# SYDNEY TECHNICAL HIGH SCHOOL

## MATHEMATICS EXTENSION 1

### YEAR 11 COMMON TEST

### MAY 2005

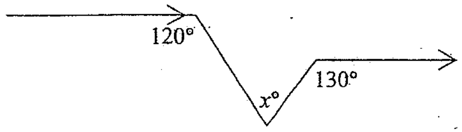
**Time allowed:** 70 minutes

**Instructions:**

- Show all necessary working in every question
- Start each question on a new page
- Attempt all questions
- All questions are not of equal value
- Marks shown are approximate & may be changed
- Full marks may not be awarded for careless or badly arranged work
- Your sketches must be neat. Use a ruler to draw axes.
- Approved calculators may be used
- These questions are to be handed in with your answers.

**Question 1** **Marks**

a)




Find the value of $x$ (no reasons necessary). 1

b) Factorise $a^2 - b^2 - (a-b)^2$ 2

c) The hyperbola $y = \frac{3}{a-x}$ has a vertical asymptote at $x = 1$. What is the value of $a$? 1

d) If $\tan a = -\frac{1}{3}$ and $\cos a > 0$, find the exact value of $\sin a$. 2

e) Given that $n$ is a positive number indicate 2

(i) the largest

(ii) the smallest

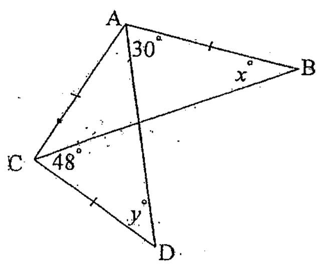
of the following numbers:

$3^{\frac{-n}{3}},\ 3^{\frac{n}{3}},\ 3^n,\ 3^{-n}$

f) Solve $|5x-3| = |3x+1|$ 2

**Question 2 (start a new page)**

a)




$\Delta ABC$ and $\Delta ACD$ are isosceles. 3

By forming a pair of simultaneous equations or otherwise, find the value of $x$.

b) If $\frac{a^n + a^{n+2}}{a^n} = 10$ find $a$ given that $a > 0$. 2

c) Solve $\frac{x-2}{x} \geq 1$ 3

d) There are two values of $\theta$ in the domain $0° \leq \theta \leq 360°$ where $\sin\theta$ and $\cos\theta$ are numerically equal. Find these two values. 2

**Question 3 (start a new page)**

a) If $p = \frac{\sqrt{3}}{4-\sqrt{3}}$ and $q = \frac{\sqrt{3}}{4+\sqrt{3}}$ evaluate $\frac{p+q}{1-pq}$ 3

b) How many solutions does the equation $(\cos x - 2)(\sin^2 x - 1) = 0$ have in the domain $0° \leq x \leq 360°$? 3

There is no need to solve the equation.

Justify your answer.

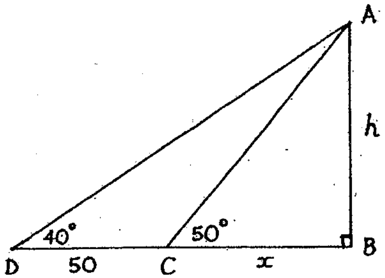
c) i) Sketch the graph of $y = x^2 + \frac{1}{2}$ 4

ii) On a separate diagram sketch $y = \frac{2}{2x^2+1}$.

iii) Use your diagram or otherwise to write down the range of $y = \frac{2}{2x^2+1}$

**Question 4 (start a new page)**

a)




We wish to find the height AB of a vertical cliff. From a point D the angle of elevation of A is $40°$. From a point C 50m nearer the base of the cliff the angle of elevation is $50°$. 4

i) Show that $h = (50 + x) \tan 40°$

ii) Show that $h = x \tan 50°$

iii) Using simultaneous equations find $h$

b) i) Sketch the graph of $y = |x+1|$. 4

ii) By using your graph or otherwise solve $\frac{2}{x} > |x+1|$

**Question 5 (start a new page).**

a) i) Sketch the graph of $y = \cos x$ for $0° \le x \le 360°$ 4

ii) Hence solve $-\frac{\sqrt{3}}{2} \le \cos x \le \frac{\sqrt{3}}{2}$ for $0° \le x \le 360°$

b) i) If $xy = c^2$ prove that $\frac{1}{c+x} + \frac{1}{c+y} = \frac{1}{c}$. 4

ii) Hence or otherwise simplify $\frac{1}{6+\sqrt{51}+\sqrt{15}} + \frac{1}{6+\sqrt{51}-\sqrt{15}}$

**Question 6 (start a new page)**

a) Find the point/s of intersection for the graphs of $y = x^2 - 1$ and $y = \frac{1}{x^2-1}$ 3

b) i) Sketch the graph of $y = -\sqrt{2-x^2}$ 5

ii) On the same diagram shade the region where $y \ge -\sqrt{2-x^2}$, $|x| \le 1$ and $y \le 0$ hold simultaneously.

iii) Find the exact value for the area of the shaded region.

