Sydney Technical High School

Year 12
2 Unit Mathematics

HSC Assessment Task 2 - March 2003

Name: ____________________ **Class:**

Time Allowed: 70 minutes

Instructions:
1. Answer questions on paper provided
2. Begin each question on a fresh page.
3. Marks may be deducted for careless or untidy work
4. Show all working
5. Marks for each question are indicated next to the question These marks are a guide and may be adjusted slightly if necessary
6. Approved calculators may be used.

Question	1	2	3	4	5
Mark	12	12	[illegible]		[illegible]
Total					

47/56

5 (a) Find: i) $\int (4x^2 + 6)\, dx$

ii) $\int \frac{1}{x^2}\, dx$

iii) $\int \sqrt{2x+1}\, dx$

iv) $\int_1^4 (x^2 + 4)\, dx$

4 (b) Find the equation of a curve for which $y'' = 6x - 4$ and when $x = 1$, $y = 12$ and $y' = 7$.

y'

3 (c) If $f(x) = \sqrt{2x^2 + 4}$, find $f''(x)$.

1m = 10 cm

Question 2 (12 marks) (Begin a new page)

2 (a) Use calculus to find the values of x for which the curve $y = 4 + x - x^2$ is decreasing

4 (b) Evaluate $\sum_{n=1}^{40} 3n - 1$

3 (c) For a certain function, $f'(x) = \frac{(x-2)(x-4)^2}{\sqrt{x(x+2)^3}}$

i) Give a reason why the function has turning points when $x = 2$ and $x = 4$.

ii) Determine the nature of the turning point at $x = 2$.

3 (d) Julie is building a huge deck using 151 timber planks which decrease uniformly in length from 2500 mm to 400 mm so that the lengths of the planks form an arithmetic sequence. Find

$n = \underline{151}$

i) the difference in length between adjacent planks

ii) the total length (in metres) of planks needed.

Question 3 (10 marks) (Begin a new page)

4 (a) Find the values of x for which the curve $y = 4x^3 - 12x^2 + 2$ is

i) concave up

ii) concave down

4 (b) The ground floor of a twenty story office block will cost \$200 000 to construct. The next floor will cost \$230 000, and the next, \$264 500. The cost of the remaining 17 floors will follow the same pattern. Find the total cost of building the twenty floors.

2 (c) The point (1, 6) lies on the curve $y = f(x)$. If $f''(x) = 12(x-1)^2$, determine whether or not (1, 6) is a point of inflexion.

Question 4 (11 marks) (Begin a new page)

For the curve $y = x^3 - 6x^2 + 9x, \ -1 \le x \le 4$.

2 (a) Find y' and y''.

4 (b) Find the coordinates of any stationary points and determine their nature.

2 (c) Find where the curve touches the x axis.

2 (d) Sketch the curve in the given domain showing all features determined above.

1 (e) Find the minimum value of the curve in the given domain.

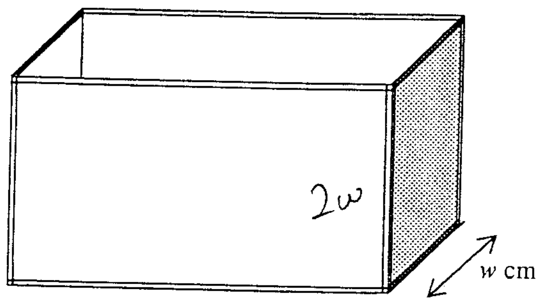
Question 5 (10 marks) (Begin a new page)

4 (a) On a half page number plane diagram, sketch a possible curve for $y = f(x)$ which satisfies the conditions given in the following table:

(1,0)

x	0	1	2	3
$f(x)$		0	-1	
$f'(x)$	-1	0	-1	0
$f''(x)$		0	0	1

6 (b) An open cardboard box is twice as long as it is wide. The volume is 24cm^3 and all edges of the box are to be taped.



