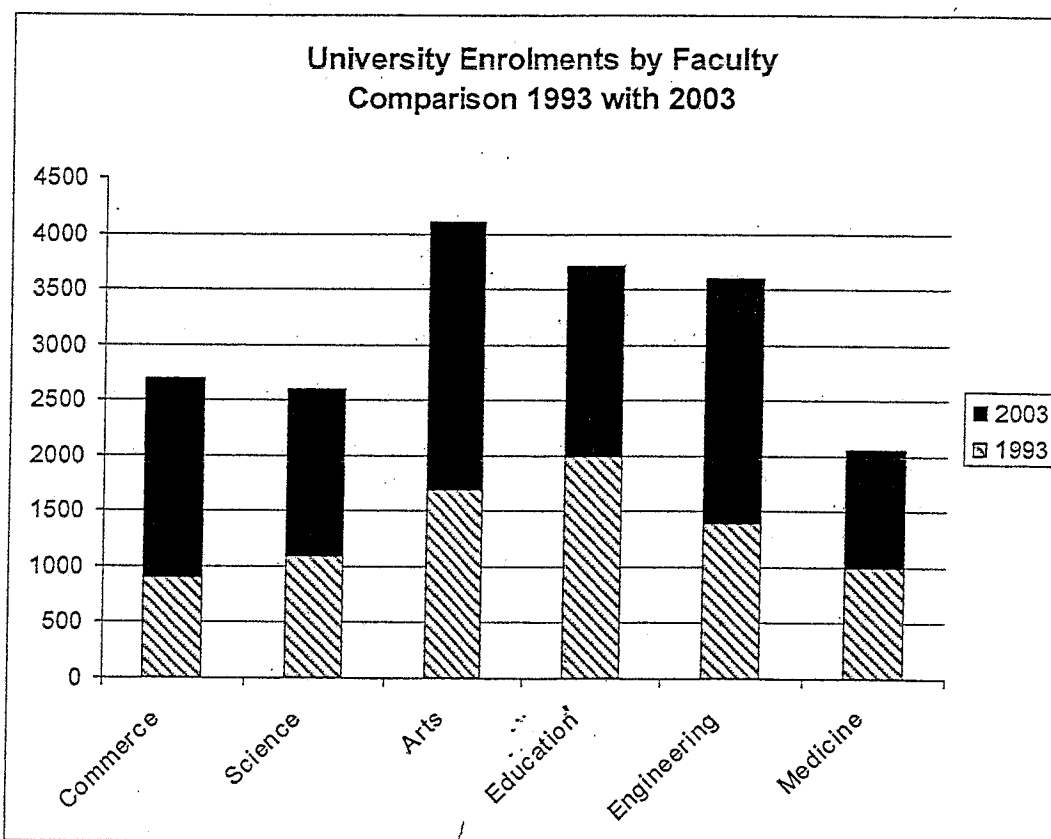
a) In which faculty have enrolments remained fairly constant? _____

b) In which faculty have enrolments declined? _____

c) Why is this graph difficult to interpret? _____

4. The results for a Half Yearly Exam were:

English: Mean = 59, Standard deviation = 8

Maths: Mean = 65, Standard deviation = 6

a) Ellen scored **75** in Maths and Paul scored **65** in English. State which is the better mark and <u>give a reason why</u>. _____

_____

b) The English teacher added 5 to everyone's English mark to get a new set of marks.

   i) What is the new mean? _____

   ii) What is the new standard deviation? _____

c) If a score of 66 was later added to the Maths results,

   i) the mean would   **increase / decrease**   (circle the correct answer)

   ii) the standard deviation would   **increase / decrease**   (circle the correct answer)

6

End of Exam

Name	Rifat Chowdhury
Class	10 M&H, Mr. Hempel

SOLUTIONS

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 10 TERM 3 EXAMINATION
AUGUST 2004

ADVANCED MATHEMATICS

Time Allowed: *70 minutes*

Directions to Candidates

- Attempt all questions.
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- Use a ruler when drawing straight lines.
- Marks may be deducted for careless or poorly arranged work.

