Name:_____ Maths Class:_____

SYDNEY TECHNICAL HIGH SCHOOL

Year 11

MATHEMATICS

Term 3 Examination

September 2003

Time allowed: 2 hours

Instructions:

- Write your name and class at the top of this page.
- At the end of the examination this examination paper must be attached to the front of your answers
- All questions are of equal value and may be attempted
- All necessary working must be shown. Marks will be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Total
10	10	10	9	10	9	10	10	78/80

Question 1 (10 marks)

a) Evaluate $|-2|-|-4|$ (1)

b) Write $49^{-3/2}$ as a simple fraction (1)

c) Find cot 102°13' correct to 3 decimal places. (2)

d) Factorise fully $x^4 - 4x^2$ (2)

e) Solve $5 = \frac{2}{5}(w+4)$ (2)

f) Find the values of a and b if $(2+\sqrt{3})^2 = a+\sqrt{b}$ (2)

Question 2 (10 marks)

a) For $f(x) = \frac{2}{x+1}$

i) Write down the domain of the function (1)

ii) Find $f(\frac{1}{a})$ as a simple fraction (2)

b) Solve $x^2 = 2x$ (1)

c) Solve $\sin\theta = \frac{-\sqrt{3}}{2}$ for $0° \leqslant \theta \leqslant 360°$ (2)

d) i) Simplify $(2x+h)^2 - 4x^2$ (1)

ii) Hence evaluate $\lim_{h \to 0} \frac{(2x+h)^2 - 4x^2}{h}$ (1)

e) Solve $\frac{|x|}{2} < 1$ (2)

Question 3 (10 marks)

a) Differentiate the following

i) $y = \frac{1}{3}x^4 + k$ (1)

ii) $y = \frac{4x^3 + x^4}{x^2}$ (2)

b) The points (2, 7) and (-4, -5) are the end points of the diameter of a circle

i) Find the coordinates of the centre of the circle (1)

ii) Find the length of the radius (2)

c) If $x^2+2x+m=0$ has roots α *and* β

i) Without finding the roots, find the value of

I) $\alpha+\beta$ (1)

II) $\alpha\beta$ (1)

ii) If $\beta=2\alpha$. Find the value of m (2)

Question 4 (10 marks)

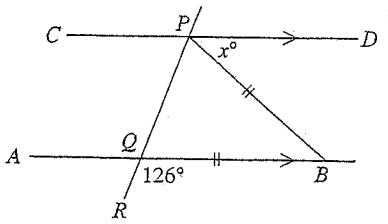
a) Find the equation of the tangent to the curve $y=3x^2+x$ *at* $x=1$ (2)

b) i) Explain why the lines $y=2x-1$ *and* $6x-3y+5=0$ are parallel. (2)

ii) If $(a,5)$ lies on $y=2x-1$, find the value of a (1)

iii) Hence find the distance between the parallel lines in part (i) (2)

c)



NOT TO SCALE

In the diagram CD is parallel to AB and PB = QB.

i) Find the size of $< PQB$ in degrees (1)

ii) Find the value of x giving reasons (2)

Question 5 (10 marks)

a) i) Write $\sqrt[3]{x}$ in index form (1)

ii) If $f(x)=\sqrt[3]{x}$. Find $f'(8)$ as a fraction (2)

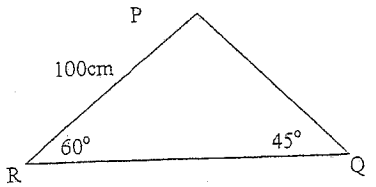
b) For the parabola $y=x^2-6x+4$

i) Find the coordinates of the vertex (2)

ii) Sketch the parabola showing the vertex and y – intercept (1)

iii) Use your graph or otherwise determine the ~~smallest~~ values of k so that x^2-6x+k is positive for all values of x (2)

c) In ΔPQR (2)

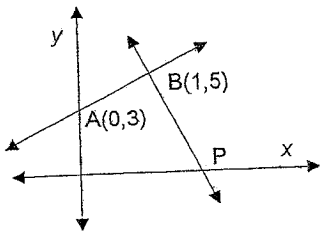


Show that the length of PQ *is* $50\sqrt{6}cm$

Question 6 (10 marks)

a) If $y=(x^2-5)^5$. Find $\frac{dy}{dx}$ (2)

b)



