SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS EXTENSION 1

YEAR 11 YEARLY EXAMINATION

2002

Time allowed: 90 minutes

Instructions:

- Show all necessary working
- Start each question on a new page
- All questions are of equal value
- Marks will not be awarded for careless or badly arranged work
- Non-programmable calculators may be used
- This paper must be handed in with your answer sheets
- Answers must be written in blue or black pen

Name:_

Class: _

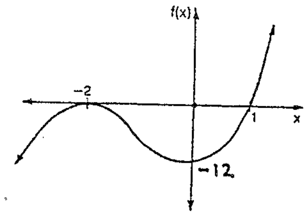
Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	TOTAL

Question 1 **Marks**

a) If $(x+1)$ is a factor of $P(x) = x^3 - ax + 3$. Find the value of a. 1

b) 2

P(x) = $(x+2)^2$ (·



Write down the equation of the polynomial function (in factored form)

c) A parabola is symmetrical about the line $y = 2$ it has a focal length 3 units and the equation of the directrix is $x = 1$. 3

i) How many parabolas satisfy these conditions?

ii) If the vertex is (4, 2) find the equation of the parabola

d) Solve $\dfrac{x+1}{x-1} \leq 0$ 2

e) The roots of the quadratic equation $(k+2)x^2 - 4x + k^2 = 0$ are reciprocals. Find the value/s of k. 3

Question 2 **Marks**

a) A polynomial of degree 7 is divided by the polynomial Q(x), the remainder is $x^2 + x + 2$. What is the least degree of Q(x). **1**

$\frac{x^7}{x^3}$

b) For the quadratic equation $x^2 + (k-3)x + 2 - k = 0$ **3**

i) Find the value of the discriminant in the terms of k

ii) Explain why the roots of this quadratic equation are real for all values of k

c) If $a + b = 1$ and $a^2 + b^2 = 2$ **3**

i) Find the value of ab

ii) Hence find the value of $a^3 + b^3$

d) i) Write $x^{-\frac{1}{2}}$ with a positive index **4**

ii) Solve $x^{\frac{1}{2}} + 10x^{-\frac{1}{2}} = 7$

Question 3 **Marks**

a) For the function $y = \sqrt{x^2 - 4}$ **3**

Write down

i) the domain

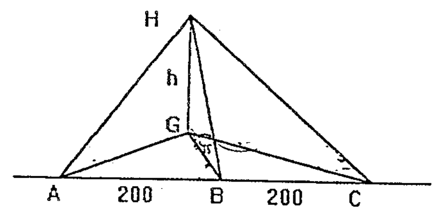
ii) the range

b) The points P($12t$, $6t^2$) and Q (36, 54) are points on a parabola **3**

i) Find the cartesian equation of the parabola

ii) If PQ is a focal chord find the value of t

c)



A cyclist riding along a straight flat road passes by three stop signs A, B and C spaced 200m apart. From these three signs the respective angles of elevation to the top of a mobile phone tower are 45°,45° and 30°. If 'h' is the height of the tower GH. **5**

i) Show that CG = $\sqrt{3}\,h$.

ii) If <GBA = α. Find two different expressions for cos α in terms of h.

iii) Hence find the height of the tower.

Question 4

Marks

a) i) Simplify
$(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$ — 3

ii) The roots of $x^3-4x^2-8=0$ are α, β and γ. Use the result in part (i) to find The value of $\alpha^2+\beta^2+\gamma^2$.

b) i) Show that $\dfrac{1+\cos 2A}{\sin 2A}=\cot A$ — 4

ii) Hence find the exact value of cot 15°

c) T $(2t, t^2)$ is a point on the parabola $x^2=4y$ with focus S. — 4
P is the point which divides ST internally in the ratio 1 : 2.

i) Write down the coordinates of P in terms of t.

ii) Hence show that as T moves on the parabola $x^2=4y$ that the locus of P is the parabola $9x^2=12y-8$

Question 5

a) The roots of the equation $x^3-6x^2+5x+8=0$ are α, β, γ — 3

The roots of the equation $x^3+ax^2+bx+512=0$ are $k\alpha, k\beta, k\gamma$

i) Find the value of k

ii) Hence find the value of b.

b) Consider the points A(-2, 3) B(6, 5) the point P(x, y) moves so that the angle APB = 90° — 4

i) Write down an expression for the gradient of AP

ii) Show that the locus of P is a circle

iii) Find its centre and radius.

c) i) Expand tan $(A+B)$

ii) The roots of $x^2-2x-1=0$ are tan A and tan B. If A and B are acute find the size of $A+B$. — 4