Section 1	Section 2	Section 3	Section 4	Section 5	TOTAL
11 /13	8 /13	10 /13	10 /13	10 /13	49 /65

Section 1

Questions and working space.

		Answers Only
1.	Simplify $\dfrac{(2x^2)^3}{2x}$ $\quad \dfrac{8x^6}{2x}$	$4x^5$
2.	Estimate the surface area of a tennis ball: (A) 50 cm^2 (B) 75 cm^2 (C) 140 cm^2 (D) 450 cm^2 $r = 3.$ 113.1	B.
3.	By factorising or otherwise find which is the factor of $6x^2 + x - 15$? (A) $2x - 3$ (B) $2x + 3$ (C) $2x - 5$ (D) $2x + 5$ $3x$ $2x$	A.
4.	Expand and simplify $(5\sqrt{2}+3)^2$ $(5\sqrt{2}+3)(5\sqrt{2}+3)$ $25 \times 2 + 15\sqrt{2} + 15\sqrt{2} + 9$ $59 + 30\sqrt{2}$	$59 + 30\sqrt{2}$
5.	Factorise $x - xy - 4 + 4y$ $x(1-y) - 4(1-y)$	$(x-4)(1-y)$
6.	Evaluate $\dfrac{\sqrt{16}+\sqrt{9}}{\sqrt{16+9}}$ $\quad \dfrac{8}{5}$	$\dfrac{8}{5}$ or $1\tfrac{3}{5}$

2

7. Simplify $\dfrac{x+2}{x^2-5x+6} - \dfrac{1}{x-3}$

2

$\dfrac{x+2}{(x-3)(x-2)} - \dfrac{x-2}{}$

$\overline{x+2} \quad x$

$-1(x-2)$

$\dfrac{x+2 \quad -x+2}{(x-3)\,(x-2)}$

$\dfrac{4}{(x-3)(x-2)}$ **2**

8. A computer purchased for \$2400 depreciates at a rate of 21% p.a.
i) Find its value after 10 years

$2400\left(\cdot 79\right)^{10}$

i) \$227.24 **1**

ii) By how much did it depreciate in the second year?

1497.84

$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

3

ii)

\$902.16 **1**

\$398.16

---

1. Solve the equation $x^2 - 3x - 9 = 0$ leaving the roots in simplest surd form.

2

$\dfrac{3 \pm\sqrt{9-4\times1\times-9}}{2} = \dfrac{3\pm\sqrt{45}}{2}$

$\dfrac{3\pm\sqrt{45}}{2}$ **1**

2. Write another score to make the range equal to the mode.

16 19 19 19 24 26 30

$19.$

1

$35.$ ✓ **1**

3. Solve   (a) $4 \!\!\!\! \diagup x - \dfrac{4(x+2)}{4} = 1+4$

$4x - x - 2 = 4$

$3x = 6$

$x = 2$

$4x - x + 2 = 4$

$3x + 2 = 4$

$3x = 2$

$x = \tfrac{2}{3}$

4

(a) $x = \tfrac{2}{3}$ ✓ **1**

$x = 2$

(b) $4 - 2x \geq x + 6$

$4 - 2x = x + 6$

$-2 > 3x$

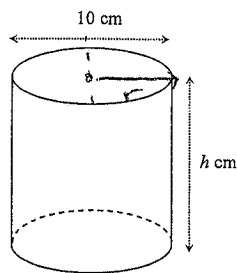$x > -\tfrac{2}{3}$

(b) $x \geq -\tfrac{2}{3}$ **1**

(4)

4. a) Calculate the total area of the flat
surfaces of this closed cylinder
(leave your answer in terms of π).

