+Soln's

CRANBROOK
SCHOOL

Year 12 Mathematics Extension 1

HSC Trial Examination- July, 2011

General Instructions
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84
- Attempt Questions 1–7
- All questions are of equal value
- Each student will need 7 **writing booklets**
- Each questions should be started in a new booklet

STANDARD INTEGRALS

$\int x^n dx = \frac{1}{n+1}x^{n+1}, n \neq -1; x \neq 0, \text{ if } n < 0$

$\int \frac{1}{x} dx = \ln x, \quad x > 0$

$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$

$\int \cos ax dx = \frac{1}{a}\sin ax, \quad a \neq 0$

$\int \sin ax dx = -\frac{1}{a}\cos ax, \quad a \neq 0$

$\int \sec^2 ax dx, = \frac{1}{a}\tan ax, \quad a \neq 0$

$\int \sec ax \tan ax dx = \frac{1}{a}\sec ax, \quad a \neq 0$

$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$

$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \ -a < x < a$

$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln(x+\sqrt{x^2-a^2}), \quad x > a > 0$

$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln(x+\sqrt{x^2+a^2})$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1 (12 marks) Use a separate page/booklet **Marks**

(a) Differentiate: $\sin^{-1} 4x$ **2**

(b) Solve for x: $\dfrac{x-3}{x-5} \le 6, \quad x \ne 5$ **3**

(c) Find the acute angle between the lines $3x-4y+1=0$ and $5x+3y-2=0$
Give answer to the nearest degree. **3**

(d) For the function, $y = 2\cos^{-1} 3x$

(i) State the domain. **1**

(ii) State the range. **1**

(iii) Sketch the curve. **1**

(e) Evaluate: $\sin\left[2\tan^{-1}(1)\right]$ **1**

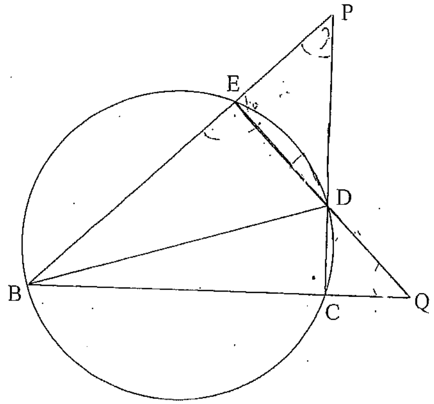
Question 2 (12 marks) Use a separate page/booklet **Marks**

(a) Prove by Mathematical Induction that $6^n - 1$ is divisible by 5 for $n \ge 1$ **3**

(b) In the figure BCQ, EDQ, CDP and BEP are straight lines and
$\angle BQE = \angle CPB$.

(i) Prove $\angle BCD = \angle BED$. **3**

(ii) Hence prove DB is a diameter. **2**



(c) The polynomial $f(x) = 2x^3 + ax^2 + bx + 6$ has a remainder of - 6 when divided by $(x-1)$ and $f(-2) = 0$.

Find the values of a and b. **2**

(d) Using the substitution $u = e^{2x}$, or otherwise, find $\displaystyle\int \frac{e^{2x}\,dx}{\sqrt{16-e^{4x}}}$ **2**

Question 3 (12 marks) Use a separate page/booklet **Marks**

(a) i) By using $t=\tan\frac{\overset{x}{\cancel{\theta}}}{2}$ show that $\frac{\cos x}{1+\sin x}=\frac{1-t^2}{(1+t)^2}$ **2**

ii) Hence or otherwise solve $\frac{\cos x}{1+\sin x}=1$ for $0\le x\le 2\pi$ **2**

(b) Find the locus of a point that is equidistant from the line $x=2$ and the point $(-2,-2)$ **3**

(c) Evaluate $\int_{\frac{1}{4}}^{\frac{1}{2}}\frac{-2}{\sqrt{1-4x^2}}dx$ **3**

(d) Find the point that divides the interval between A $(-4,1)$ and B $(-1,7)$ externally in the ratio 2:1 **2**

Question 4 (12 marks) Use a separate page/booklet
Marks

(a) i) Show that $\sin^2 x\cos^2 x=\frac{\sin^2 2x}{4}$ **1**

ii) Hence or otherwise find $\int \sin^2 x\cos^2 x\ dx$ **2**

(b) A particle moves along the x axis. The velocity (v m/s) of the particle is described by $v=\cos^2 t$ where t is the time in seconds and x metres is the displacement from the origin 0.
If $x=\frac{\pi}{4}$ when $t=\pi$, find x when $t=\frac{\pi}{2}$. **2**

(c) Solve $x^3-21x^2+126x-216=0$, given that the roots are in geometric progression. **3**

(d) A spherical bubble is expanding so that its volume is increasing at 6 cm^3s^{-1}. Find the rate of increase of its radius when the surface area is 750 cm^2. **2**

(e) The velocity $v\,ms^{-1}$ of a particle moving in simple harmonic motion along the x axis is given by $v^2=18+3x-x^2$.

