SYDNEY TECHNICAL HIGH SCHOOL

YEAR 12

MATHEMATICS EXTENSION 1

ASSESSMENT TASK TWO

2003

Time allowed: 70 minutes

Instructions

*Attempt all questions.
*Answers to be written on the paper provided.
*Start each question on a new page.
*Marks may not be awarded for careless or badly arranged working.

Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
5 /9	2 /8	9 /9	7 /8	8 /9	6 /8	

37/51

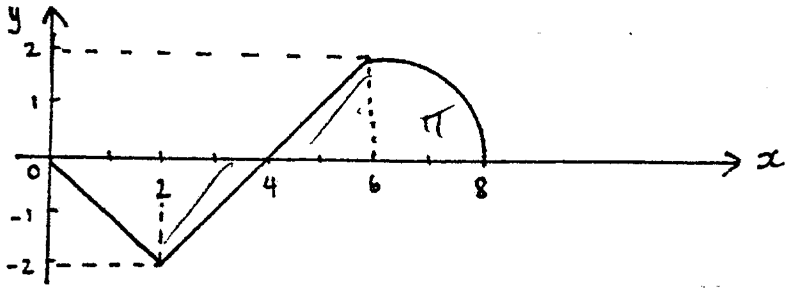
Question 1

(9 Marks)

a. Find i. $\int 4(1-2x)^3\,dx$ ii. $\int (x^2-1)^2\,dx$ (3)

b. Evaluate $\int_{-1}^{4} \frac{x^2+x}{2x}\,dx$ (3)

c. The graph below shows the curve $y=f(x)$ between $x=0$ and $x=8$



Find the value of $\int_0^8 f(x)dx$ (1)

d. Evaluate $\int_{-4}^{4} \sqrt{16-x^2}\,dx$ (2)

Question 2

(8 Marks)

a. Evaluate $\int_{-2}^{2} \frac{x^3}{1+x^2}\,dx$, giving a reason to support your answer (2)

b Find the area bounded by the curve $y=x-x^2$, the x axis, and the lines $x=1$ and $x=-1$ (3)

c. Find the value of $\int_0^2 \frac{dx}{\sqrt{1+2x}}$ (3)

Question 3

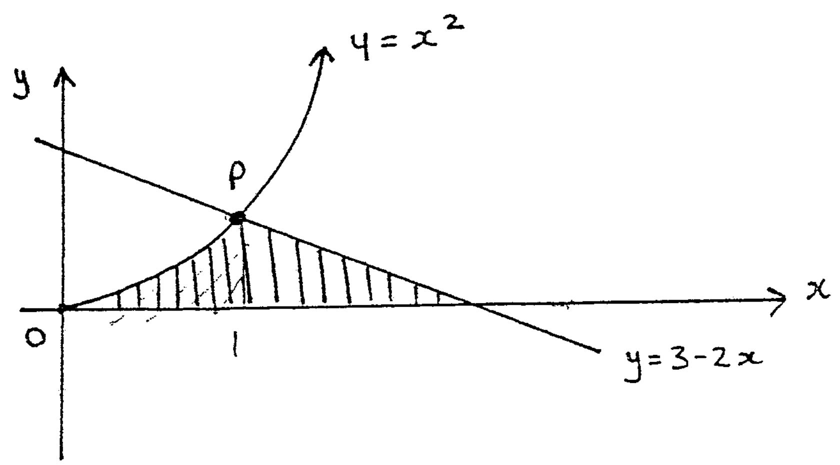
(9 Marks)

a. By use of the substitution $t = x + 1$ (4)

i.. Show $\int \frac{x}{\sqrt{x+1}}\,dx = \int (\sqrt{t} - \frac{1}{\sqrt{t}})dt$

ii. Find the exact value of $\int_3^8 \frac{x}{\sqrt{x+1}}dx$

b. (5)



The diagram shows the parabola $y = x^2$ and the line $y = 3 - 2x$ intersecting at the point P, in the first quadrant.

i. Show that P is the point (1, 1)

ii The shaded region is rotated about the $x - axis$. Find the volume of the solid formed.

