Students Name 12029339 Teacher Sender

SYDNEY TECHNICAL HIGH SCHOOL

2002 TRIAL HIGHER SCHOOL CERTIFICATE

MATHEMATICS

Time Allowed - Three Hours

(plus 5 minutes reading time)

Direction to Candidates

- All questions may be attempted
- All questions are of equal value
- Approximate marks are shown
- All working should be shown in every question
- Full marks may not be awarded for careless or badly arranged work
- Standard Integrals are printed on the last page of this examination
- Approved calculators may be used
- Each question attempted is to be started ON A NEW PAGE, clearly marked with the number of the question and your name and class on the top right hand side of the page.
- This question paper must be handed in with your answers at the conclusion of the examination.

Question	Mark
Question 1	12
Question 2	11
Question 3	9
Question 4	11
Question 5	11
Question 6	9
Question 7	9
Question 8	9
Question 9	11
Question 10	8
TOTAL	100

Question 1 Use a separate writing page.

(a) Factorise $100 - 4x^2$ **(2)**

(b) Express $200°$ as an exact radian value **(1)**

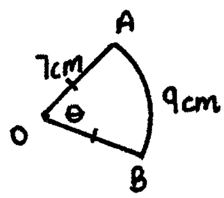
(c) Find the value of e^5, correct to 4 significant figures **(2)**

(d) Solve $5 - 2x \le 17$ **(2)**

(e) Find $\frac{d}{dx}(x^2 - 1)$ **(1)**

(f) Find the integers a and b such that $(3 + \sqrt{2})^2 = a + \sqrt{b}$ **(2)**

(g)

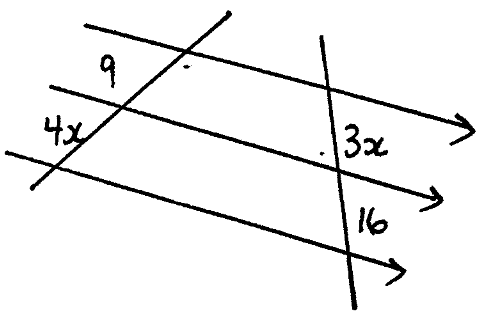


NOT TO SCALE

In the diagram, AB is an arc of a circle with centre 0. The radius OA is 7cm. The arc length AB is 9 cm. Find the size of angle AOB to the nearest degree. **(2)**

Question 2 Use a separate writing page

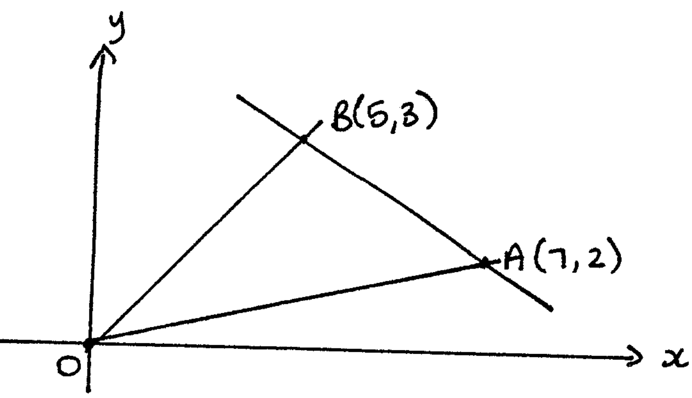
(a) Find the value/s of x **(3)**



(b) Find, (i) $\int\left(\frac{1}{x^2}-\frac{1}{x}\right)dx$ **(4)**

(ii) $\int 2\cos(3x-1)dx$

(c) **(5)**



NOT TO SCALE

The diagram shows the points A(7,2) and B(5,3) and the origin O.

(i) Show that the gradient of AB is $-\frac{1}{2}$.

(ii) Find the length of AB.

(iii) Find the equation of the line ℓ, passing through the points A and B.

(iv) Show that the area of ΔAOB is $5\frac{1}{2}$ square units.