(i) Find its acceleration in terms of displacement. **1**

(ii) Find the amplitude. **1**

Question 5 (12 marks) Use a separate page/booklet **Marks**

(a) A parabola, with parametric equation $x = a(t^2+1),\ y = 2a(2t+1)$,

is cut by the line $y = mx + 5a$ in distinct points P and Q.

(i) Show that the parameters of P and Q are the roots of the equation

$mt^2 - 4t + (m+3) = 0$. **2**

(ii) Show that the possible values for m are $-4 < m < 1$. **2**

(iii) Hence, or otherwise, find the equations to the parabola from the point $(0, 5a)$ **2**

(b) Newton's law of cooling states that for an object placed in surroundings at constant temperature, the rate of change of the temperature of the object is proportional to the difference between the temperature of the object and the surroundings i.e.

$$\frac{dT}{dt} = k(T - A)$$

where A is the temperature of the surroundings, T is the temperature of the object at any time.

(i) Show that

$$T = A + Ce^{kt}$$

satisfies Newton's law of cooling. C and k are constants. **1**

(ii) A liquid drops in temperature from 80^0C to 55^0C in 45 minutes. The room in which the liquid has been placed has a constant temperature of 8^0C.

(α) Find the values of C and k. **2**

(β) How long will it take the liquid to reach a temperature of 35^0C? **2**

(c) Find the general solution to $\cos x = \cos\frac{\pi}{4}$ **1**

Question 6 (12 marks) Use a separate page/booklet **Marks**

(a) (i) Express $3\sin\theta + 4\cos\theta$ in the form $R\sin(\theta+\alpha)$. **2**

(ii) Hence solve the equation $3\sin\theta + 4\cos\theta = -4$ for $0 < \theta < 360^0$. **2**

~~(a) If $y = \frac{(2x+1)^2}{4x(1-x)}$~~

(i) Show that the curve $y = \frac{(2x+1)^2}{4x(1-x)}$ has three asymptotes. **2**

(ii) The curve has a relative maximum at $\left(-\frac{1}{2}, 0\right)$ and a relative minimum at $\left(\frac{1}{4}, 3\right)$. Sketch the curve showing the asymptotes and turning points. **2**

The line $y = x$ and the curve $y = \frac{(2x+1)^2}{4x(1-x)}$ intersect at the point B, which has x coordinate equal to β.

(iii) Show that β is a root of the equation $4x^3 + 4x + 1 = 0$. **1**

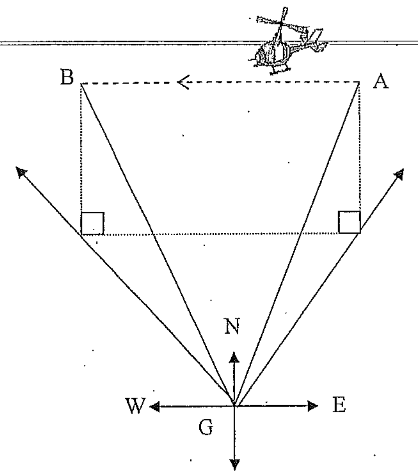
(iv) Show that β lies in the interval $-\frac{1}{2} < \beta < 0$. **1**

(v) By taking $-\frac{1}{2}$ as the first approximation for β, use Newton's Method once to find a second approximation for β. **2**

Question 7 (12 marks) Use a separate page/booklet **Marks**

(a) A helicopter flies due west from A to B at a constant speed of 420 *km/h*. From a point *G* on the ground the bearing of the helicopter when it is at A is $079^{0}T$ with an angle of elevation β. Four minutes later the helicopter is at B with a bearing from G being $302^{0}T$ and an angle of elevation 32^{0}. The altitude of the helicopter is h *metres*.