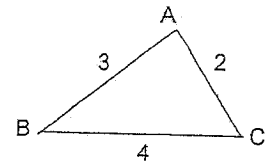
i) Find gradient of line AB (1)

ii) If PB is perpendicular to AB find the equation of PB in general form. (3)

iii) Find the coordinates of P (1)

iv) If ABPQ form a rectangle find the coordinates of Q (1)

c)



i) Use the cosine rule to find the exact value of $\cos A$ (2)

ii) Hence find the exact value of $\sin A$ (1)

Question 7 (10 marks)

a) i) Write down the discriminant of $x^2 + px + (p+3)$ (1)

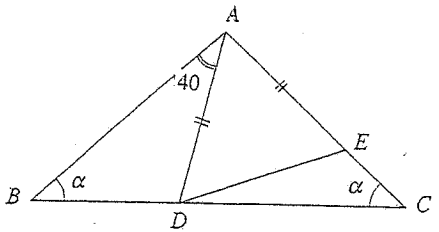
ii) If the equation $x^2 + px + (p+3) = 0$ has equal roots find the values of p. (2)

b) If $y = \dfrac{x}{x^2+1}$

i) Find $\dfrac{dy}{dx}$ (2)

ii) Find the x values of the points where $\dfrac{dy}{dx} = 0$ (1)

c)



In the isosceles triangle ABC $< ABC = < ACB = \alpha$ AD = AE

i) Explain why $< ADC = \alpha + 40$ (1)

ii) Find $< DAC$ *in terms of* α (1)

iii) Hence or otherwise find $< EDC$ giving reasons (2)

Question 8 (10 marks)

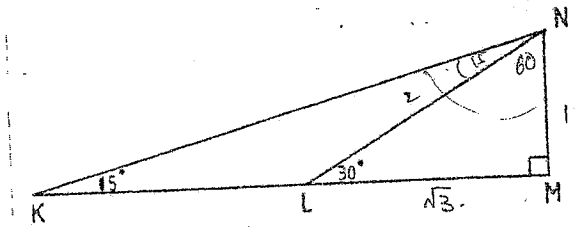
a) i) Sketch the curve $y = \dfrac{8}{x}$ (1)

ii) Find $\dfrac{dy}{dx}$ (1)

iii) Find the equation of the normal to $y = \dfrac{8}{x}$ *at* (4,2) (2)

iv) The normal cuts the curve again at P. Find the coordinates of P. (2)

b)



i) Explain why $KL = LN$ (1)

ii) If $NM = 1$ deduce that $\tan 15° = 2 - \sqrt{3}$ (3).

SOLUTIONS

Teacher's Name: Parrish **Student's Name/N°:** Paul Shieh

Question 1.

a) $2-4=-2$. ✓

b) $49^{-\frac{3}{2}}$

$= \frac{1}{49^{\frac{3}{2}}}$

$= \frac{1}{7^3} = \frac{1}{343}$. ✓

c) $\cot 102°13'$

$= \frac{1}{\tan 102°13'} = -0.217$ ✓

d) $x^4 - 4x^2$

$= x^2(x^2-4)$

$= x^2(x-2)(x+2)$ ✓

e) $5 = \frac{2}{5}(w+4)$

$25 = 2(w+4)$.

$25 = 2w+8$.

$2w = 25-8$

$= 17$ ✓

$w = \frac{17}{2}$

f) $(2+\sqrt{3})^2$

$= 4 + 4\sqrt{3} + 3$

$= 7 + 4\sqrt{3}$.

$= 7 + \sqrt{48}$ ✓

$\therefore a=7 \quad b=48$

10

Teacher's Name: Parrish **Student's Name/N°:** Paul Shieh

Question 2.

i) domain: all real x but $x \neq -1$ ✓

ii) $f(\frac{1}{a}) = \frac{2}{\frac{1}{a}+1} \times \frac{a}{a}$

$= \frac{2a}{a+1}$ ✓

b) $x^2 - 2x = 0$.