Question 4

(8 Marks)

a. i.. Estimate the area between the function $y = \sqrt{x}$, the *x-axis* and the ordinates $x=0$ and $x= 1$, correct to 3 decimal places by using Simpson's rule with 3 function values. (4)

ii Find the percentage error in (i) correct to two decimal places

b. The graphs $y = 16 - x^2$ and y = $6x$ intersect at P and Q. (4)

i. Find the x-values of P and Q

ii. Hence, find the area between the two curves

Questions 5. **(9 Marks)**

a. Find $\int \frac{2x}{\sqrt{x^2-4}}dx$ using the substitution **(3)**

$u = x^2 - 4.$

b. Consider the function $y = x + 2 + \frac{4}{x-1}$ **(6)**

i. For what values of x is the function underfined ?

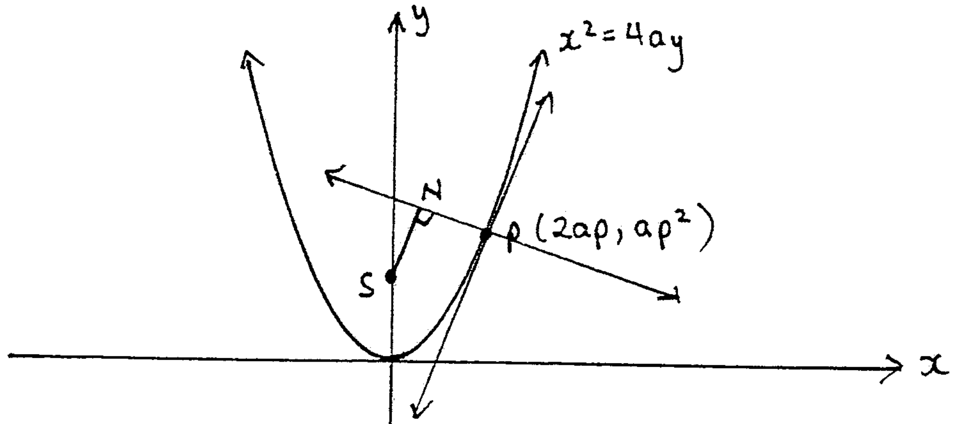
ii. What is the equation of the oblique asymptote ?

iii Find the co-ordinates of any stationary points and determine their nature

iv . Sketch the curve $y = x + 2 + \frac{4}{x-1}$.

Question 6 **(8 Marks)**

P$(2ap, ap^2)$ is a point on the parabola at $x^2 = 4ay$. SN is perpendicular to the normal at P, where S is the focus of the parabola and N the foot of the perpendicular from SN to the normal



a. Show that the equation of the normal at P is

$$x + py = 2ap + ap^3$$

b. Find the equation of SN

c. Show that the co-ordinates of the point N are $(ap, ap^2 + a)$

d. Find the locus of N as P moves on the parabola

stion 1

$\int 4(1-2x)^3\,dx = \frac{4(1-2x)^4}{-8}+c$

$= \frac{(1-2x)^4}{-2}+c$

$\int (x^2-1)^2\,dx = \int (x^4-2x^2+1)\,dx$

$= \frac{x^5}{5}-\frac{2x^3}{3}+x+c$

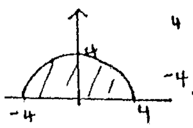
$\int_{-1}^{4} \frac{x^2+x}{2x}\,dx = \int_{-1}^{4}\left(\frac{x}{2}+\frac{1}{2}\right)dx$

$= \left[\frac{x^2}{4}+\frac{x}{2}\right]_{-1}^{4}$

$= \left(\frac{16}{4}+\frac{4}{2}\right)-\left(\frac{1}{4}-\frac{1}{2}\right)$

$= 6\frac{1}{4}$

$\int_0^8 f(x)\,dx = -2+\pi$



$\int_{-4}^{4}\sqrt{16-x^2}\,dx$

$= \frac{\pi\times 4^2}{2}$

$= 8\pi$

stion 2

$f(x) = \frac{x^3}{1+x^2}$ $\quad -f(-x) = -\left(\frac{-x^3}{1+x^2}\right)$

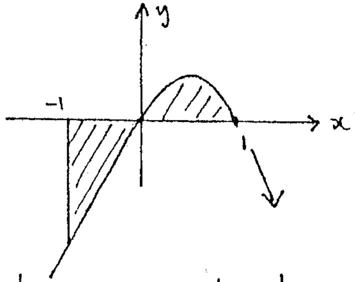
$= \frac{x^3}{1+x^2}$

function is odd

symmetry about origin

nce $\int_{-2}^{2}\frac{x^3}{1+x^2}\,dx = 0$

b)