(i) Copy and complete the sketch below showing the above information. **1**

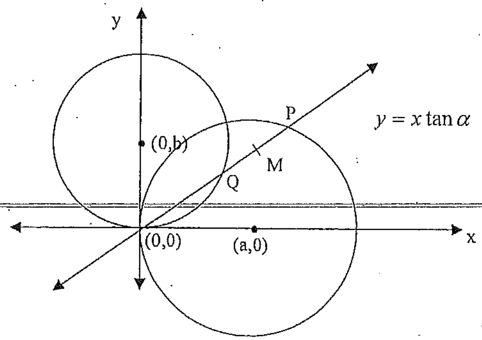


(ii) Calculate the height of the plane to the nearest metre. **2**

(iii) Calculate the value of β to the nearest degree. **2**

Question 7 (continued)

(b) Two circles are drawn. The first circle has centre (a, 0) and the second circle has centre (0, b). Both circles pass through the origin. The line $y = x\tan\alpha$ cuts the first circle at P and the second circle at Q.



(i) Show that the coordinates of P are $\left(2a\cos^2\alpha,\ 2a\sin\alpha\cos\alpha\right)$ **3**

(ii) Show that the coordinates of Q are $\left(2b\sin\alpha\cos\alpha, 2b\sin^2\alpha\right)$ **3**

(iii) Show that M, the midpoint of PQ is **1**

$$[\cos\alpha(a\cos\alpha + b\sin\alpha), \sin\alpha(a\cos\alpha + b\sin\alpha)]$$

End of paper

CRANBROOK - TRIAL HSC (EXT 1) SOLUTIONS - 2011

1(a) $\frac{d}{dx}\sin^{-1}4x = \frac{1}{\sqrt{1-(4x)^2}}\times\frac{d}{dx}(4x) = \frac{4}{\sqrt{1-16x^2}}$

(b) $\frac{x-3}{x-5}\times(x-5)^2 \le 6(x-5)^2$

$(x-3)(x-5)\le 6(x-5)^2 \Rightarrow 6(x-5)^2-(x-3)(x-5)\ge 0$

$(x-5)(6x-30-x+3)\ge 0$

$(x-5)(5x-27)\ge 0$

$\therefore x<5,\ x\ge 5\frac{2}{5}$

(c) $3x-4y+1=0 \therefore$ grad. $=\frac{3}{4}$

$5x+3y-2=0 \quad \therefore$ grad. $-\frac{5}{3}$

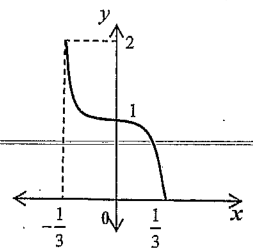
$\tan\theta = \left|\frac{\frac{3}{4}-\frac{-5}{3}}{1+\frac{3}{4}\times\frac{-5}{3}}\right| = 9\frac{2}{3} \Rightarrow \theta = 84^0 6'$ (acute angle)

(d) $2\cos^{-1}3x$

(i)Domain: $-1\le 3x\le 1$ i.e. $-\frac{1}{3}\le x\le\frac{1}{3}$

(ii)Range: $0\le y\le 2$

(iii)



(e) $\sin(2\tan^{-1}1)=\sin\frac{2\pi}{4}=1$

NOTES

1a) No need here to use $\sin^{-1}(\frac{x}{a})$ rule here simply use chain rule ie $\frac{d}{dx}\sin^{-1}f(x) = \frac{1}{\sqrt{1-(f(x))^2}}\times f'(x)$

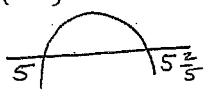
b) KEEP FACTORISED AS ABOVE & REMEMBER $x\ne 5$ *

OR $\frac{x-3}{x-5} - 6 \le 0$ (ie 1ST MAKE RHS ZERO. PUT ON common denominator Now $\times(x-5)^2$

$\frac{x-3-6(x-5)}{x-5} \le 0$

$\frac{(-5x+2)}{(\cancel{x-5})}\times(x-5)^{\cancel{2}} \le 0\times(x-5)^2$

$(-5x+27)(x-5)\le 0$ DRAW



$\therefore x<5^* \quad x\ge 5\frac{2}{5}$

c) a simple question requiring accurate substitution!
d) and correct rules to be known

e) Keep it simple $\tan^{-1}1=\frac{\pi}{4}$ $\therefore \sin 2(\frac{\pi}{4}) = \sin\frac{\pi}{2} = 1$

Solutions. Ext 1 Trial 2011

Q2 a) Prove 6^n-1 divisible by 5 $n\ge 1$

Prove true for $n=1$

$6^1-1=5$ Which is divisible by 5.