V=

i) Show that the length of tape needed is $12w + \frac{48}{w^2}$ where w is the width (in cm) of the box.

ii) Find the dimensions of the box which will give a minimum amount of tape.

End of Test

2003 HS.C 2UNIT TASK 2

QUESTION 1

a) i) $\int 4x^2+6\,dx = \dfrac{4x^3}{3} + 6x + c$

ii) $\int x^{-2}\,dx = \dfrac{x^{-1}}{-1} + c = \dfrac{-1}{x} + c$

iii) $\int (2x+1)^{1/2}\,dx = \dfrac{(2x+1)^{3/2}}{\frac{3}{2}\times 2} + c$

$= \dfrac{\sqrt{(2x+1)^3}}{3} + c$

iv) $\int_1^4 (x^2+4)\,dx$

$= \left[\dfrac{x^3}{3} + 4x\right]_1^4$

$= \left(\frac{64}{3} + 16\right) - \left(\frac{1}{3} + 4\right)$

$= 33$

b) $\dfrac{d^2y}{dx^2} = 6x - 4$

$\dfrac{dy}{dx} = 3x^2 - 4x + c$ sub $y' = 7$, $x = 1$

$7 = 3 - 4 + c$

$c = 8$

$\therefore \dfrac{dy}{dx} = 3x^2 - 4x + 8$

$y = x^3 - 2x^2 + 8x + k$

sub $x = 1$ $y = 12$

$12 = 1 - 2 + 8 + k$

$k = 5$

$\therefore y = x^3 - 2x^2 + 8x + 5$

c) $f(x) = (2x^2+4)^{1/2}$

$f'(x) = \cancel{4}^{2}x \cdot \frac{1}{\cancel{2}} (2x^2+4)^{-1/2}$

$= 2x \cdot (2x^2+4)^{-1/2}$

$u = 2x$ $\quad v = (2x^2+4)^{-1/2}$

$u' = 2$ $\quad v' = 4x\left(-\frac{1}{2}\right)(2x^2+4)^{-3/2}$

$f''(x) = \dfrac{2}{\sqrt{2x^2+4}} - \dfrac{4x^2}{\sqrt{(2x^2+4)^3}}$

QUESTION 2

a) $y = 4 + x - x^2$

$\dfrac{dy}{dx} = 1 - 2x$

decreasing if $\dfrac{dy}{dx} < 0$ $\therefore 1 - 2x < 0$

$x > \frac{1}{2}$

b) $\sum_{n=1}^{40} 3n - 1 = 2 + 5 + 7 + \cdots + 11$ (T_1, T_2, T_4)

$S_{40} = \frac{40}{2}(2 + 119)$

$S_{40} = 2420$

c) $f'(x) = 0$ gives turning pts

i) $\therefore (x-2)(x-4)^2 = 0$

$\therefore x = 2$ $\quad x = 4$

ii)

x	1	2	3
$f'(x)$	$-ve$	0	$+ve$

$-\searrow$ min $\underline{0}$ $\nearrow +$ $+$

$\therefore$ min turning pt

d) T_1 $\quad T_2$ $\quad T_{151}$

$2500 + 2486 + \quad + 400$

i) difference 14 mm

ii) $S_{151} = \frac{151}{2}(2500 + 400)$

$= 218950$ mm

$= 218.95$ m

QUESTION 3

a) $y = 4x^3 - 12x^2 + 2$

$y' = 12x^2 - 24x$

$y'' = 24x - 24$

i) con ↑ $y'' > 0$ $\quad 24x - 24 > 0$

$24x > 24$

$\underline{\underline{x > 1}}$

ii) con ↓ $y'' < 0$ $\quad \underline{\underline{x < 1}}$

b) T_1 $\quad T_2$ $\quad T_3$ $\quad T_{20}$

200,000 $\quad$ 230,000 $\quad$ 264,500 $\quad$ —

$r = 1.15$ $\quad a = 200{,}000$ $\quad$ G.P

$$S_{20} = 200{,}000\left[\frac{1.15^{20} - 1}{1.15 - 1}\right]$$

$= \underline{\underline{\$20488716.52}}$

Test

c) Point of inflexion

i) $f''(1) = 0 \quad \therefore (1,6)$

ii)