$x(x-2) = 0$.

$\therefore x=0$ or $x=2$ ✓

c) $\sin\theta = \frac{-\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$ if $\theta < 60°$ (acute angle)

but -ve in 3rd and 4th

$\therefore \theta = 240°, 300°$ ✓

d i) $(2x+h)^2 - 4x^2$

$= 4x^2 + 4xh + h^2 - 4x^2$

$= 4xh + h^2$

$= h(4x+h)$ ✓

ii) $\lim_{h\to 0} \frac{(2x+h)^2 - 4x^2}{h}$

$= \lim_{h\to 0} \frac{h(4x+h)}{h}$

$\therefore = 4x$ ✓

10

e) $\frac{|x|}{2} < 1$

$\therefore |x| < 2$

$\therefore x < 2$ or $-x < 2$.

$\therefore x < 2$ or $x > -2$

$-2 < x < 2$ ✓

Teacher's Name: Parrish Student's Name/N°: Paul Mich

Question 3.

i) $y = \frac{1}{3}x^4 + k$

$y' = \frac{4}{3}x^3$ ✓ (1)

ii) $y = \frac{4x^3 + x^4}{x^2}$

$= \frac{x^2(4x + x^2)}{x^2}$

$= 4x + x^2$ ✓

$y' = 2x + 4$. ✓ (2)

b) i) centre = midpoint.

$\therefore x = \frac{-4+2}{2} = \frac{-2}{2} = -1$.

$y = \frac{-5+7}{2} = \frac{2}{2} = 1$.

$\therefore$ centre of circle = $(-1, 1)$ ✓ (1)

ii) length $= \sqrt{(-1-2)^2 + (1-7)^2}$

$= \sqrt{(-3)^2 + (-6)^2}$

$= \sqrt{9+36}$ ✓

$= \sqrt{45}$ (2)

c) i) I) $\alpha + \beta = \frac{-b}{a} = \frac{-2}{1} = -2$ ✓

II) $\alpha\beta = \frac{c}{a} = m$ ✓ (2)

ii) $\beta = 2\alpha$

$\therefore \alpha + 2\alpha = -2$.

$3\alpha = -2$

$\alpha = \frac{-2}{3}$

$\alpha\beta = m$.

$2\alpha^2 = m$.

$m = 2\left(\frac{-2}{3}\right)^2$ ✓

$= 2\left(\frac{4}{9}\right)$

$= \frac{8}{9}$ (2)

10

10

Teacher's Name: Parrish Student's Name/N°: Paul Mich

Question 4.

a) $y = 3x^2 + x$

$y' = 6x + 1$

at $x = 1$, m of tangent $= 7$ ✓

$\therefore$ at $x = 1$, $y = 3(1) + 1 = 4$

tangent: $(y-4) = 7(x-1)$

$y - 4 = 7x - 7$

$7x - y - 3 = 0$ ✓ 2

b i) $y = 2x - 1$; $3y = 6x + 5$; $y = 2x + \frac{5}{2}$

$\rightarrow$ m of 2 ✓ $\rightarrow$ m of 2. ✓

both have same gradients thus lines are parallel. 2

ii) $y = 2x - 1$

$5 = 2a - 1$

$2a = 6$

$a = 3$ ✓ 1

iii) distance $= \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$= \left| \frac{6(3) + -3(5) + 5}{\sqrt{6^2 + 3^2}} \right|$

$= \left| \frac{18 - 15 + 5}{\sqrt{45}} \right|$

$= \frac{8}{\sqrt{45}}$ ✓ 2

(9)

c) i) $\angle PQB = 180 - 126 = 54°$ ✓ 1

ii) $\angle QPB = 54°$ (base $\angle$ of isosceles $\triangle$)

$\angle DPQ = 180 - 54 = 126$ (co-interior) ✓ not sufficient.

$\therefore x = 126 - 54 = 72°$ ✓ angle on parallel lines

Teacher's Name: Parrish Student's Name/N°: Paul Shieh

Question 5.

a i) $\sqrt[3]{x} = x^{\frac{1}{3}}$ 1

ii) $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$\therefore f'(8) = \frac{1}{3}(8)^{-\frac{2}{3}}$

$= \frac{1}{3} \times \frac{1}{8^{\frac{2}{3}}}$

$= \frac{1}{3} \times \frac{1}{4}$

$= \frac{1}{12}$ 2.