$\boxed{1}$ Showing $n=1$

Assume true for $n=k$

let $6^k-1 = \cancel{4m}\ 5m$ ...① where m is an integer.

Prove true for $n=k+1$

$6^{k+1}-1 = 6^k.6-1$ from ① $6^k = 5m+1$

$= 6(5m+1)-1$

$\boxed{2}$ - for working and substitution

$= 30m+6-1$

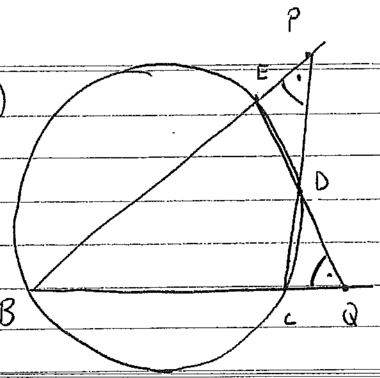
$= 30m+5$

$= 5(6m+1)$ As m is an integer, $6m+1$ is an integer

$\therefore 6^{k+1}-1$ is divisible by 5.

As this is true for $n=1$ and it has been proven for $n=k+1$ it must be true for $n=2$ and so on.

$\therefore$ true for all integers $n\ge 1$

i) 

$\angle EDP = \angle QDC$ (vertically opposite) $\boxed{1}$

$\angle DEP = \angle QCD = x$ (Angle sum of a triangle) $\boxed{1}$

$\angle BED = 180 - x$ (Angles on a straight line) $\boxed{1}$
$\angle BCD = 180 - x$

$\therefore \angle BED = \angle BCD$

ii) $\angle BED = \angle BCD$ (part i))

AND $\angle BED + \angle BCD = 180°$ (opposite $\angle$'s in a cyclic quadrilateral)

$\therefore \angle BED = \angle BCD = 90°$ $\boxed{1}$

$\therefore$ BD is a diameter as a diameter subtends a $90°$ angle at the circumference. $\boxed{1}$

c) $f(x) = 2x^3 + ax^2 + bx + 6$

$f(1) = -6$
$f(-2) = 0$

$f(1) = 2 + a + b + 6$

$\therefore a+b+8 = -6$
$a+b = -14 \quad \ldots (1)$

$f(-2) = -16 + 4a - 2b + 6 = 0$

$\therefore 4a - 2b = 10$
$2a - b = 5 \quad \ldots (2)$

$\boxed{1}$ for correct substitutions

$(1) + (2)$: $3a = -9$
$a = -3 \quad \ldots (3)$

$(3) \Rightarrow (1)$: $-3 + b = -14$
$b = -11$

$\boxed{1}$ correct answers.

d) $u = e^{2x}$ $\qquad \frac{du}{dx} = 2e^{2x} \qquad \frac{du}{2e^{2x}} = dx$

$u^2 = e^{4x}$

$$\int \frac{e^{2x}}{\sqrt{16-e^{4x}}}\,dx = \int \frac{\cancel{e^{2x}}}{\sqrt{16-u^2}} \frac{du}{2\cancel{e^{2x}}} \quad \boxed{1}$$

correct substitution

$$= \frac{1}{2}\int \frac{1}{\sqrt{16-u^2}}\,du$$

$$= \frac{1}{2}\sin^{-1}\left[\frac{u}{4}\right] + C$$

$$= \frac{1}{2}\sin^{-1}\left[\frac{e^{2x}}{4}\right] + C \quad \boxed{1}$$

correct answer.

Question 3

a) i) $t = \tan\frac{x}{2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad \sin x = \frac{2t}{1+t^2}$ (1).

$$\frac{\cos x}{1+\sin x} = \frac{1-t^2}{(1+t)^2}$$

$$\text{LHS} = \frac{1-t^2}{1+t^2} \div \frac{1+2t}{1+t^2}$$

$$= \frac{1-t^2}{1+t^2} \div \frac{1+t^2+2t}{1+t^2}$$

$$= \frac{1-t^2}{\cancel{1+t^2}} \times \frac{\cancel{1+t^2}}{(t+1)^2}$$

$$= \frac{1-t^2}{(1+t)^2} \quad (1).$$

$= \text{RHS}$

ii) $\frac{1-t^2}{(1+t)^2} = 1$

$1-t^2 = 1+2t+t^2$

$2t^2+2t = 0$

$2t(t+1) = 0$

$t = 0, -1$ (1)

$t = 0$ $\qquad t = -1$

$\tan\frac{x}{2} = 0 \qquad \tan\frac{x}{2} = -1$

$\frac{x}{2} = 0, \pi, 2\pi \qquad \frac{x}{2} = \frac{3\pi}{4}, \frac{7\pi}{4}$ (1)

$x = 0, 2\pi \qquad x = \frac{3\pi}{2}$

$x = 0, \frac{3\pi}{2}, 2\pi.$

Many left out at least one angle or gave answer in degrees not radians.