Question 6

Marks

a) i) Derive the equation of the tangent to the parabola $x^2=4ay$ at the point P $(2at, at^2)$ — 7

ii) The tangent cuts the y axis at R. Find the co-ordinates of R.

iii) If S is the focus of the parabola. Find the length of PS.

iv) Prove that the triangle PSR is isosceles

v) If $< PSR = 120°$. Find the numerical value of t.

b) If $P(x) = 4x^3+9x-4$ — 4

i) Find $P(\alpha+1)$

ii) If α is a root of $P(x)$ use part (i) to help show that $P(\alpha+1)>0$

Teacher's Name:

(1) a) $P(-1) = -1 + a + 3$

$= 2 + a$

If factor $P(-1) = 0$ (1)

$\therefore 2 + a = 0 \qquad a = -2$

b) $f(x) = A(x+2)^2(x-1)$

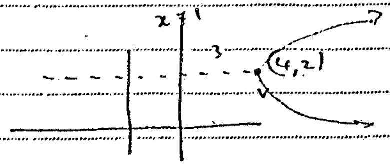
$x = 0,\ f(0) = -12$ (1)

$-12 = -4A$ (1)

$A = 3$

$f(x) = 3(x+2)^2(x-1)$

c)

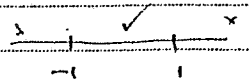


i) 2. (1)

ii) $(y-2)^2 = 4 \times 3(x-4)$ (1)

$(y-2)^2 = 12(x-4)$ (1)

d) $\frac{x+1}{x-1} \leq 0$



$-1 \leq x < 1$ (1) (1)

e) For reciprocals

$\frac{k^2}{k+2} = 1$ (1)

$k^2 = k + 2$ (1)

$k^2 - k - 2 = 0$

$(k-2)(k+1) = 0$

$k = 2 \text{ or } -1$ (1)

(2)

a) 3 (1)

b) i) $\Delta = (k-3)^2 - 4.1.(2-k)$

$= k^2 - 6k + 9 - 8 + 4k$ (2)

$= k^2 - 2k + 1$

ii) $\Delta = (k-1)^2$

For real roots $\Delta \geq 0$

$(k-1)^2 \geq 0$ for all k (1)

$\therefore$ roots are always real

c) i) $(a+b)^2 = a^2 + b^2 + 2ab$

$1 = 2 + 2ab$ (1)

$\therefore ab = -\frac{1}{2}$

ii) $a^3 + b^3 = (a+b)[a^2 - ab + b^2]$

$= 1[2 + \frac{1}{2}]$

$= \frac{5}{2}$ (2)

d) i) $\frac{1}{x^{1/2}}$ (1)

ii) $x^{1/2} + \frac{10}{x^{1/2}} = 7$ (1)

Let $m = x^{1/2}$

$\therefore m + \frac{10}{m} = 7$

$m^2 - 7m + 10 = 0$

$(m-5)(m-2) = 0$ (1)

$\therefore m = 5 \text{ or } 2$

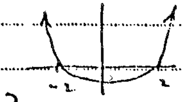
$\therefore x^{1/2} = 5 \text{ or } 2$

$\therefore x = 25 \text{ or } 4.$ (1)

Teacher's Name: Student's Name/N°:

(3)

a) i) $x^2 - 4 \geq 0$ (1)



$\therefore x \leq -2,\ x \geq 2$ (1)

ii) $y \geq 0$ (1)

b) i) $x = 12t \implies t = \frac{x}{12}$

$y = 6t^2$

$\therefore y = 6 \times \frac{x^2}{144}$ (1)

$x^2 = 24y$

ii) At Q $12t = 36$

$t = 3.$ (2)

For focal chord $t_1 t_2 = -1$

$\therefore t_2 = -\frac{1}{3}.$

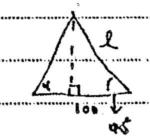
c) i) $\tan 30° = \frac{h}{GC}$

$\therefore GC = \frac{h}{\tan 30°}$

$= \sqrt{3}h$ (1)

ii) In $\triangle ABC$

$\cos\alpha = \frac{100}{h}.$



In $\triangle AGC$

$\cos\alpha = \frac{h^2 + 400^2 - 3h^2}{800h}$ (3)

iii) $\frac{100}{h} = \frac{160000 - 2h^2}{800h}$

$80000 = 160000 - 2h^2$

$2h^2 = 80000$

(4)

a) $(\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ (1)

$= \alpha^2 + \beta^2 + \gamma^2$

ii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$= 16 - 0$ (2)