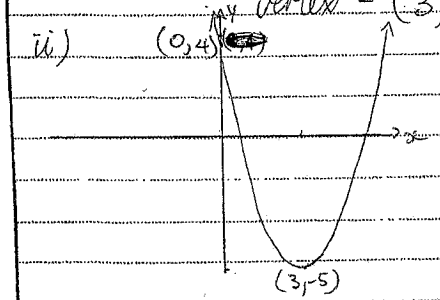
b) axis of symmetry $= \frac{-b}{2a} = \frac{6}{2} = 3$

$\therefore y = (3)^2 - 6(3) + 4$

$= 9 - 18 + 4 = -5$.

$\therefore$ vertex $= (3, -5)$ 2

ii)



1

iii) ~~for~~ for +ve for all x $\Delta < 0$

$\therefore \Delta = (6)^2 - 4 \times 1 \times k$

$\therefore 36 - 4k < 0$

$4k > 36$

$k > 9$ 2

c) $\frac{100}{\sin 45} = \frac{x}{\sin 60}$

$x = \frac{100 \times \sin 60}{\sin 45}$

$= \frac{100 \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}} = 100 \times \frac{\sqrt{3}}{2} \div \frac{1}{\sqrt{2}}$

$= 100 \times \frac{\sqrt{3}}{2} \times \sqrt{2}$

$= \frac{100\sqrt{6}}{2} = 50\sqrt{6}$ 2

10

Teacher's Name: Parrish Student's Name/N°: Paul Shieh

Question 6

a) $y = (x^2 - 5)^5$

$y' = 5(x^2-5)^4 \times 2x$

$= 10x(x^2-5)^4$ ✓✓

b) i) $m = \frac{5-3}{1-0} = \frac{2}{1} = 2$ ✓

$\therefore$ line $= (y-3) = 2(x-0)$

$y - 3 = 2x$.

$2x - y + 3 = 0$. ← why?

9

ii) if PB is perpendicular, then m of PB $= -\frac{1}{2}$

$\therefore (y-5) = \frac{-1}{2}(x-1)$

$2(y-5) = -(x-1)$

$2y - 10 = -x + 1$

$x + 2y - 11 = 0$ ✓✓✓

iii) P is when $y = 0$.

$x = 11$ $\therefore P(11, 0)$ ✓

iv) rectangle, corners joined must have same midpoint.

AP's midpoint $= \left(\frac{11}{2}, \frac{3}{2}\right)$

$\therefore$ BP midpoint $= \left(\frac{11}{2}, \frac{3}{2}\right)$

$\therefore \frac{11}{2} = \frac{1+x}{2}$ $\frac{3}{2} = \frac{5+y}{2}$.

$2(1+x) = 22$ $2(5+y) = 6$

$2 + 2x = 22$ $10 + 2y = 6$.

$2x = 20$ $2y = -4$

$x = 10$ $y = -2$.

$\therefore$ Q has the co-ordinate $(10, -2)$ ✓

c) i) $\cos A = \frac{3^2 + 2^2 - 4^2}{2 \times 3 \times 2} = \frac{9 + 4 - 16}{12} = \frac{-3}{12} = \frac{-1}{4}$.

$\cos A = \frac{-1}{4}$ ✓

ii) $\sin^2 A + \cos^2 A = 1$.

$\sin^2 A = 1 - \cos^2 A$.