(7)

This question was not attempted by many. There was not a lot of recognition that a parabola was formed.

b) $x = 2$ $(-2,-2)$

$(2, y)$

$$PA^2 = PB^2$$

$$(x-2)^2 + (y-y)^2 = (x+2)^2 + (y+2)^2$$

$$\cancel{x^2} - 4x + \cancel{4} = \cancel{x^2} + 4x + \cancel{4} + (y+2)^2$$

$$\therefore (y+2)^2 = -8x$$ (3)

c) $$\int_{\frac14}^{\frac12} \frac{-2}{\sqrt{1-4x^2}}\,dx$$

$$= -2\int_{\frac14}^{\frac12} \frac{dx}{\sqrt{4(\frac14 - x^2)}}$$ (1)

forgetting taking out the square root of 4 was the most common mistake.

$$= -\cancel{2} \times \frac{1}{\cancel{2}} \left[\sin 2x\right]_{\frac14}^{\frac12}$$ (1)

$$= -\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$$

$$= -\frac{\pi}{3}$$ (1)

d) $(-4, 1)$ $(-1, 7)$

$-2:1$

$$x = \frac{-4+2}{-1} = 2 \qquad y = \frac{1-14}{-1} = 13$$

$(2, 13)$ divides the pt externally in ratio $2:1$.

Q4 a)(i) Simply expand – must know expansions!

(8)

$$RHS = \frac14 \sin^2 2x$$

$$= \frac14 (\sin 2x)^2$$

$$= \frac14 (2\sin x \cos x)^2$$

$$= \frac14 (4\sin^2 x \cos^2 x)$$

$$= \sin^2 x \cos^2 x$$

$$= LHS$$

OR start LHS

$$= \sin^2 x \cos^2 x$$

$$= (\sin x \cos x)(\sin x \cos x)$$

$$= \frac12 \sin 2x \times \frac14 \sin 2x$$

$$= \frac14 \sin^2 2x$$

(ii) $$\frac14 \int \sin^2 (2x)\,dx = \frac14 \int \frac12 (1 - \cos 4x)\,dx$$

MUST LEARN OR SHOW $\sin^2\theta = \frac12(1-\cos 2\theta)$

$$= \frac18\left(x - \frac{\sin 4x}{4}\right) + c$$

$$= \frac{x}{8} - \frac{\sin 4x}{32} + c$$

b) $$V = \frac{dx}{dt} = \cos^2 t$$

$$\therefore x = \int \cos^2 t\,dt$$

$$= \int \frac12 (1 + \cos 2t)\,dt$$

$\left\{ x = \frac{\pi}{4},\ t = \pi \right.$

$$x = \frac12\left(t + \frac12 \sin 2t\right) + c$$

$$\frac{\pi}{4} = \frac{\pi}{2} + \frac{\sin 2\pi}{4} + c$$

$$-\frac{\pi}{4} = c$$

$$\therefore x = \frac{t}{2} + \frac{\sin 2t}{4} - \frac{\pi}{4}$$ when $t = \frac{\pi}{2}$

$$= \frac{\pi}{4} + \frac{\sin\pi}{4} - \frac{\pi}{4}$$

$$= 0$$

c) NOTE (PTO) $\frac{a}{r}, r, ar$ makes this very easy

a, ar, ar^2 will work but MUCH more working needed

Leaving as α, β, γ makes even more work

(9)

