

- show all necessary working

1. evaluate correct to 2 significant figures (in scientific notation)

$$\frac{(3.4 \times 10^8) + (6.15 \times 10^7)}{8.31 \times 10^3}$$

2. factorise $2x^3 + 1$

3. Simplify $\frac{a^2 - b^2}{ab + b^2} - \frac{a - b}{a + b}$

4. Express $0.14\bar{7}$ in the form $\frac{p}{q}$ where p and q are integers

5. Express $\frac{5}{3 - 2\sqrt{2}}$ in the form $a + \sqrt{b}$ where a, b are real.

6. Simplify, expressing your answer with positive indices:

$$\left(\frac{a^{-2}b}{a^3b^{-4}}\right)^{-3} \div \left(\frac{ab^{-1}}{a^{-3}b^2}\right)^5$$

7. Simplify $\frac{3 \cdot 2^n - 4 \cdot 2^{n-2}}{2^n - 2^{n-1}}$

8. Solve $3x^2 - x - 5 = 0$

9. Solve and graph the solution of $|3 - x| > 4$

10. Solve $\frac{4}{x-2} \geq 2$

11. Solve $|2x - 1| = 3x + 4$

12. Solve simultaneously for x and y

$$\begin{aligned} x^2 + y^2 - 2y - 4 &= 0 \\ x - y + 2 &= 0 \end{aligned}$$

13. Solve $|x+1| + |x+2| = 6$

14. Solve $\left|\frac{2x}{x-1}\right| > 4$

15. Solve $x^2 + 2x - 4 + \frac{3}{x^2 + 2x} = 0$

⑩ Solve and plot solutions on a number line :

a) $-5 < 1 - 2x \leq 4$

b) $\frac{3x}{x-2} \leq 1$

c) $\frac{x^2 - 3x}{x} \geq 0$

d) $|x-1| > 4$

⑪ Solve :

a) $|x-1| = 2x+1$

b) $|x+3| - |2-x| < 0$

c) $2 < \frac{4}{|x-1|}$

d) $\frac{|x+2|}{x+1} > 3$

⑫ Sketch : a) $|x| - |y| = 3$

b) $y = 1 + x - |x|$

c) $y = 4 - |x+1|$

SOLUTIONS - SYD TECH HS - 3 UNIT TEST.

1. $49000 = 4.9 \times 10^4$

2. $8x^3 + 1$
 $= (2x+1)(4x^2 - 2x + 1) \checkmark$

3. $\frac{a^2 - b^2}{ab + b^2} - \frac{a-b}{a+b}$
 $= \frac{(a-b)(a+b)}{b(a+b)} - \frac{b(a-b)}{b(a+b)}$
 $= \frac{(a-b)(a+b) - b(a-b)}{b(a+b)}$
 $= \frac{(a-b)(a+b-b)}{b(a+b)}$
 $= \frac{a(a-b)}{b(a+b)} \checkmark$

4. $0.\overline{147} = 0.1474747\dots$
let $x = 0.\overline{147}$
 $10x = 1.\overline{47}$
 $100x = 14.\overline{74}$
 $1000x = 147.\overline{47}\dots$
 $\therefore 990x = 146$
 $x = \frac{146}{990} \checkmark$
 $= \frac{73}{495}$

5. $\frac{5}{3-2\sqrt{2}} = \frac{5}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{5(3+2\sqrt{2})}{1} = 15 + 10\sqrt{2} \checkmark$