$= 1 - \left(\frac{1}{16}\right)$

$\sin^2 A = \frac{15}{16}$

$\therefore \sin A = +ve$.

and so $\sin A = \frac{\sqrt{15}}{4}$

Teacher's Name: Parrish Student's Name/N°: Paul Stiehl

Question 7

a i) $\Delta = b^2 - 4ac$
$= p^2 - 4\times1\times(p+3)$
$= p^2 - 4p + 12$ ✓ 1

ii) if has equal roots, $\Delta = 0$.
$\therefore p^2 - 4p + 12 = 0$.
$(p-6)(p+2) = 0$.
$p = 6$ or -2 ✓ 2

b) i) $y = \frac{x}{x^2+1}$ $y' = \frac{vu' - uv'}{v^2}$

$y' = \frac{1(x^2+1) - x(2x)}{(x^2+1)^2}$
$= \frac{x^2+1-2x^2}{(x^2+1)^2}$
$= \frac{1-x^2}{(x^2+1)^2}$ ✓ ✓

ii) $\frac{1-x^2}{(x^2+1)^2} = 0$.
$1 - x^2 = 0$.
$(1-x)(1+x) = 0$. ✓ 1
$x = 1$ or -1

c) i) $\angle BDA = 180 - (\alpha+40)$
$\angle ADC = 180 - (180-(\alpha+40))$ (∠ sum of Δ)
$= 180 - 180 + (\alpha+40)$ (∠ supplementary ∠) 1
$= \alpha + 40$. ✓

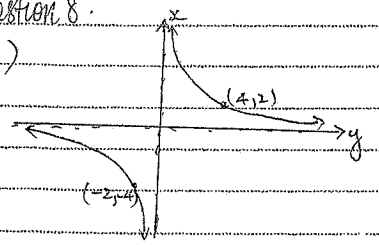
ii) $\angle BAC = 180 - 2\alpha$ (angle of isos. Δ)
$\angle DAC = 180 - 2\alpha - 40$
$= 140 - 2\alpha$ ✓ 1

iii) $\Delta EDA = \frac{(180-(140-2\alpha))}{2} = \frac{180-140+2\alpha}{2} = \frac{40+2\alpha}{2} = 20+\alpha$ (base ∠ isos. Δ)
$\Delta EDC = \angle ADC - \angle ADE = (\alpha+40) - (20+\alpha)$ ✓ ✓
$= 20°$

Teacher's Name: Parrish Student's Name/N°: Paul Stiehl

Question 8.

a i)



✓

ii) $y = 8x^{-1}$
$y' = -8x^{-2}$ ✓

iii) ~~equation~~ m of tangent at $x=4$
$= -8(x)^{-2}$
$= -8(4)^{-2} = -8 \times \frac{1}{16} = -\frac{1}{2}$
$\therefore$ ~~eq~~ m of the normal at $x=4 = 2$.
$\therefore$ equation of the normal $= (y-2) = 2(x-4)$
$y - 2 = 2x - 8$.
$2x - y - 6 = 0$ ✓

iv) $y = \frac{8}{x}$ ... ①.
$2x - y - 6$ ... ②.
$2x - \frac{8}{x} - 6 = 0$.
$2x^2 - 6x - 8 = 0$.
$(2x-8)(x+1)$
$\therefore$ normal cuts at $x=4$ and $x=-1$. ✓ ✓
$\therefore P = (-1, -8)$

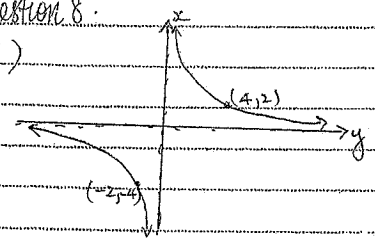
b) i) $\angle KLN = 180 - 30 = 150$. (supp. ∠).
$\angle LNK = 180 - 15 - 30 = 15°$ (∠ sum of Δ) ✓
$\therefore \Delta KLN$ is isosceles
$\therefore KL = LN$ (sides of isosceles Δ).

ii) $\tan 15 = \frac{1}{KL+LH}$. $LH = \sqrt{3}$ because $\tan 30 = \frac{1}{\sqrt{3}}$
$LN = 2$. because $\sin 30 = \frac{1}{2}$.
$\tan 15 = \frac{1}{KL+\sqrt{3}}$ $\therefore KL = 2$ (side of isos. Δ). ✓
$\therefore \tan 15 = \frac{1}{2+\sqrt{3}}$.
$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$.