(c) $x^3-21x^2+126x-216=0$ let the roots be $\frac{a}{r}, a, ar$

now $\alpha\beta\chi = a^3 = \frac{-d}{a} = \frac{216}{1} \Rightarrow a = 6$

$\alpha+\beta+\chi = \frac{-b}{a} = \frac{21}{1}$

$\frac{6}{r}+6+6r = 21 \Rightarrow 2r^2-15r+2=0$

$(2r-1)(r-2)=0$

$r=\frac{1}{2}, 2 \Rightarrow$ roots are 12, 6, 3

(d) $v=\frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dr} = 4\pi r^2$

using the chain rule $\frac{dr}{dt} = \frac{dr}{dv}\times\frac{dv}{dt}$

$= \frac{1}{4\pi r^2}\times 6$

surface area $= 750 = 4\pi r^2$

$\therefore \frac{dr}{dt} = \frac{6}{750} = \frac{1}{125} cm/s$

(e) (i) $v^2 = 18+3x-x^2$

$\frac{1}{2}(v^2) = 9+\frac{3}{2}x-\frac{x^2}{2}$

$\frac{d^2x}{dt^2} = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\left(9+\frac{3}{2}x-\frac{x^2}{2}\right) = \frac{3}{2}-x$

(ii) $v^2 = 18+3x-x^2 = (6-x)(x+3)$

The particle oscillates between -3 and 6 and the amplitude is 4.5

NOTE FOR d) Chain rule
OR
WRITE ALL INFORMATION
+ "MAKE WHAT IS ASKED, OUT OF WHAT YOU'VE GOT!!

SPHERE $V = \frac{4}{3}\pi r^3$

$\therefore \frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = 6$ given

$\therefore \frac{dr}{dt} = \frac{dr}{dV}\times\frac{dV}{dt}$

$= \frac{1}{4\pi r^2}\times 6$

(ALSO $SA = 750 = 4\pi r^2$ can be substituted as a whole)

$\frac{1}{750}\times 6$

$= \frac{1}{125}$ cm/s $= 0.008$ cm/s

e) (i) Remember acceleration is the derivative of $\frac{1}{2}V^2$ when in terms of x

$\therefore \times \frac{1}{2}$ first

(ii) $V=0$ at the ends of the path

$\therefore$ Let $V=0$ in $v^2 = 18+3x-x^2$ giving

-3 ——— 6

THIS is 9 units ie the RANGE

halve this for amplitude

(10)

Question 5

i). $x = a(t^2+1)$ $\quad y = 2a(2t+1)$

$y = mx + 5a$

$2a(2t+1) = m\,\boxed{a(t^2+1)} + 5a$ (1)

$2(2t+1) = m(t^2+1) + 5$

$4t+2 = mt^2 + m + 5$

$0 = mt^2 - 4t + m + 3$ (1)

as req.

ii). $\Delta > 0$

$\therefore b^2 - 4ac > 0$

$(-4)^2 - 4\times m(m+3) > 0$ (1)

$16 - 4m^2 - 12m > 0$

$m^2 + 3m - 4 < 0$

$(m+4)(m-1) < 0$ (1)

$\therefore -4 < m < 1$ as req.

iii) $\therefore$ if $m=-4$ $\quad y = -4x + 5a$ (1)

if $m = 1$ $\quad y = x + 5a$ (1)

this was not well done or attempted by many.

All b) + c) was well done

b) i) $T = A + Ce^{kt}$

$Ce^{kt} = T - A$

$\frac{dT}{dt} = kCe^{kt}$

$= k(T-A)$ 1

ii) $T = A + Ce^{kt}$

$80 = 8 + Ce^{0k}$

$C = 72$ 1

$55 = 8 + 72e^{45k}$

$47 = 72e^{45k}$

$e^{45k} = \frac{47}{72}$

$45k = \ln\frac{47}{72}$

$k = -0.0095$ 1

iii) $35 = 8 + 72e^{-0.0095t}$ 1

$\frac{27}{72} = e^{-0.0095t}$

$\therefore t = 103.5$ mins 1

c) $\cos x = \cos\frac{\pi}{4}$

$x = \frac{\pi}{4} \ldots$

$\therefore x = ~~\ldots~~ 2\pi n \pm \frac{\pi}{4}$ 1

Question 6

a) let $3\sin\theta + 4\cos\theta = R\sin(\theta + \alpha)$

$R\sin(\theta+\alpha) = R[\sin\theta\cos\alpha + \sin\alpha\cos\theta]$

$= R\cos\alpha\sin\theta + R\sin\alpha\cos\theta$

$\boxed{1}$ - for R

Comparing coefficients $\boxed{1}$ - for α

$R\cos\alpha = 3$ $\therefore R = 5$

$R\sin\alpha = 4$ $\tan\alpha = \frac{4}{3}$

$\alpha = \tan^{-1}\frac{4}{3}$

$\therefore 3\sin\theta + 4\cos\theta = 5\sin\left(\theta + \tan^{-1}\frac{4}{3}\right)$

b) $5\sin(\theta+\alpha) = -4$

$\theta + \alpha = \sin^{-1}\frac{-4}{5}$ (3rd & 4th quadrant)