Question 3 Use a separate writing page

(a) Find the first derivative of **(5)**

(i) $y = (x^2 - 1)^3$

(ii) $f(x) = \dfrac{2x}{x-1}$

(iii) $f(x) = \ln(3 - x)$

(b) Evaluate $\int_3^8 \sqrt{x+1}\,dx$ **(3)**

(c) If the quadratic equation $3x^2 - 4x + 7 = 0$ has roots α *and* β , find **(4)**

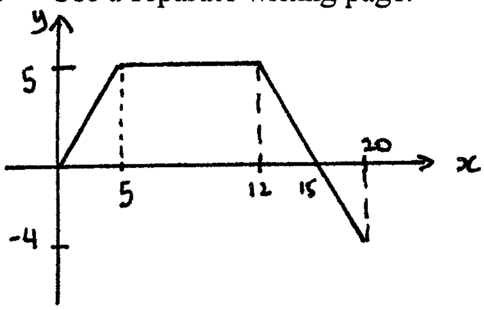
(i) $\alpha\beta$

(ii) $\alpha + \beta$

(iv) $\alpha^2 + \beta^2$

Question 4 Use a separate writing page.

(a)



The diagram shows the function $y = f(x)$ between $x = 0$ *and* $x = 20$. **(3)**

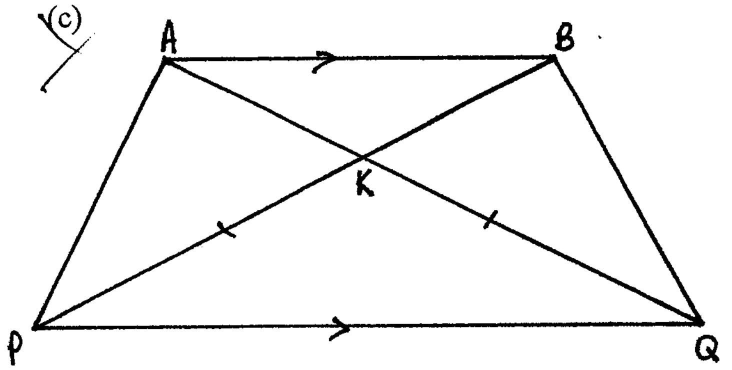
(i) Evaluate $\int_0^{20} f(x)dx$.

(ii) Is the area between the function $y = f(x)$, the x axis and the lines $x = 0$ *and* $x = 20$ greater, smaller or equal to $\int_0^{20} f(x)dx$.

Explain fully

(b) Find the equation of the normal to the curve $y = \tan x$ at the point where $x = \dfrac{\pi}{4}$. **(3)**

(c)



In the diagram AB ll PQ, AQ intersects PB at K and PK = KQ. **(6)**

Copy the diagram onto your answer page, marking on it all relevant information.

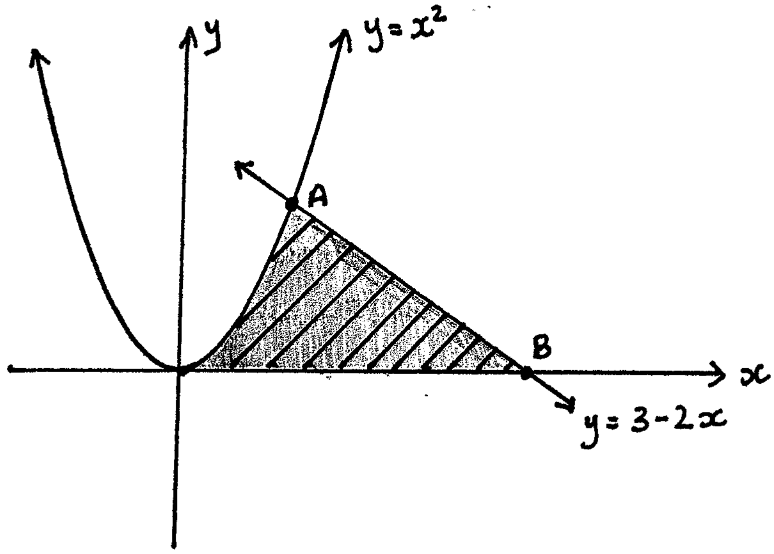
(i) Prove that ΔAKB is isosceles

(ii) Prove that $\Delta AKP \equiv \Delta BKQ$

(iii) Hence, show that $AP=BQ$

Question 5 Use a separate writing page

(a) **(5)**



The diagram shows the graph of the functions $y = x^2$ *and* $y = 3 - 2x$.

(i) Find the coordinates of A

(ii) Find the x - value of the coordinate at B

(iii) Find the area of the shaded region contained by the curves, $y = 3 - 2x,\ y = x^2$ and the x - axis.