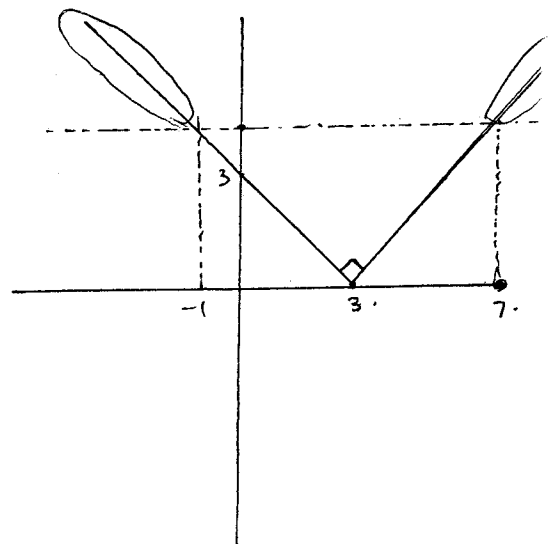
$$\begin{aligned}
 6. & \left(\frac{a^{-2}b}{a^3b^{-4}} \right)^{-3} \div \left(\frac{ab^{-1}}{a^{-3}b^2} \right)^5 \\
 & = (a^{-5}b^5)^{-3} \div (a^4b^{-3})^5 \checkmark \\
 & = (a^{15}b^{-15}) \div (a^{20}b^{-15}) \\
 & = \frac{a^{15}b^{-15}}{a^{20}b^{-15}} = a^{-5} = \frac{1}{a^5}
 \end{aligned}$$

$$\begin{aligned}
 7. & \frac{3 \cdot 2^n - 4 \cdot 2^{n-2}}{2^n - 2^{n-1}} \\
 & = \frac{3 \cdot 2^n - 4 \cdot 2^n \cdot 2^{-2}}{2^n - 2^n \cdot 2^{-1}} \\
 & = \frac{2^n(3 - 4 \cdot \frac{1}{4})}{2^n(1 - \frac{1}{2})} \checkmark \\
 & = \frac{(3-1)}{\frac{1}{2}} = \frac{2}{0.5} = 4. \checkmark
 \end{aligned}$$

$$\begin{aligned}
 8. & 3x^2 - x - 5 = 0. \\
 x & = \frac{1 \pm \sqrt{1 - 4 \cdot 3 \cdot -5}}{6} \\
 & = \frac{1 \pm \sqrt{61}}{6}
 \end{aligned}$$

$$x \doteq \begin{matrix} 1.47 \\ \text{or} \end{matrix} \doteq -1.14. \checkmark$$

$$\begin{aligned}
 9. & |3-x| > 4. \\
 & 3^2 + 2 \cdot 3x - x + x^2 > 16. \\
 & 9 - 6x + x^2 > 16. \\
 & \leftarrow x^2 - 6x + 9 - 16 > 0. \\
 & \leftarrow x^2 - 6x - 7 > 0. \\
 & \leftarrow (x-7)(x+1) > 0 \\
 & \leftarrow x > 7 \text{ or } x < -1. \checkmark \\
 & \leftarrow \begin{array}{c} + \quad - \quad + \\ \hline -1 \quad 7 \end{array}
 \end{aligned}$$



$$10. \frac{4}{x-2} \geq 2.$$

$$\Rightarrow 4(x-2) \geq 2(x-2)^2$$

$$\Rightarrow 2x-4 \geq x^2-4x+4.$$

$$\Rightarrow x^2-6x+8 \leq 0.$$

$$\Rightarrow (x-4)(x-2) \leq 0$$

$$\Rightarrow \begin{array}{c} + \quad - \quad + \\ \bullet \quad \bullet \\ 2 \quad 4 \end{array} \quad 2 < x \leq 4$$

$$11. |2x-1| = 3x+4$$

$$\Rightarrow 4x^2-4x+1 = 9x^2+24x+16$$

$$\Rightarrow 5x^2+28x+15=0.$$

$$\Rightarrow (5x+3)(x+5) = 0.$$

$$\Rightarrow x = -3/5 \text{ or } -5$$

try/check. sub $x = -3/5$ and $x = -5$.

$$|2(-3/5)-1| = 3(-3/5)+4 \quad \checkmark.$$

$$|2(-5)-1| = 3(-5)+4 \quad \checkmark.$$

\therefore both $x = -3/5$ and $x = -5$ are correct.

$$12 \text{ ① } x^2+y^2-2y-4=0.$$

$$\text{② } x-y+2=0.$$

from ② $y = x+2$ ③ \checkmark

sub ③ into ①.