$\sin 53°8' \doteqdot \frac{4}{5}$ (nearest minute) $\boxed{1}$ correct quadrants

$\therefore \theta + \alpha = 233°8'$ & $306°52'$

$\therefore \theta = 180°$ & $253°44'$ $\boxed{1}$ Both answers

(13)

b) i) $y = \dfrac{(2x+1)^2}{4x(1-x)}$

Vertical asymptotes at $x=0$ & $x=1$ as both would mean dividing by zero. $\boxed{1}$

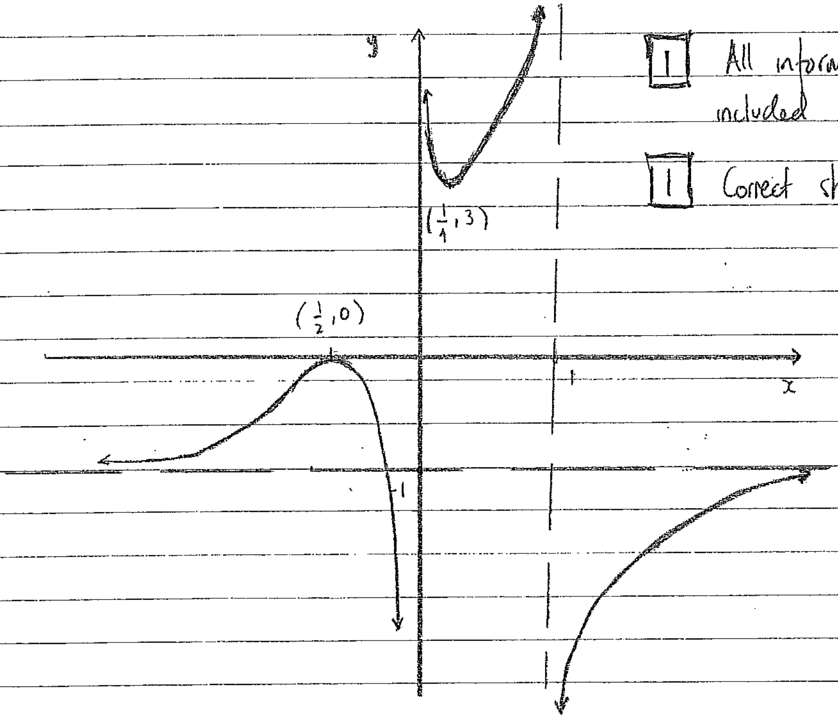
ii) $y = \dfrac{4x^2+4x+1}{4x-4x^2}$

$\lim\limits_{x \to \infty} \dfrac{4x^2+4x+1}{-4x^2+4x} = \dfrac{4}{-4}$

$= -1$

$\therefore$ Asymptote at $y=-1$ $\boxed{1}$

ii)



$\boxed{1}$ All information included

$\boxed{1}$ Correct shape.

(14)

iii) $y = x \qquad y = \dfrac{(2x+1)^2}{4x(1-x)}$

$x = \dfrac{(2x+1)^2}{4x(1-x)}$

$4x^2(1-x) = 4x^2+4x+1$ $\boxed{1}$ equating & rearranging.

$\cancel{4x^2} - 4x^3 = \cancel{4x^2} + 4x + 1$

$4x^3 + 4x + 1 = 0$

$\therefore$ β is the solution to $4x^3+4x+1$

v) let $f(x) = 4x^3+4x+1$

$f(-\frac{1}{2}) = -1.5$ $\boxed{1}$ Showing sign change

$f(0) = 1$ $\therefore$ A root must lie between $-\frac{1}{2}$ & 1

i) $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$ $\boxed{1}$

$x_1 = -\frac{1}{2}$ $\qquad f'(x) = 12x^2+4$

$f(x_1) = -1.5$

$f'(x_1) = 7$

$\therefore$ $x_2 = \cancel{-1.5} -\frac{1}{2} - \left[\dfrac{-1.5}{7}\right]$ $\boxed{1}$