$A = \left|\int_{-1}^{0}(x-x^2)\,dx\right| + \int_0^1 (x-x^2)\,dx$

$= \left|\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_{-1}^{0}\right| + \left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1$

$= \frac{5}{6}+\frac{1}{6}$

$= 1$ unit^2

c) $\int_0^2 (1+2x)^{-1/2}\,dx$

$= \left[\frac{(1+2x)^{1/2}}{2\times\frac{1}{2}}\right]_0^2$

$= \left[\sqrt{1+2x}\right]_0^2$

$= \sqrt{5}-1$

Question 3

a) i) $t = x+1$

$\frac{dt}{dx} = 1 \quad \therefore \quad dt = dx$

$\int \frac{x}{\sqrt{x+1}}\,dx = \int \frac{t-1}{\sqrt{t}}\,dt$

$= \int (t-1)\,t^{-1/2}\,dt$

$= \int t^{1/2} - t^{-1/2}\,dt$

$= \int \sqrt{t} - \frac{1}{\sqrt{t}}\,dt$

ii) $x = 8 \rightarrow t = 9$

$x = 3 \rightarrow t = 4$

$\int_4^9 \sqrt{t} - \frac{1}{\sqrt{t}}\,dt$

$= \left[\frac{2t^{3/2}}{3} - 2t^{1/2}\right]_4^9$

$= \left[\frac{2}{3}\sqrt{t^3} - 2\sqrt{t}\right]_4^9$

$= 12 - \frac{4}{3}$

$= 10\frac{2}{3}$

b) i) point intersection $x^2 = 3-2x$

$x^2+2x-3=0$

$(x+3)(x-1)=0 \quad \therefore x=-3 \quad x=1$

$\therefore$ in first quadrant $P(1,1)$

ii) $V_x = \pi\int_0^1 (x^2)^2\,dx + \pi\int_1^{3/2}(3-2x)^2\,dx$

$= \pi\int_0^1 x^4\,dx + \pi\int_1^{3/2}(9-12x+4x^2)\,dx$

$= \pi\left[\frac{x^5}{5}\right]_0^1 + \pi\left[9x-6x^2+\frac{4x^3}{3}\right]_1^{3/2}$

$= \frac{\pi}{5} + \pi\left[\left(\frac{27}{2}-6\times\frac{9}{4}+\frac{4}{3}\times\frac{27}{8}\right)-\left(9-6+\frac{4}{3}\right)\right]$

$= \frac{\pi}{5} + \pi\left[\frac{9}{2}-\frac{13}{3}\right]$

$= \frac{\pi}{5}+\frac{\pi}{6}$

$= \frac{11\pi}{30}$ unit^3

Question 4

a) i)

x	0	$1/2$	1
y	0	$\sqrt{1/2}$	1

$A \doteqdot \frac{1/2}{3}\left[0+1+4\sqrt{\frac{1}{2}}\right]$

$A \doteqdot 0.638$ (3 dec pl)

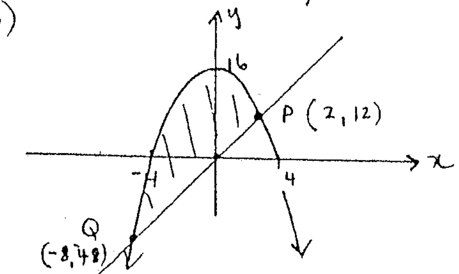
ii) $A = \int_0^1 x^{1/2}\,dx$

$= \left[\frac{2x^{3/2}}{3}\right]_0^1$

$= \frac{2}{3}$

$\therefore$ % error $\left(\frac{0.028\dot{6}}{2/3}\right)\times 100 = 4.3\%$

b)