(b)

(i) Write $0.\dot{7}\dot{3}$ as a geometric series and write down the value of the first term and the common ratio. **(3)**

(ii) Hence or otherwise, write $0.\dot{7}\dot{3}$ in the form of $\frac{x}{y}$ where x and y are integers.

(c) Evaluate $\sum_{t=3}^{100}(3t-5)$ **(2)**

(d) If $\int_0^{\ln 4} \frac{e^x}{e^x+1}\,dx = \ln A$, find the value of A **(2)**

Question 6 Use a separate writing page.

(a) For what values of K is the quadratic $Kx^2 - 6x + (6K+3) = 0$ positive definite **(3)**

(b) Consider the curve $f(x) = x^3 - 12x$ **(9)**

(i) Where does the curve cross the x axis

(ii) Find the coordinates of any stationary points and determine their nature

(iii) Sketch the curve $y = f(x)$ in the domain $-4 \le x \le 4$.

(iv) What is the minimum value of $y = f(x)$ in the domain $-4 \le x \le 4$.

Question 7 Use a separate writing page.

(a) If $\log_2 x = 4.716$ and $\log_2 y = 0.631$ find the value of **(3)**

$\log_2 \left(\frac{\sqrt{xy}}{4}\right)$

(b) For $y = 3\sin 4x,\ 0 \le x \le \pi$ **(6)**

(i) State the (α) Period

(β) Amplitude.

(ii) Sketch the curve $y = 3\sin 4x,\ 0 \le x \le \pi$, clearly showing where the curve cuts the x axis.

(iii) Hence, or otherwise, find the **number** of solutions to

$\sin 4x = \cos x$ *where* $0 \le x \le \pi$

(c) Solve $\sin^2 x - \sin x = 0$ *for* $0 \le x \le \pi$ **(3)**

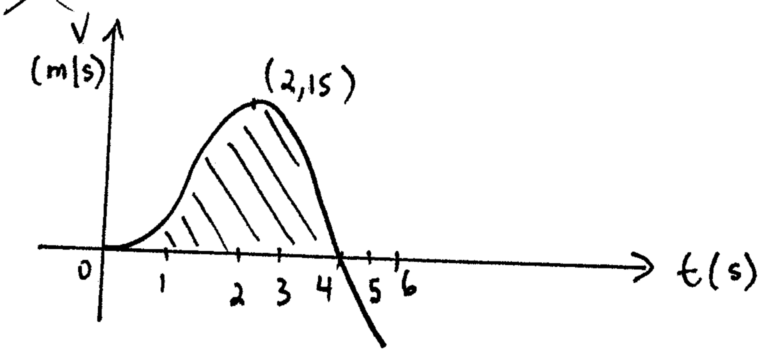
Question 8 Use a separate writing page.

(a) The region bounded by the curve $y = e^{-x} + 2$, the coordinates axes and the line $x=1$, is rotated about the x axis. **(4)**

(i) Sketch the region described above

(ii) Calculate the exact volume generated.

(b) **(8)**



A particle is observed as it moves in a straight line in the period between t = 0 and t = 6. Its velocity V at time t is shown on the graph above.

(i) What is the velocity of the particle after 2 seconds?

(ii) What is the particle's acceleration after 2 seconds?

(iii) At what time/s is the particle at rest?

(iv) At what time does the particle change direction?

(v) What does the shaded area represent?

(vi) If the particle starts at the origin, sketch the graph of the displacement x as a function of t.

Question 9 Use a separate writing page.

(a) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin \frac{1}{2}x}{x}$ **(1)**

(b) The population of koalas in a park was estimated to be 1000 at the start of 1990. **(5)**
Ten years later the population had fallen to 900. The rate of decrease of the population is proportional to the population at any time t.

(i) show that $P = P_0 e^{kt}$ is a solution to the differential equation.

$$\frac{dP}{dt} = kP$$

(ii) After how many years will the population of Koalas in the park be half that of 1990.

(c) A 4 metre piece of wire is cut into 3 pieces, which are bent to form a square and two congruent circles. **(6)**

(i) If the radius of each circle is r metres, show that the total area (A square metres) of the 3 figures is given by $A = 2\pi r^2 + (1 - \pi r)^2$

(ii) Find the value of r which will make this total area a minimum (give your answer in **exact** form).

Question 10 Use a separate writing page.