Teacher's Name: Parrish Student's Name/N°: Paul Stitch

Question 8.

a i)



✓

ii) $y = 8x^{-1}$

$y' = -8x^{-2}$ ✓

iii) ~~equation~~ m of tangent at $x=4$

$= -8(x)^{-2}$

$= -8(4)^{-2} = -8 \times \frac{1}{16} = -\frac{1}{2}$

∴ m of the normal at $x=4 = 2$. ✓

∴ equation of the normal $= (y-2) = 2(x-4)$

$y-2 = 2x-8$.

$2x-y-6=0$ ✓

iv) $y = \frac{8}{x}$ ... ①.

$2x-y-6$ ... ②.

$2x - \frac{8}{x} - 6 = 0$.

$2x^2 - 6x - 8 = 0$.

$(2x-8)(x+1)$

∴ normal cuts at $x=4$ and $x=-1$. ✓ ✓

∴ $P = (-1, -8)$

b) i) $\angle KLN = 180-30 = 150$. (supp. $\angle$).

$\angle LNK = 180-15-30 = 15^\circ$ ($\angle$ sum of Δ) ✓

∴ ΔKLN is isosceles

∴ $KL =$ ~~$\angle$~~ LN (sides of isosceles Δ).

ii) $\tan 15 = \frac{1}{KL+LM}$. $LM = \sqrt{3}$ ~~$\tan$~~ because $\tan 30 = \frac{1}{\sqrt{3}}$.

$\tan 15 = \frac{1}{KL+\sqrt{3}}$ $LN = 2$. because $\sin 30 = \frac{1}{2}$.

∴ $\tan 15 = \frac{1}{2+\sqrt{3}}$. ∴ $KL = 2$ (side of isos. Δ). ✓

$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$.

Teacher's Name: Parrish Student's Name/N°: Paul Stitch

Question 7

a i) $\Delta = b^2 - 4ac$

$= p^2 - 4 \times 1 \times (p+3)$

$= p^2 - 4p + 12$ ✓ |

ii) if has equal roots, $\Delta = 0$.

∴ $p^2 - 4p + 12 = 0$.

$(p-6)(p+2) = 0$. ✓ 2 ✓

$p = 6$ or -2

b) i) $y = \frac{x}{x^2+1}$ $y' = \frac{vu' - uv'}{v^2}$

$y' = \frac{1(x^2+1) - x(2x)}{(x^2+1)^2}$

$= \frac{x^2+1-2x^2}{(x^2+1)^2}$

$= \frac{1-x^2}{(x^2+1)^2}$ ✓ ✓

ii) $\frac{1-x^2}{(x^2+1)^2} = 0$

$1-x^2 = 0$. ✓ |

$(1-x)(1+x) = 0$.

$x = 1$ or -1

c) i) $BDA = 180 - (\alpha+40)$

$\angle ADC = 180 - (180-(\alpha+40))$ ($\angle$ sum of Δ)

$= 180 - 180 + (\alpha+40)$ ($\angle$ supplementary $\angle$) ✓ |

$= \alpha + 40$.

ii) $\angle BAC = 180 - 2\alpha$ (angle of isos. Δ)

$\angle DAC = 180 - 2\alpha - 40$ ✓ |

$= 140 - 2\alpha$

iii) $\Delta EDA = \frac{(180-(140-2\alpha))}{2} = \frac{180-140+2\alpha}{2} = \frac{40+2\alpha}{2} = 20+\alpha$ ✓ (base $\angle$ isos. Δ)

$\Delta EDC = \angle ADC - \angle ADE = (\alpha+40) - (20+\alpha)$ 2

$= \alpha + 40 - 20 - \alpha = 20^\circ$

Teacher's Name: Parrish Student's Name/N°: Paul Shieh

Question 6

a) $y = (x^2-5)^5$

$y' = 5(x^2-5)^4 \times 2x$

$= 10x(x^2-5)^4$ ///

b) i) $m = \frac{5-3}{1-0} = \frac{2}{1} = 2$ ✓

$\therefore$ line $= (y-3) = 2(x-0)$

$y-3 = 2x$.

$2x - y + 3 = 0$. ← why?

ii) if PB is perpendicular, then m of PB $= -\frac{1}{2}$

$\therefore (y-5) = -\frac{1}{2}(x-1)$

$2(y-5) = -(x-1)$

$2y - 10 = -x + 1$

$x + 2y - 11 = 0$ ///

iii) P is when $y = 0$.