$2\pi r^2$
$2 \times \pi \times 5^2$
50π

10 cm

h cm

a) 50π ✓

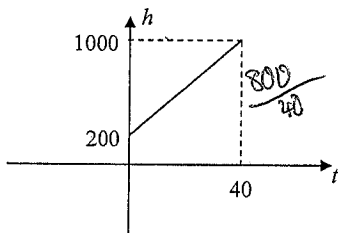b) Write down an expression for the
area of the curved surface

$2\pi rh$

$2 \times \pi \times 5x$

b) $2\pi rh$
$100\pi h$.

c) What should the value of h be so that the curved surface area is the same
as the flat surface area?

c) $h=5$ ✓

5. Find the equation of the line shown below.

$\frac{800}{40}$
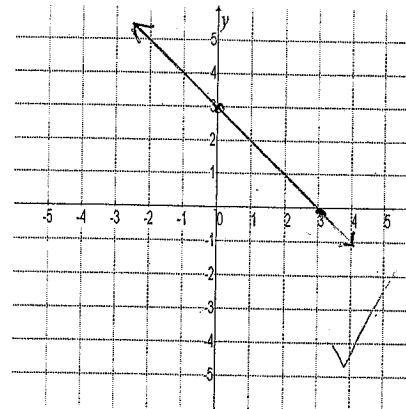
$y = 20x + 200$

$h + 2$
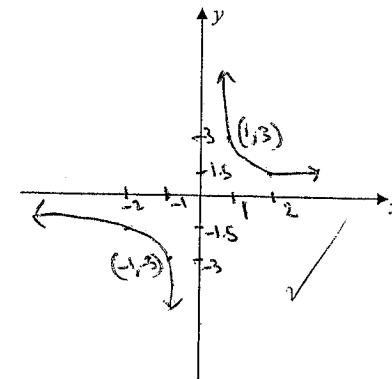
$y = 20x + 200.$

Section 3 answers area markers: 3, 3

1. Sketch the graphs of the following relations.

(a) $y + x - 3 = 0$ $\quad y = -x + 3$

(b) $xy = 3$ (label a point on the graph) $\quad y = \frac{3}{x}$

$(1,3)$
$(-1,-3)$

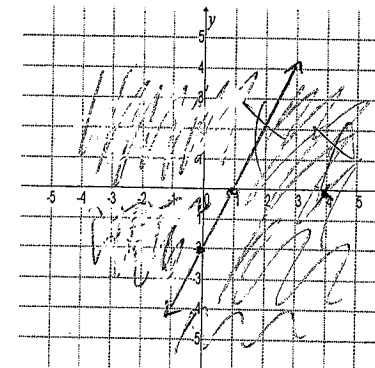**2.** (a) Sketch the region $y \geq \frac{x}{2} - 2$

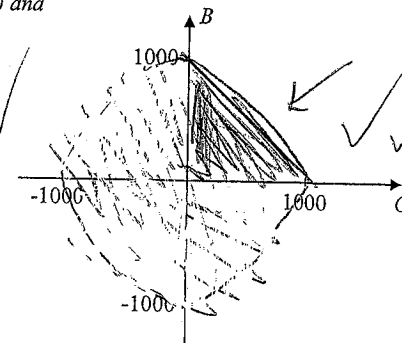$y = \frac{x}{2} - 2$
$2y = x - 2$
$y = \frac{x-2}{2}$

$0 = -2$

not real !

$(4,0) (0,-2)$

(b) In a school the total number of boys (B) and
girls (G) is no more than 1000.

i) Write an inequation for this situation
in terms of B and G.

$\Rightarrow B + G \leq 1000$ ✓

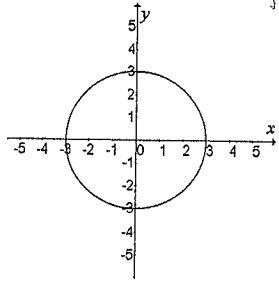ii) Sketch the region given by your
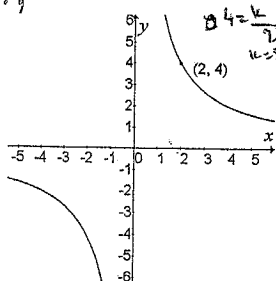inequation on the diagram at the right.