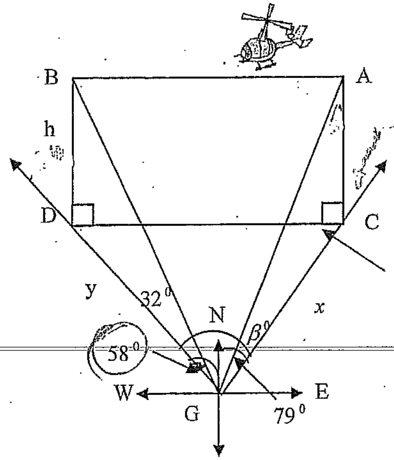
$= -0.29 \left(\frac{-2}{7}\right)$ (2dp)

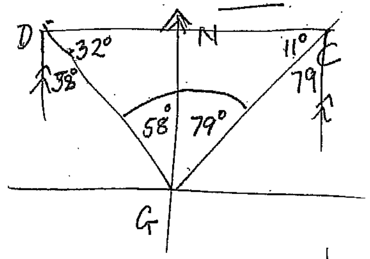
T

(15)

DC = 28000 m
ON the GROUND

(i)





$(90^0 - 79^0) = 11^0$

(ii) let $x = GC$ and $y = GD$. In ΔBDG $\cot 32^0 = \frac{y}{h} \Rightarrow y = h\cot 32^0$ .....A $BA = 420km \times \frac{1}{15} = 28km = 28000m$

From ΔCDG $\frac{y}{\sin 11^0} = \frac{28000}{\sin(79^0 + 58^0)} \Rightarrow y = \frac{28000 \sin 11^0}{\sin 137^0}$ .....B

Sub A into B $\therefore h\cot 32^0 = \frac{28000 \sin 11^0}{\sin 137^0} \Rightarrow h = \frac{28000\sin 11^0}{\sin 137^0 \cot 32^0} = 4895.11 = 4895\,m$

(iii) from ΔDGC $\frac{y}{\sin 11^0} = \frac{x}{\sin 32^0} \Rightarrow \frac{h\cot 32^0}{\sin 11^0} = \frac{h \cot \beta}{\sin 32^0}$

$\Rightarrow \cot\beta = \frac{\cot 32^0 \sin 32^0}{\sin 11^0}$ $\therefore \beta = 12.68^0 = 13^0$

7(b)(i) The equation of the first circle is

$(x-a)^2 + y^2 = a^2$ sub. $y = x\tan\alpha$

$\therefore (x-a)^2 + x^2\tan^2\alpha = a^2$

$x^2 - 2ax + a^2 + x^2\tan^2\alpha = a^2$

$x^2(1+\tan^2\alpha) - 2ax = 0$

$x^2\sec^2\alpha - 2ax = 0$

$x(x\sec^2\alpha - 2a) = 0$

$x\sec^2\alpha - 2a = 0$ we ignore $x = 0$ as it is the origin

$x = 2a\cos^2\alpha$

$y = x\tan\alpha \Rightarrow y = 2a\cos^2\alpha \times \frac{\sin\alpha}{\cos\alpha} = 2a\sin\alpha\cos\alpha$

$\therefore P(2a\cos^2\alpha,\ 2a\sin\alpha\cos\alpha)$

(ii) The equation of the second circle is

$x^2 + (y-b)^2 = b^2$

sub. $y = x\tan\alpha$ $\therefore x^2 + (x\tan\alpha - b)^2 = b^2$

$x^2 + x^2\tan^2\alpha - 2bx\tan\alpha + b^2 = b^2$

$x^2(1+\tan^2\alpha) - 2bx\tan\alpha + b^2 = b^2$

$x^2\sec^2\alpha - 2bx\tan\alpha = 0$

$x(x\sec^2\alpha - 2b\tan\alpha) = 0$ we ignore $x = 0$ as it is the origin

$\therefore x\sec^2\alpha - 2b\tan\alpha = 0$

$x = \frac{2b\tan\alpha}{\sec^2\alpha} = 2b\sin\alpha\cos\alpha$

$y = x\tan\alpha \Rightarrow y = 2b\sin\alpha\cos\alpha \times \frac{\sin\alpha}{\cos\alpha} = 2b\sin^2\alpha$

$\therefore Q(2b\sin\alpha\cos\alpha, 2b\sin^2\alpha)$

PLEASE READ & DO ⇑ even if you

(iii) The midpoint M has coords

$\left(\frac{2a\cos^2\alpha + 2b\sin\alpha\cos\alpha}{2}, \frac{2a\sin\alpha\cos\alpha + 2b\sin^2\alpha}{2}\right)$

$= [\cos\alpha(a\cos\alpha + b\sin\alpha), \sin\alpha(a\cos\alpha + b\sin\alpha)]$

HAVE NOT DONE (i) & (ii)