$$\Rightarrow x^2 + (x+2)^2 - 2(x+2) - 4 = 0.$$

$$\Rightarrow x^2 + x^2 + 4x + 4 - 2x - 4 - 4 = 0.$$

$$\Rightarrow 2x^2 + 2x - 4 = 0. \Rightarrow x^2 + x - 2 = 0$$

$$\Rightarrow (2x-4)(x+1) = 0. \quad (x+2)(x-1) = 0$$

$$\therefore x=2 \text{ or } x=-1.$$

$$\text{if } x=2 \quad y=4.$$

$$\text{if } x=-1 \quad y=1$$

only solution

$$x=2, y=1.$$

$$\therefore x = -2 \text{ or } 1$$

$$y = 0 \text{ or } 3$$

$$13. |x+1| + |x+2| = 6.$$

$$\Rightarrow x+1 + x+2 = 6.$$

$$\Rightarrow 2x+3=6.$$

$$\Rightarrow 2x=3.$$

$$\Rightarrow x = \frac{3}{2} \quad \checkmark$$

$$\therefore x = \frac{3}{2} \text{ or } x = -\frac{9}{2} \quad \checkmark$$

$$+(x+1) - (x+2) = 6$$

$$\text{or } -x-1 -x-2 = 6.$$

$$\Rightarrow -2x-3=6.$$

$$\Rightarrow -2x=9.$$

$$\Rightarrow x = -\frac{9}{2}$$

$$-(x+1) + (x+2) =$$

$$14. \frac{4x^2}{(x-1)^2} > 16.$$

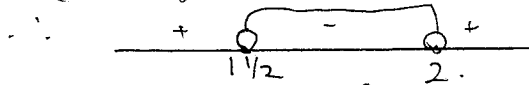
$$\Rightarrow 4x^2 > 16(x-1)^2$$

$$\Rightarrow x^2 > 4(x^2 - 2x + 1)$$

$$\Rightarrow x^2 > 4x^2 - 8x + 4 \quad \checkmark$$

$$\Rightarrow 3x^2 - 8x + 4 < 0.$$

$$\Rightarrow (3x-2)(x-2) < 0$$



$$\therefore x: \frac{2}{3} < x < 2 \quad \checkmark$$

$$15. x^2 + 2x - 4 + \frac{3}{x^2 + 2x} = 0. \quad \text{let } u = x^2 + 2x.$$

$$= u - 4 + \frac{3}{u} = 0.$$

$$\Rightarrow u^2 - 4u + 3 = 0.$$

$$\Rightarrow (u-3)(u-1) = 0.$$

$$\therefore u = 3 \text{ or } 1.$$

$$\therefore x^2 + 2x = 3.$$

$$\Rightarrow x^2 + 2x - 3 = 0.$$

$$(x+3)(x-1) = 0$$

$$\therefore x = -3 \text{ or } 1.$$

$$\text{or } x^2 + 2x = 1.$$

$$x^2 + 2x - 1 = 0.$$

$$= \frac{-2 \pm \sqrt{4 - 4 \times (-1)}}{2}$$

$$= \frac{-2 \pm \sqrt{8}}{2}$$

$$= \frac{2(-1 \pm \sqrt{2})}{2}$$

$$= -1 \pm \sqrt{2}$$

$$= \sqrt{2} - 1 \text{ or } -\sqrt{2} - 1.$$

$$16a) -5 < 1-2x \leq 4.$$

$$-6 < -2x \leq 3.$$

$$3 > x \geq -\frac{3}{2} \quad \checkmark$$

$$\Rightarrow -\frac{3}{2} \leq x < 3.$$



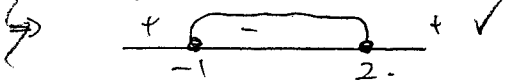
$$b) \frac{3x}{x-2} \leq 1$$

$$\Leftrightarrow 3x(x-2) \leq (x-2)^2$$

$$\Leftrightarrow 3x^2 - 6x \leq x^2 - 4x + 4.$$

$$\Leftrightarrow 2x^2 - 2x - 4 \leq 0.$$

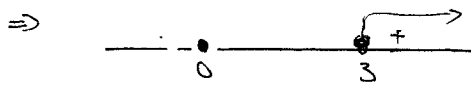
$$\Leftrightarrow (2x+2)(x-2) \leq 0.$$



$$c) \frac{x^2-3x}{x} \geq 0.$$

$$\Rightarrow x(x^2-3x) \geq 0. \Rightarrow \text{times both sides by } x^2$$

$$\Rightarrow x^2(x-3) \geq 0.$$



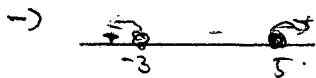
$$x \neq 0; \quad x \geq 3. \quad \checkmark$$