④

2

5

5

6

3. Under each graph write the letter which corresponds to its equation.

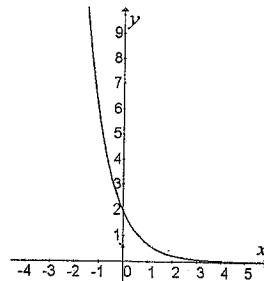A) $y = \frac{2}{x}$ B) $x^2 + y^2 = 9$ C) $y = \frac{4}{x}$ D) $y = 3^{-x+1}$

E) $x^2 + y^2 = 3$ F) $y = 2(3^{-x})$ G) $y = \frac{8}{x}$

y = ²/₉

(2, 4)

$x^2 + y^2 = 9$

B ✓   G ✓   E ✓

4.

Mortgage payments as % of total male average earnings

90%
80%
70%
60%
50%
40%
30%
20%
10%

1983 1985 1987 1989 1991 1993 1995 1997 1999 2001 2003
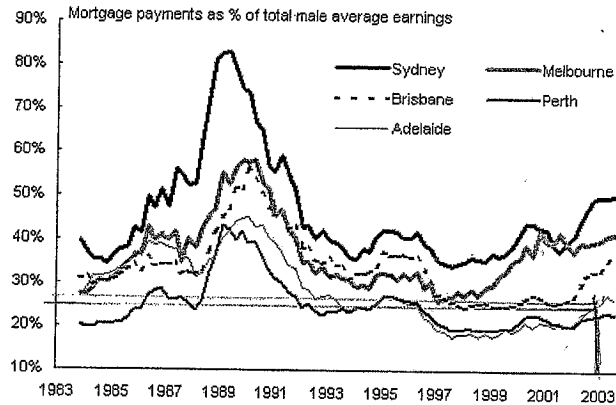
Sydney   Melbourne
Brisbane   Perth
Adelaide

a) In which city is it usually most difficult to pay off a mortgage? ...Sydney...

b) The average male earner in Adelaide in December 2003 earned $700/week. How much (approximately) would he have had left after paying the mortgage? (25%)
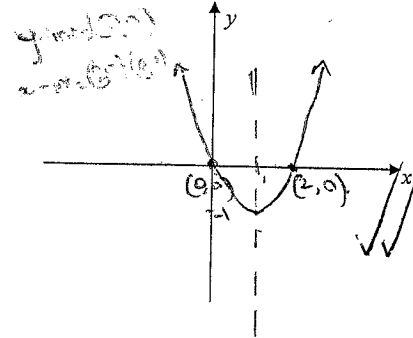
Approx. $525

c) State one aspect of the graph which is usually true over time. compared.

Over time mortgage rates will keep growing compared to previous years

Write answers in the spaces provided

1. Sketch the following curves, showing the **x and y intercepts for each one**:

(a) $y = x(x - 2)$
$y = x^2 - 2x$

(b) $y = (x - 2)^2$ $(x-2)(x-2)$
$y = x^2 - 2x - 2x + 4$

(0,0)   (2,0)
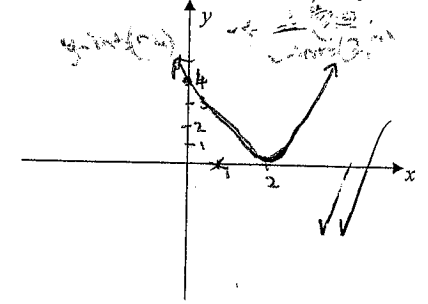
(c) $y = -x^2 + 1$
$-x^2 + 1$
$(x+1)(-x+1)$

$4 - 4(2) + 4$ $\frac{4}{2}$

-1   1

2. a) What is the maximum value of the expression $120 - (x - 3)^2$? $(x-3)(x-3)$
$120 + x^2 - 9x + 9$ $x^2 - 3x - 3x + 9$ 120

111 ✗

b) For what value of x does this occur?

0 ✗ 3.