(a) (i) Sketch the curve $y = \log_{10} x$ and state its domain. **(6)**

(ii) Write an expression for the volume of the solid generated when $y = \log_{10} x$ is rotated about the x axis between $x = 2$ and $x = 4$.

(iii) Hence, use Simpson's rule, with three function values to find the approximate volume of the solid formed, correct to 3 significant figures.

(b) Mr Loaner borrows \$P to fund his house extensions. The term of the loan is 10 years with a monthly reducible rate of 6%p.a. **(6)**

Mr Loner repays the loan in equal monthly instalments of \$750.

(i) Write an expression for the amount Mr Loaner owes immediately after his first repayment.

(ii) Show that at the end of n months the amount owing is given by $A = P(1.005)^n - 150000(1.005)^n + 150000$.

(iii) If at the end of 10 years the loan has been repaid, calculate the amount that Mr Loaner originally borrowed, correct to the nearest dollar.

END OF EXAM

Teacher's Name: Student's Name/N°:

Answers 2unit trial 2002.

Question 1

a) $100 - 4x^2$

$= 4(25 - x^2)$

$= 4(5-x)(5+x)$

b) $200° = \frac{200\pi}{180}$

$= \frac{10\pi}{9}$

c) $e^5 = 148.413...$

$= 148.4$ (4 sig fig)

d) $5 - 2x \leq 17$

$-2x \leq 12$

$x \geq -6$

e) $\frac{d}{dx}(x^2 - 1) = 2x$

f) $(3+\sqrt{2})^2 = a + \sqrt{b}$

LHS $= 9 + 6\sqrt{2} + 2$

$= 11 + 6\sqrt{2}$

$= 11 + \sqrt{72}$

$\therefore a = 11 \quad b = 72$

g) $l = r\theta$ (θ in rad)

$9 = 7\theta$

$9/7 = \theta$

$\therefore \theta = \frac{9}{7} \times \frac{180°}{\pi}$

$= 74°$ nearest degree

or $9 = \frac{\theta}{360°} \times 2 \times \pi \times 7$

$\theta = 74°$

Question 2

a) $\frac{9}{4x} = \frac{3x}{16}$

$12x^2 = 144$

$x^2 = 12$

$x = \pm 2\sqrt{3}$ but $x > 0$

$\therefore x = 2\sqrt{3}$ mention why only 1 ans.

b) i. $\int x^{-2} - 1/x \, dx$

$= -x^{-1} - \ln x + c$

$= -1/x - \ln x + c$

ii. $\int 2\cos(3x-1)\, dx$

$= \frac{2}{3}\sin(3x-1) + c$

c) i. $M_{AB} = \frac{3-2}{5-7} = \frac{-1}{2}$

ii. $d_{AB} = \sqrt{(7-5)^2 + (2-3)^2}$

$= \sqrt{5}$

iii. eq AB: $y - 2 = -\frac{1}{2}(x - 7)$

$2y - 4 = -x + 7$

$x + 2y - 11 = 0$

iv. $d_{\perp} = \frac{|0 + 2\times 0 - 11|}{\sqrt{1+4}} = \frac{11}{\sqrt{5}}$

$\therefore A = \frac{1}{2}hb$

$= \frac{1}{2} \times \frac{11}{\sqrt{5}} \times \sqrt{5}$

$= \frac{11}{2}$

$= 5\frac{1}{2}$ units^2

Teacher's Name: Student's Name/N°:

Question 3

a. i. $y = (x^2 - 1)^3$

$y' = 6x(x^2 - 1)^2$

ii. $f(x) = \frac{2x}{x-1}$

$f'(x) = \frac{(x-1).2 - 2x.1}{(x-1)^2}$

$= \frac{-2}{(x-1)^2}$

iii. $f(x) = \ln(3 - x)$

$f'(x) = \frac{-1}{3-x}$

b. $\int_3^8 (x+1)^{1/2}\, dx$

$= \frac{2}{3}(x+1)^{3/2}\Big]_3^8$

$= \frac{2}{3}\left[9^{3/2} - 4^{3/2}\right]$

$= \frac{2}{3}\left[27 - 8\right]$

$= 12\,^2/_3 \quad (38/3)$

c) $3x^2 - 4x + 7 = 0$

i. $\alpha \times \beta = c/a$

$= 7/3$

ii. $\alpha + \beta = -b/a$

$= 4/3$

iii. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(\frac{4}{3}\right)^2 - 2 \times \frac{7}{3}$