x	0	1	2
$f''(x)$	+ve	0	+ve

no concavity change

$\therefore$ (1,6) not pt of inflexion

both conditions not satisfied

QUESTION 4

a) $y = x^3 - 6x^2 + 9x$

$\underline{\underline{y' = 3x^2 - 12x + 9}}$

$\underline{\underline{y'' = 6x - 12}}$

b) st pts $y' = 0$ $\quad 3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 3 \quad x = 1$

at $(3,0)$ $y'' > 0 \quad \therefore$ min

at $(1,4)$ $y'' < 0 \quad \therefore$ max

c) cut x axis $y = 0$

$x^3 - 6x^2 + 9x = 0$

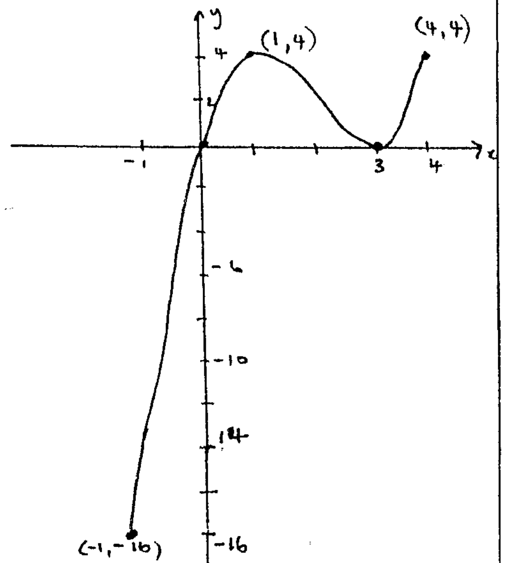
$x(x^2 - 6x + 9) = 0$

$x(x - 3)^2 = 0$

at $\underline{\underline{x = 0 \quad x = 3}}$

d) end pts $(-1, -14)$

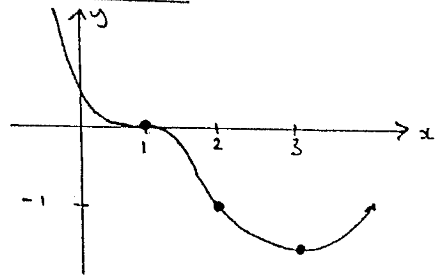
$(4, 4)$



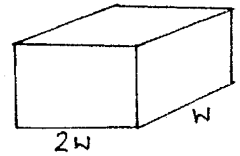
e) min value $\quad y = -16$

QUESTION 5

a)



b)



$h = \frac{12}{w^2}$

$V = 2w^2 h$

$24 = 2w^2 h$

$\therefore h = \frac{12}{w^2}$

i) Length of edges

$= 4(2w) + 4(w) + 4(h)$

$L = 12w + 4\left[\frac{12}{w^2}\right]$

$L = 12w + \frac{48}{w^2} = 4w + 48w^{-2}$

ii) $\frac{dL}{dw} = 12 - 96w^{-3} = -\frac{96}{w^3}$

$\frac{d^2L}{dw^2} = 288w^{-4} = \frac{288}{w^4}$

st pts $\frac{dL}{dw} = 0 \quad 12 = \frac{96}{w^3}$

$12w^3 = 96$

$w^3 = 8$

$w = 2$

$w = 2 \quad \frac{d^2L}{dw^2} > 0 \quad \therefore$ min

$\therefore$ Box Dimensions $\quad \underline{\underline{2 \times 4 \times 3 \text{ cm}}}$