$= 16$

b) i) $\frac{1 + \cos 2A}{\sin 2A} = \frac{1 + 2\cos^2 A - 1}{2\sin A \cos A}$ (2)

$= \cot A = RHS$

ii) $\cot 15° = \frac{1 + \cos 30°}{\sin 30°}$

$= \frac{1 + \frac{\sqrt{3}}{2}}{\frac{1}{2}}$ (2)

$= 2 + \sqrt{3}$

c) Focus $(0,1)$

$(0,1) \quad (x,y) \quad (2t, t^2)$

1 2

$x = \frac{2 \times 0 + 2t}{3} \qquad y = \frac{2 \times 1 + t^2}{3}$

$P\left(\frac{2t}{3}, \frac{2+t^2}{3}\right)$ (2)

Now $x = \frac{2t}{3} \implies t = \frac{3x}{2}$

$\therefore y = \frac{2 + \left(\frac{3x}{2}\right)^2}{3}$

$3y = 2 + \frac{9x^2}{4}$ (2)

$12y = 8 + 9x^2$

$\therefore 9x^2 = 12y - 8$

5 a) i) $k\alpha . k\beta . k\gamma = -512$

$k^3\alpha\beta\gamma = -512$

but $\alpha\beta\gamma = -8$

$\therefore k^3 = 64$

$k = 4$ 1

ii $4\alpha.4\beta + 4\alpha.4\gamma + 4\beta.4\gamma = b$

$16(\alpha\beta + \beta\gamma + \alpha\gamma) = b$

but $\alpha\beta + \beta\gamma + \alpha\gamma = 5$

$\therefore b = 80$ 2

b i) $mAP = \frac{y-3}{x+2}$ 1

ii) $mAP . mBP = -1$

$\frac{y-3}{x+2} \times \frac{y-5}{x-6} = -1$

$(y-3)(y-5) + (x+2)(x-6) = 0$

$y^2 - 8y + 15 + x^2 - 4x - 12 = 0$ 2

$y^2 - 8y + x^2 - 4x + 3 = 0$

iii $y^2 - 8y + 16 + x^2 - 4x + 4 = -3 + 16 + 4$

$(y-4)^2 + (x-2)^2 = 17$ 1

centre $(2,4)$ radius $\sqrt{17}$

c) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ 1

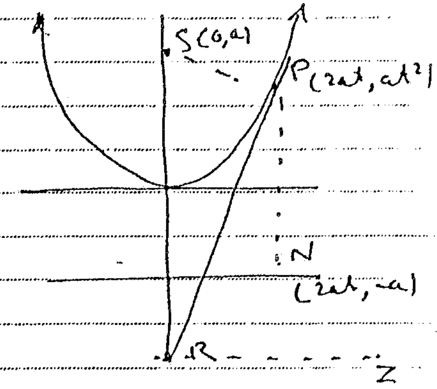
$\tan A + \tan B = 2$ 1

$\tan A \tan B = -1$ 1

$\therefore \tan(A+B) = \frac{2}{2}$

$(A+B) = 45^\circ$ 1

Question 6



i) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a}$

at $(2at, at^2)$

$\frac{dy}{dx} = t$

$\therefore y - at^2 = t(x - 2at)$

$y = tx - at^2$ 2

ii) $x = 0, y = -at^2$

$\therefore R(0, -at^2)$ 1

iii) $PS = PN$ (by definition)

$= at^2 + a$ 2

iv) $SR = a + at^2$ 1

$\therefore SR = SP$ $\therefore \Delta SPR$ is isosceles.

v) $\angle PSR = 120^\circ$, $\angle SRP = 30^\circ$

$\therefore \angle PRZ = 60^\circ$

$\therefore t = \tan 60^\circ$ 1

$= \sqrt{3}$

b) i) $P(\alpha+1) = 4(\alpha+1)^3 + 9(\alpha+1) - 4$

$= 4(\alpha^3 + 3\alpha^2 + 3\alpha + 1) + 9\alpha + 9 - 4$

$= 4\alpha^3 + 12\alpha^2 + 21\alpha + 9$ 1

ii) If α is a root.

$4\alpha^3 + 9\alpha - 4 = 0$

$\therefore P(\alpha+1) = 12\alpha^2 + 12\alpha + 13.$

$\Delta = 144 - 4 \times 12 \times 13$ 3

< 0

and since $a > 0$ $12\alpha^2 + 12\alpha + 13$ is a P.D.Q.F

$\therefore 12\alpha^2 + 12\alpha + 13 > 0$ for all α.

$\therefore P(\alpha+1) > 0$