$x = 11$ $\therefore$ P(11,0) ✓

iv) rectangle, corners joined must have same midpoint.

AP's midpoint $= (\frac{11}{2}, \frac{3}{2})$

$\therefore$ BP midpoint $= (\frac{11}{2}, \frac{3}{2})$

$\therefore \frac{11}{2} = \frac{1+x}{2}$ $\quad \frac{3}{2} = \frac{5+y}{2}$.

$2(1+x) = 22$ $\quad 2(5+y) = 6$

$2 + 2x = 22$. $\quad 10 + 2y = 6$.

$2x = 20$ $\quad 2y = -4$

$x = 10$ $\quad y = -2$.

$\therefore$ Q has the co-ordinate (10, -2) ✓

c) i) $\cos A = \frac{3^2+2^2-4^2}{2 \times 3 \times 2} = \frac{9+4-16}{12} = \frac{-3}{12} = \frac{-1}{4}$.

$\cos A = \frac{-1}{4}$ ✓

ii) $\sin^2 A + \cos^2 A = 1$.

$\sin^2 A = 1 - \cos^2 A$.

$= 1 - (\frac{1}{16})$

$\sin^2 A = \frac{15}{16}$. $\quad \therefore \sin A = +ve$.

$\sin A = \frac{\sqrt{15}}{4}$

(9)

Teacher's Name: Parrish Student's Name/N°: Paul Shieh

Question 5.

a. i) $\sqrt[3]{x} = x^{\frac{1}{3}}$ 1

ii) $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$\therefore f'(8) = \frac{1}{3}(8)^{-\frac{2}{3}}$

$= \frac{1}{3} \times \frac{1}{8^{2/3}}$

$= \frac{1}{3} \times \frac{1}{4}$

$= \frac{1}{12}$ 2.

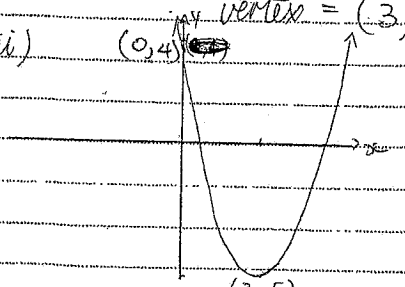
b) axis of symmetry $= \frac{-b}{2a} = \frac{6}{2} = 3$.

$\therefore y = (3)^2 - 6(3) + 4$

$= 9 - 18 + 4 = -5$.

$\therefore$ vertex $= (3, -5)$ 2

ii)



1

iii) $ax^2 + bx + c$ for +ve for all x $\quad D < 0$

$\therefore D = (6)^2 - 4 \times 1 \times k$

$\therefore 36 - 4k < 0$

$4k > 36$ 2

$k > 9$

c) $\frac{100}{\sin 45} = \frac{x}{\sin 60}$

$x = \frac{100 \times \sin 60}{\sin 45}$

$= \frac{100 \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}}$ $= 100 \times \frac{\sqrt{3}}{2} \div \frac{1}{\sqrt{2}}$ 2

$= 100 \times \frac{\sqrt{3}}{2} \times \sqrt{2}$

$= \frac{100\sqrt{6}}{2} = 50\sqrt{6}$

(10)

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Question 4.

a) $y = 3x^2 + x$

$y' = 6x + 1$

at $x = 1$, m of tangent $= 7$ ✓

∴ at $x = 1$, $y = 3(1) + 1 = 4$

tangent: $(y-4) = 7(x-1)$

$y - 4 = 7x - 7$ ✓ 2

$7x - y - 3 = 0$

b i) $y = 2x - 1$; $3y = 6x + 5 \Rightarrow y = 2x + \frac{5}{3}$

→ m of 2 ✓ → m of 2 ✓

both have same gradients thus lines are parallel. 2

ii) $y = 2x - 1$

$5 = 2a - 1$

$2a = 6$ ✓ 1

$a = 3$

iii) distance $= \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$= \left| \frac{6(3) + -3(5) + 5}{\sqrt{6^2 + 3^2}} \right|$ (9)

$= \left| \frac{18 - 15 + 5}{\sqrt{45}} \right|$

$= \frac{8}{\sqrt{45}}$ ✓ 2

c) i) $\angle PQB = 180 - 126 = 54°$ ✓ 1

ii) $\angle QPB = 54°$ (base ∠ of isosceles △)

$\angle DPQ = 180 - 54 = 126$ (co-interior) not sufficient.