## Question 1

a) $x = 70$

b) $a^2 - b^2 - (a-b)^2$

$(a-b)(a+b) - (a-b)^2$

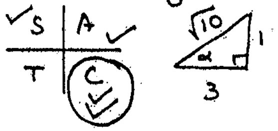
$(a-b)((a+b) - (a-b))$

$2b(a-b)$

c) $y = \frac{3}{a-x}$ $\quad a - x = 0$

$\therefore a = 1$

d) $\tan\alpha = -\frac{1}{3}$ $\quad \cos\alpha > 0$




$\therefore \sin\alpha = -\frac{1}{\sqrt{10}}$

e) $3^{-\frac{n}{3}}$, $3^{\frac{n}{3}}$, $3^n$, $3^{-n}$

i) largest $3^n$

ii) smallest $3^{-n}$

} n a +ve integer

f) $|5x-3| = |3x+1|$

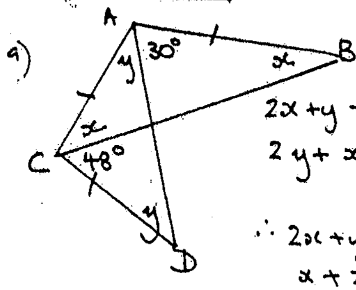
$5x - 3 = 3x + 1$ or $5x - 3 = -3x - 1$

$2x = 4$ $\quad 8x = 2$

$x = 2$ or $x = \frac{1}{4}$

## Question 2

a)




$2x + y + 30 = 180$

$2y + x + 48 = 180$

$\therefore 2x + y = 150$ — ①

$x + 2y = 132$ — ②

$2x + 4y = 264$

$\therefore 3y = 114$ $\quad y = 38°$, $x = 56°$

b) $\frac{a^n + a^{n+2}}{a^n} = 10$

$\frac{a^n(1 + a^2)}{a^n} = 10$

$\therefore a^2 = 9$

$a = 3$ only $a > 0$

c) $\frac{x-2}{x} \geq 1$

$x(x-2) \geq x^2$

$x(x-2) - x^2 \geq 0$

$x(x - 2 - x) \geq 0$

$-2x \geq 0$

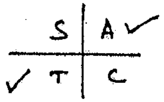
$x \leq 0$ but $x \neq 0$

$\therefore x < 0$

d) $\sin\theta = \cos\theta$

$\tan\theta = 1$

acute $\theta = 45$




$\therefore \theta = 45°, 225°$

## Question 3

a) $p = \frac{\sqrt{3}}{4-\sqrt{3}} \times \frac{4+\sqrt{3}}{4+\sqrt{3}} = \frac{4\sqrt{3}+3}{13}$

$q = \frac{\sqrt{3}}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{4\sqrt{3}-3}{13}$

$p + q = \frac{4\sqrt{3} + 3 + 4\sqrt{3} - 3}{13}$

$= \frac{8\sqrt{3}}{13}$

$1 - pq = 1 - \frac{\sqrt{3}}{4-\sqrt{3}} \times \frac{\sqrt{3}}{4+\sqrt{3}}$

$= 1 - \frac{3}{13}$

$= 10/13$

$\frac{p+q}{1-pq} = \frac{8\sqrt{3}}{13} \div \frac{10}{13}$

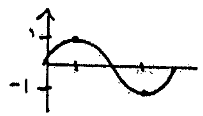
$= \frac{8\sqrt{3}}{10}$

$= \frac{4\sqrt{3}}{5}$

b) $(\cos x - 2)(\sin^2 x - 1) = 0$

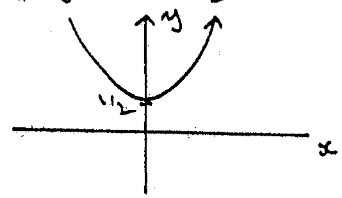
$\cos x = 2$ no solutions, $\quad \sin^2 x = 1$