$= -2\,^8/_9$

Teacher's Name: **Student's Name/Nº:**

Question 4

a) i. $\int_0^{20} f(x)\,dx = 12\frac{1}{2} + 35 + \frac{15}{2} + (-10)$

$= 45$

ii. The area would be more as the section under the x axis becomes +ve with area increasing the value ie $Area = 12\frac{1}{2} + 35 + \frac{15}{2} + |-10|$

b) $y = \tan x$ $\quad y = \tan\frac{\pi}{4}$

$y' = \sec^2 x$ $\quad = 1$

$= \frac{1}{\cos^2 x}$

$\therefore m = \frac{1}{(\cos\frac{\pi}{4})^2}$

$= \frac{1}{(\frac{1}{\sqrt{2}})^2}$

$= 2 \quad \therefore M_N = -\frac{1}{2}$

$\therefore y - 1 = -\frac{1}{2}(x - \frac{\pi}{4})$

$2y - 2 = -x + \frac{\pi}{4}$

$x + 2y - 2 - \frac{\pi}{4} = 0$

c) i. ΔPKQ is isosceles, $PK = KQ$

$\therefore \angle KPQ = \angle KQP$ (angles opp equal sides)

$\angle KPQ = \angle ABK$ (alt $\angle$'s $AB \parallel PQ$)

and $\angle KQP = \angle KAB$ (alt $\angle$'s $AB \parallel PQ$)

$\therefore \angle KAB = \angle KBA$

and $AK = KB$ (sides opp equal $\angle$'s)

$\therefore \Delta AKB$ is isosceles

ii. In ΔAKP and ΔBKQ

$AK = KB$ (part (i))

$PK = KQ$ (given)

$\angle PKA = \angle BKQ$ (vert opp $\angle$'s)

$\therefore \Delta AKP \equiv \Delta BKQ$ (SAS)

iii. $AP = BQ$ corresp. sides of congruent triangles.

Teacher's Name: **Student's Name/Nº:**

Question 5

a. i. $x^2 = 3 - 2x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3 \quad x = 1$

at A $x = 1$ $y = 1$.

ii. at B $x = \frac{3}{2}$

iii. $A = \int_0^1 x^2\,dx + \frac{1}{2}hb$ $\longrightarrow$ or $\int_1^{1.5} 3 - 2x\,dx$

$= \left.\frac{x^3}{3}\right]_0^1 + \frac{1}{2} \times 1 \times \frac{1}{2}$

$= \frac{1}{3} - 0 + \frac{1}{4}$

$= \frac{1}{3} + \frac{1}{4}$

$= \frac{7}{12}\ u^2$

b. i) $0.\dot{7}\dot{3} = \frac{73}{100} + \frac{73}{10000} + \frac{73}{1000\,000} + \ldots$

$\therefore a = 0.73$

$r = 0.01$

ii) $S_\infty = \frac{a}{1-r}$

$= 0.73/0.99$

$= 73/99$

c) $\sum_{t=3}^{100} (3t - 5) = 4 + 7 + 10 + \ldots + 295$

$a = 4 \quad d = 3 \quad n = 100 - 3 + 1$

$= 98$

$\therefore S_n = \frac{n}{2}(a + l)$

$= \frac{98}{2}(4 + 295)$

$= 14651$

5(d) $\int_0^{\ln 4} \frac{e^x}{1+e^x}\,dx = \ln A$

LHS $= \left.\ln(1+e^x)\right]_0^{\ln 4}$

$= \ln(1 + e^{\ln 4}) - \ln(1+1)$

$= \ln 5 - \ln 2$

$= \ln\left(\frac{5}{2}\right)$

$\therefore A = \frac{5}{2}$

Teacher's Name: Student's Name/Nº:

Question 6

a) $Kx^2 - 6x + (6K+3) = 0$

$a > 0$ but $\Delta < 0$

$b^2 - 4ac < 0$

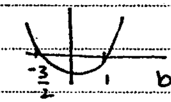
$36 - 4 \times K \times (6K+3) < 0$

$36 - 24K^2 - 12K < 0 \quad (\div -12)$

$-3 + 2K^2 + K > 0$

$2K^2 + K - 3 > 0$

$(2K+3)(K-1) > 0$



$\therefore K < -\frac{3}{2}$, $K > 1$ both answers

but

$a = K$ and $a > 0$

$\therefore K > 1$ only

b) $f(x) = x^3 - 12x$

i. $f(x) = 0$

$x(x^2 - 12) = 0$

$x = 0$, $x = 2\sqrt{3}$, $x = -2\sqrt{3}$

ii. Stat pts $f'(x) = 3x^2 - 12 = 0$

$3(x^2 - 4) = 0$

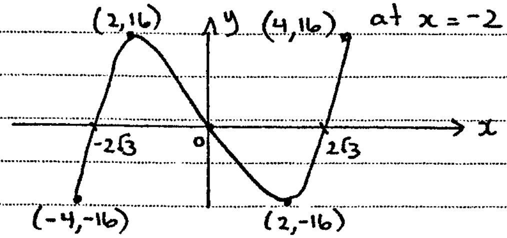
$\therefore x = 2$, $x = -2$

ie Stat pts $(2, -16)$ and $(-2, 16)$

Nature $f''(x) = 6x$ at $x = 2$ $f''(x) > 0$ $\therefore$ min at $(2, -16)$

at $x = -2$ $f''(x) < 0$ $\therefore$ max at $(-2, 16)$

iii.



iv. min value is -16

Teacher's Name: Student's Name/Nº:

Question 7.

a. $\log_2 x = 4.716$ and $\log_2 y = 0.631$

$\log_2\left(\frac{\sqrt{xy}}{4}\right) = \log_2(xy)^{1/2} - \log_2 4$

$= \frac{1}{2}(\log x + \log y) - 2\log_2 2$

$= \frac{1}{2}(4.716 + 0.631) - 2 \times 1$

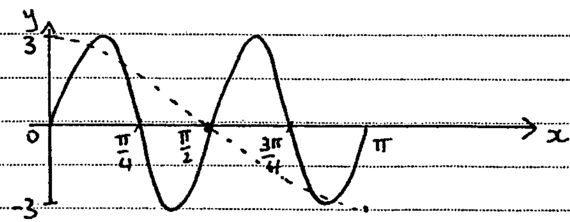
$= 0.6735$

b. $y = 3\sin 4x$ i. α. period $= \frac{2\pi}{4}$

$= \pi/2$

β. amp $= 3$

ii.



iii. $\sin 4x = \cos x$ ie $3\sin 4x = 3\cos x$ ($3\cos x$ dotted graph

ie 5 Solutions.

c. $\sin^2 x - \sin x = 0$

$\sin x(\sin x - 1) = 0$

$\sin x = 0$ $\quad$ $\sin x = 1$

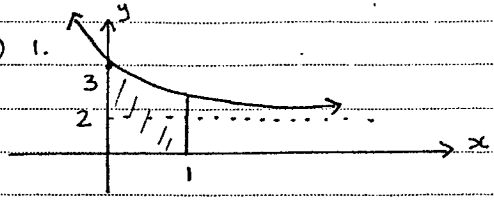
$x = 0, \pi$ $\quad$ $x = \frac{\pi}{2}$

$\therefore x = 0, \pi/2, \pi$

Teacher's Name: **Student's Name/N°:**

Question 8

a) I.



II. $V_x = \pi \int y^2 \, dx$

$= \pi \int_0^1 (e^{-x} + 2)^2 \, dx$

$= \pi \int_0^1 e^{-2x} + 4e^{-x} + 4 \, dx$

$= \pi \left[\frac{e^{-2x}}{-2} - 4e^{-x} + 4x \right]_0^1$

$= \pi \left[\frac{e^{-2}}{-2} - 4e^{-1} + 4 - \left(\frac{-1}{2} - 4 + 0 \right) \right]$

$= \pi \left[\frac{e^{-2}}{-2} - \frac{4}{e} + \frac{17}{2} \right]$

b) I. 15 m/s

II. Acceleration $= 0$

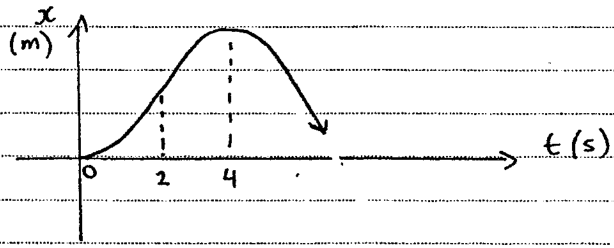
III. At rest when $v = 0$

<u>ie</u> $t = 0$ and $t = 4$ seconds.