∴ $x = 126 - 54 = 72°$ ✓ angle on parallel lines.

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Question 3.

i) $y = \frac{1}{3}x^4 + k$

$y' = \frac{4}{3}x^3$ ✓ (1)

ii) $y = \frac{4x^3 + x^4}{x^2}$

$= \frac{x^2(4x + x^2)}{x^2}$

$= 4x + x^2$ ✓

$y' = 2x + 4$. ✓ (2)

b) i) centre = midpoint.

∴ $x = \frac{-4+2}{2} = \frac{-2}{2} = -1$.

$y = \frac{-5+7}{2} = \frac{2}{2} = 1$.

∴ centre of circle = $(-1, 1)$ ✓ (1)

ii) length $= \sqrt{(-1-2)^2 + (1-7)^2}$

$= \sqrt{(-3)^2 + (-6)^2}$

$= \sqrt{9 + 36}$ ✓

$= \sqrt{45}$ (2)

c) i) I) $\alpha + \beta = \frac{-b}{a} = \frac{-2}{1} = -2$ ✓

II) $\alpha\beta = \frac{c}{a} = m$ ✓ (2)

ii) $\beta = 2\alpha$

∴ $\alpha + 2\alpha = -2$.

$3a = -2$

$a = \frac{-2}{3}$

$\alpha\beta = m$.

$2a^2 = m$.

$m = 2\left(\frac{-2}{3}\right)^2$

$= 2\left(\frac{4}{9}\right)$ ✓ (2)

$= \frac{8}{9}$

10

10

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Question 2.

i) domain: all real x but $x \neq -1$ ✓

ii) $f\left(\frac{1}{a}\right) = \frac{2}{\frac{1}{a}+1} \times \frac{a}{a}$ ✓

$= \frac{2a}{a+1}$ ✓

b) $x^2 - 2x = 0$.

$x(x-2) = 0$.

$\therefore x = 0$ or $x = 2$ ✓

c) $\sin\theta = -\frac{\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$ if $\theta = 60°$ (acute angle) ✓

but -ve in 3rd and 4th

$\therefore \theta = 240°, 300°$

d.i) $(2x+h)^2 - 4x^2$

$= 4x^2 + 4xh + h^2 - 4x^2$

$= 4xh + h^2$

$= h(4x+h)$ ✓

ii) $\lim_{h\to 0} \frac{(2x+h)^2 - 4x^2}{h}$

$= \lim_{h\to 0} \frac{h(4x+h)}{h}$

$\therefore = 4x$ ✓

10

e) $\frac{|x|}{2} < 1$.

$\therefore |x| < 2$

$\therefore x < 2$ or $-x < 2$.

$\therefore x < 2$ or $x > -2$

$-2 < x < 2$ ✓

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Question 1.

a) $2 - 4 = -2$. ✓

b) $49^{-\frac{2}{3}}$

$= \frac{1}{49^{\frac{2}{3}}}$

$= \frac{1}{7^2} = \frac{1}{343}$. ✓

c) $\cot 102°13'$

$= \frac{1}{\tan 102°13'} = -0.217$. ✓

d) $x^4 - 4x^2$

$= x^2(x^2 - 4)$

$= x^2(x-2)(x+2)$ ✓

e) $5 = \frac{2}{5}(w+4)$

$25 = 2(w+4)$.

$25 = 2w + 8$.

$2w = 25 - 8$

$= 17$. ✓

$w = \frac{17}{2}$

f) $(2+\sqrt{3})^2$

$= 4 + 4\sqrt{3} + 3$

$= 7 + 4\sqrt{3}$.

$= 7 + \sqrt{48}$ ✓

$\therefore a = 7 \quad b = 48$

10