i) Solve sim. eq $y = 16-x^2$ $\quad y = 6x$

$16-x^2 = 6x$

$x^2+6x-16=0$

$(x+8)(x-2)=0$

$\therefore$ let $P(2,12)$ $\quad Q(-8,-48)$

ii) $A = \int_{-8}^{2}(16-x^2-6x)\,dx$

$= \left[16x-\frac{x^3}{3}-3x^2\right]_{-8}^{2}$

$= \left[32-\frac{8}{3}-12\right]-\left[-128+\frac{512}{3}-192\right]$

$= 17\frac{1}{3}+149\frac{1}{3}$

$= 166\frac{2}{3}$

Question 5

a) $u = x^2 - 4$

$\frac{du}{dx} = 2x \quad \therefore \quad dx = \frac{du}{2x}$

$\therefore \int \frac{2x}{\sqrt{x^2-4}}\,dx = \int \frac{\cancel{2x}}{\sqrt{u}}\,\frac{du}{\cancel{2x}}$

$= \int u^{-1/2}\,du$

$= 2u^{1/2} + c$

$= 2\sqrt{x^2-4} + c$

b i) $x = 1$

ii) $y = x + 2$

iii) $y = x + 2 + 4(x-1)^{-1}$

$\frac{dy}{dx} = 1 - 4(x-1)^{-2} = 1 - \frac{4}{(x-1)^2}$

$\frac{d^2y}{dx^2} = 8(x-1)^{-3} = \frac{8}{(x-1)^3}$

st pts $\frac{dy}{dx} = 0 \qquad 1 = \frac{4}{(x-1)^2}$

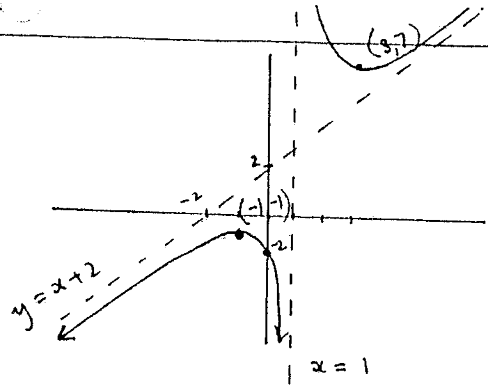
$(x-1)^2 = 4$

$x - 1 = 2 \qquad x - 1 = -2$

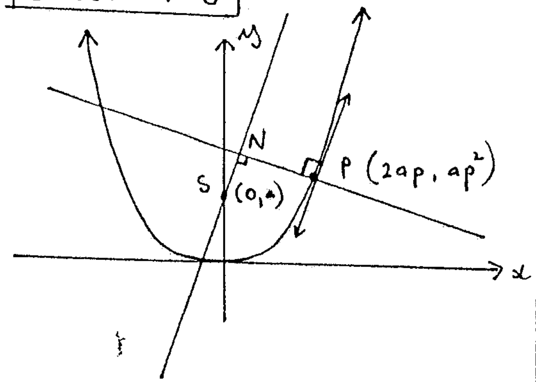
$x = 3 \qquad x = -1$

at $(3,7)$ $\quad y'' > 0 \quad$ MIN

at $(-1,-1)$ $\quad y'' < 0 \quad$ MAX



Question 6



i) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$

$m_T = \frac{2ap}{2a} = p \quad \therefore \quad m_N = -\frac{1}{p}$

eqn of normal

$y - ap^2 = -\frac{1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap$

$x + py = 2ap + ap^3$ —— ①

ii) gradient SN is p

eqn SN $\quad y = px + a$ —— ②

iii) solve sim eq ① & ②

$x + p(px + a) = 2ap + ap^3$

$x + p^2x + ap = 2ap + ap^3$

$x + p^2x = ap + ap^3$

$x(1 + p^2) = ap(1 + p^2)$

$\therefore x = ap$

$y = p(ap) + a$

$= ap^2 + a$

$\therefore N\,(ap,\ ap^2 + a)$

iv) $x = ap \qquad y = ap^2 + a$

$\frac{x}{a} = p$

$\therefore y = a\left(\frac{x}{a}\right)^2 + a$

$y = \frac{x^2}{a} + a$ parabola