$$d) |x-1| \geq 4$$

$$\Rightarrow x^2 - 2x + 1 \geq 16$$

$$\Rightarrow x^2 - 2x - 15 \geq 0.$$

$$\Rightarrow (x-5)(x+3) \geq 0.$$



$$\Rightarrow x \leq -3 \quad \text{or} \quad x \geq 5. \quad \checkmark$$

$$(17) (d) \frac{|x+2|}{x+1} > 3.$$

$$\frac{x+1}{|x+2|} < \frac{1}{3}.$$

$$\frac{1}{3}|x+2| > x+1.$$

$$|x+2| > 3x+3.$$

$$x+2 > 3x+3.$$

$$\Rightarrow -2x - 1 < 0.$$

$$-2x < 1.$$

$$x < -\frac{1}{2}.$$

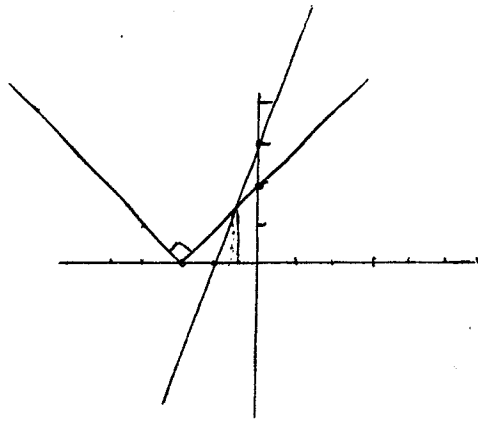
or.

$$-x-2 > 3x+3.$$

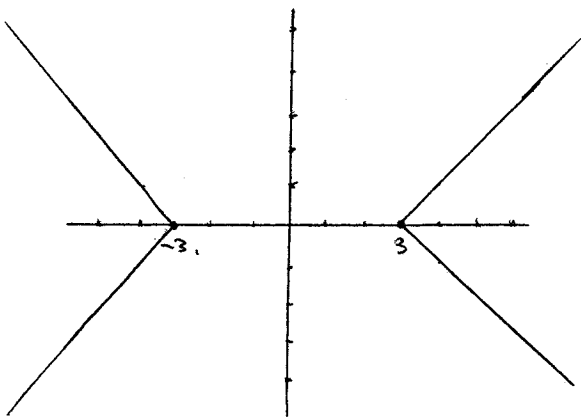
$$\Rightarrow 4x+5 < 0.$$

$$\Rightarrow 4x < -5.$$

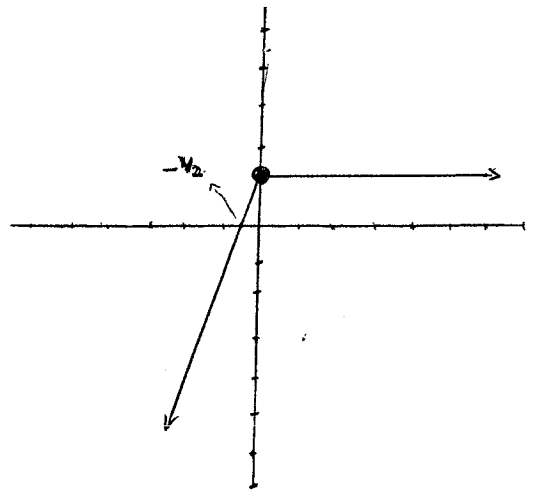
$$\Rightarrow x < -\frac{5}{4}.$$



$$18. a) |x| - |y| = 3.$$



$$(b) y = 1 + x - |x|.$$



$$c). (-1, 4)$$