IV. Changes direction when $t = 4$ s

V. distance travelled in the first 4 seconds.

VI.



Teacher's Name: **Student's Name/N°:**

Question 9.

a) $\lim_{x \to 0} \frac{\sin \frac{1}{2} x}{x} = \frac{1}{2}$

b) I. $P = P_0 e^{kt}$

$\frac{dP}{dt} = k \cdot P_0 e^{kt}$

$= k.P$

$\therefore P = P_0 e^{kt}$ is a sol^n to.

II. $P = 1000\, e^{kt}$

$900 = 1000\, e^{k \times 10}$

$\frac{9}{10} = e^{10k}$

$\ln \frac{9}{10} = \ln e^{10k}$

$10k = \ln \frac{9}{10}$

$k = \frac{1}{10} \ln \frac{9}{10}$ $(\doteqdot -0.0105)$

$\therefore 500 = 1000\, e^{kt}$

$\frac{1}{2} = e^{kt}$

$\ln 0.5 = \ln e^{kt}$

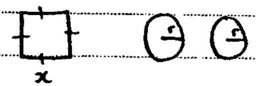
$kt = \ln 0.5$

$t = \ln 0.5 \div k$

$= 65.788\ldots\ldots$

$\therefore$ 66 years.

c)



I.

$4x + 2 \times 2\pi r = 4$ $(\div 4)$

$x + \pi r = 1$

$x = 1 - \pi r$

$\therefore A = \pi r^2 + \pi r^2 + x^2$

$= 2\pi r^2 + (1 - \pi r)^2$

II. $\frac{dA}{dr} = 4\pi r + 2(1 - \pi r) \times -\pi$

$0 = 4\pi r - 2\pi(1 - \pi r)$ $(\div 2\pi)$

$0 = 2r - (1 - \pi r)$

$0 = 2r - 1 + \pi r$

$1 = r(2 + \pi)$

$r = \frac{1}{2 + \pi}$ $(r \doteqdot 0.194)$

Nature

$\frac{d^2A}{dr^2} = 4\pi + 2\pi^2 > 0$

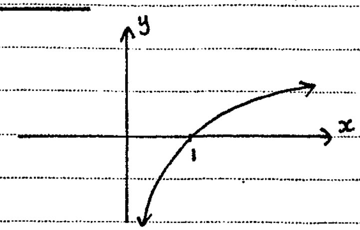
$\therefore$ min when

$r = \frac{1}{2 + \pi}$.

Teacher's Name: **Student's Name/N°:**

Question 10

a. i.



Domain $x > 0$

ii. $V_x = \pi \int y^2 \, dx$

$= \pi \int_2^4 (\log_{10} x)^2 \, dx$

iii.

x	2	3	4
$(\log_{10} x)^2$	0.0906	0.2276	0.3625

$h = 1$

$V \doteqdot \pi \times \frac{h}{3}\left(f + L + 4 \times M\right)$

$= \pi \times \frac{1}{3}\left(0.0906 + 0.3625 + 4 \times 0.2276\right)$

$= \frac{\pi}{3}(1.3635)$

$= 1.43$ (3 sig fig)

b. i. $P(1.005) - 750$

ii. $A_1 = P(1.005) - 750$

$A_2 = P(1.005)^2 - 750(1.005) - 750$

$A_3 = P(1.005)^3 - 750(1.005)^2 - 750(1.005) - 750$

$\therefore A_n = P(1.005)^n - 750(1.005)^{n-1} - 750(1.005)^{n-2}$

$\ldots\ldots - 750$

$A = P(1.005)^n - 750\left[1.005^{n-1} + 1.005^{n-2} + \ldots + 1\right]$

$A = P(1.005)^n - 750\left[\frac{a(r^n - 1)}{r - 1}\right]$

$= P(1.005)^n - 750\left[\frac{1(1.005^n - 1)}{0.005}\right]$

$= P(1.005)^n - 150\,000(1.005^n - 1)$

$= P(1.005)^n - 150\,000(1.005)^n$

$+ 150000.$

iii) $P = \frac{150\,000(1.005)^{120} - 150\,000}{1.005^{120}}$

$= 67555.09$

$= \$67555.$