$\sin x = \pm 1$

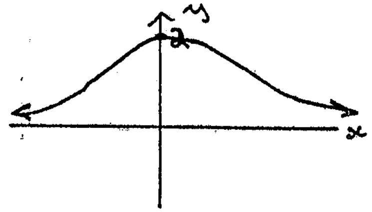



2 solutions

c) i) $y = x^2 + \frac{1}{2}$



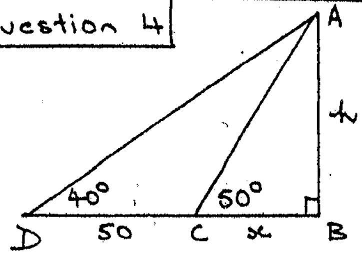

ii) $y = \frac{2}{2x^2 + 1}$




iii) Range $0 < y \leq 2$

## Question 4

a)




ii) $\tan 50° = \frac{h}{x}$

$\therefore h = x\tan 50$

i) $\tan 40 = \frac{h}{50 + x}$

$(50 + x)\tan 40° = h$

iii) $x\tan 50° = (50 + x)\tan 40°$

$x\tan 50° = 50\tan 40° + x\tan 4$

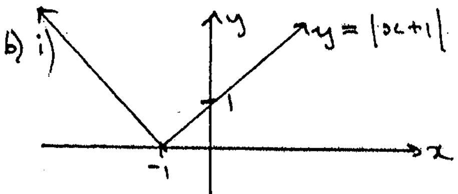
$x\tan 50° - x\tan 40° = 50\tan 40°$

$x(\tan 50 - \tan 40) = 50\tan 40°$

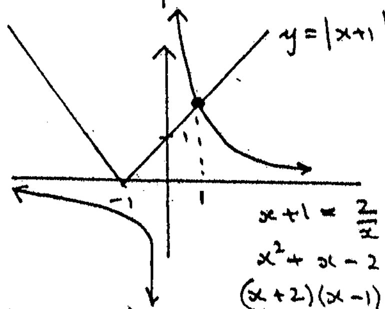
$x = \frac{50\tan 40°}{\tan 50 - \tan 40}$

$x = 118.97$ units (2 dec pl)

b) i)




ii)




$x + 1 = \frac{2}{x}$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = 1$ only abov

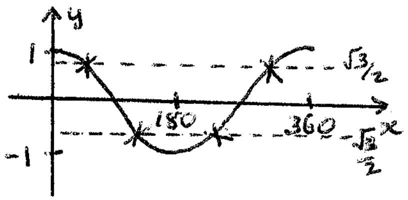
$\frac{2}{x} > |x+1|$

$\therefore$ Solution : $0 < x < 1$

## QUESTION 5

(a)

i)




ii) Solve $\cos x = \pm \sqrt{3}/2$

$x = 30°, 150°, 210°, 330°$

$\therefore \{x : 30° < x < 150°\} \cup \{x : 210° < x < 330°\}$

(b)

i) $LHS = \frac{1}{c+x} + \frac{1}{c+y}$

$= \frac{c+y+c+x}{c^2+cx+cy+xy}$ ← subs $xy = c^2$

$= \frac{2c+x+y}{2c^2+cx+cy}$

$= \frac{2c+x+y}{c(2c+x+y)} = \frac{1}{c} = RHS$.

ii) Let $c = 6$, $x = \sqrt{51}+\sqrt{15}$, $y = \sqrt{51}-\sqrt{15}$

Note $c^2 = 36$ & $xy = (\sqrt{51}+\sqrt{15})(\sqrt{51}-\sqrt{15})$
$= 51 - 15 = 36$

$\therefore c^2 = xy$

$\therefore$ Ans: $\frac{1}{6}$

## QUESTION 6

(a) $\therefore x^2 - 1 = \frac{1}{x^2-1} \quad (x \neq \pm 1)$

$\therefore (x^2-1)^2 = 1$

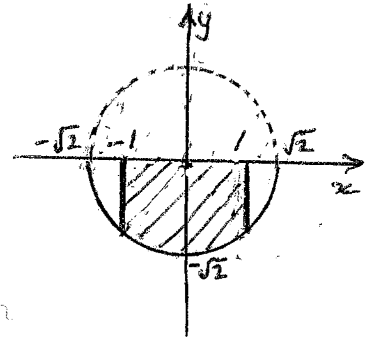
$\therefore x^2 - 1 = 1$ or $x^2 - 1 = -1$

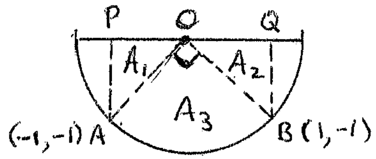
$\therefore x^2 = 2$ or $x^2 = 0$

$\therefore x = \pm\sqrt{2}$ or $x = 0$

(b)

i)




ii)

$A = A_1 + A_2 + A_3$ (Note $\angle AOB = 90° = \pi/2$)

$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2}(\sqrt{2})^2 \times \frac{\pi}{2}$

$= 1 + \frac{\pi}{2}$ units$^2$