7   8

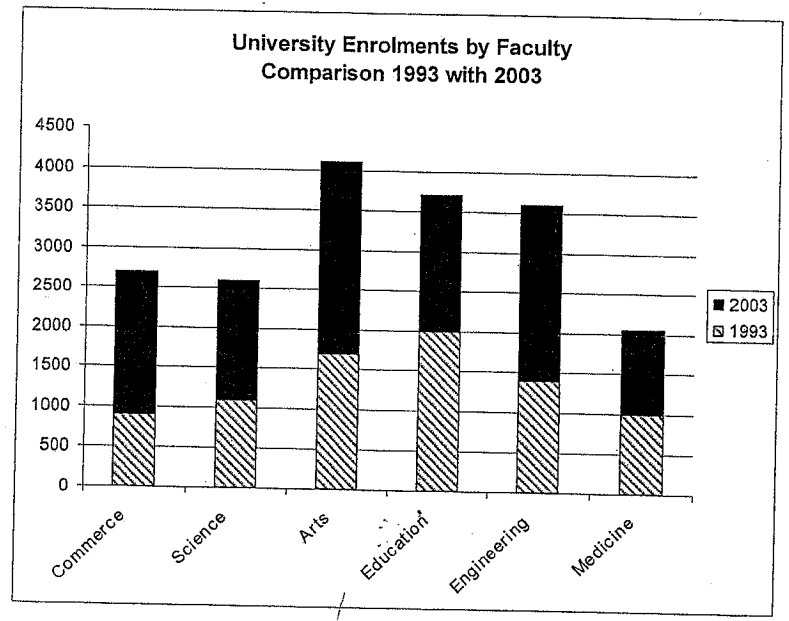3. For the curve $y = 2x^2 - 8x + 5$

a) Find the equation of the axis of symmetry.

(a) $x = \dfrac{8}{2 \cdot 2} = 2$ ✓

b) Find the coordinates of the vertex.

(b) $(2, -3)$ ✓

2

4. Write the equation $y = x^2 - 8x + 25$ in the form $y = (x - A)^2 + B$.

$y = (x - 4)^2 + 25 - 9$

$A = -4$ $B = 9$

$y = (x - 4)^2 + 9$ ✓

2

5. A farmer can sell 50 lambs for $20 each. For every $1 she adds to the price, one lamb will remain unsold.

If x is the number of unsold lambs, write an expression for the amount of money she receives.

$S = $ lamb price.

$50(s) - x$

$50s - xs$ ✗

$(50 - x)(20 + x)$ 10

1

Write answers in the spaces provided

1. Name a measure of spread that is not affected by outliers (extremely high or low scores).

1   Inter quartile. median

2. For these scores $5, 2, 12, 4, 11, 6, 9, 10$ use your calculator to find (to 1 dec. place)

a) the mean

b) the median

c) the standard deviation

3

a) 5.6 ✓

b) 5.5 ✓

c) 2.8 ✓

3. This *stacked bar graph* shows the numbers of students enrolled in different faculties in 1993 and in 2003.

University Enrolments by Faculty
Comparison 1993 with 2003

3

a) In which faculty have enrolments remained fairly constant? Medicine

b) In which faculty have enrolments declined? Education.

c) Why is this graph difficult to interpret? This graph is difficult to interpret because it is not detailed, and y-axis is not labelled.

4. The results for a Half Yearly Exam were:

English: Mean = 59, Standard deviation = 8

Maths: Mean = 65, Standard deviation = 6   4.

a) Ellen scored **75** in Maths and Paul scored **65** in English. State which is the better mark and <u>give a reason why</u>. *Ellen's score because it above the standard deviation, so she did much better compared to rest of the class.*

b) The English teacher added 5 to everyone's English mark to get a new set of marks.

   i) What is the new mean? .... *64.*   ✓

   ii) What is the new standard deviation? ... *Some — 8.* ✓

c) If a score of 66 was later added to the Maths results,

   i) the mean would [**increase**] / **decrease** (circle the correct answer)

   ii) the standard deviation would [**increase**] / **decrease** (circle the correct answer)

   ✗

End of Exam

10

11