Centre Number

Student Number

CSSA

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NSW

2011
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Morning Session
Friday, 12 August 2011

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this page
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1–7
- All questions are of equal value

6300-1

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Total marks – 84
Attempt Questions 1–7
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Solve $4^{x-3} = 27$. Give your answer correct to two decimal places. **2**

(b) Evaluate $\int_0^1 \frac{1}{\sqrt{2-x^2}}\,dx$. **2**

(c) How many arrangements of the letters of the word AUSTRALIA are possible? **2**

(d) The graphs of the line $x+2y-1=0$ and the curve $y=1-x^3$ intersect at (1, 0). **3**

Find the size of the acute angle between the line and the tangent to the curve at the point of intersection.

(e) Use the substitution $u = 1+2x$, to find $\int \frac{x}{\sqrt{1+2x}}\,dx$. **3**

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\int_0^{\frac{\pi}{6}} \sin^2 3x\,dx$. **3**

(b) The polynomial $P(x)$ is given by $P(x) = 2x^3 + 7x^2 - 46x + 21$.

(i) Show that $P(x)$ is divisible by $(x-3)$. **1**

(ii) Hence, fully factorise $P(x)$. **2**

(c) A school swim squad consists of six girls and four boys. A team consisting of four of these students is to be chosen.

(i) In how many ways can the team be chosen? **1**

(ii) Among the squad are a brother and sister. **2**
A team of two boys and two girls is chosen at random. What is the probability that it contains the brother and sister?

(d) The polynomial equation $2x^3 - 4x^2 + 5x - 1 = 0$ has 3 roots α, β and γ.

(i) Find $\alpha\beta + \beta\gamma + \alpha\gamma$. **1**

(ii) Find $\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma}$. **2**

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) State the domain and range of $y = 3\cos^{-1} 2x$. **2**

(b) A pot of soup is to be refrigerated overnight. It needs to be cooled from its temperature of 100°C before it can be placed in the fridge.

The pot of soup is cooled in a sink full of cold water kept at a temperature of 5°C. The temperature of the pot of soup, $T^\circ\text{C}$, after t minutes in the sink satisfies the equation

$$\frac{dT}{dt} = -k(T-5).$$

(i) Show that $T = 5 + 95e^{-kt}$ satisfies both this equation and the initial condition. **2**

(ii) After 10 minutes in the sink, the temperature of the pot of soup reduces from 100°C to 60°C. It must be cooled to a temperature of 20°C before it can be placed in the fridge. **3**

How long (to the nearest minute) must the pot of soup remain in the sink full of cold water?

(c) (i) Find the vertical and horizontal asymptotes of the hyperbola $y = \dfrac{x-1}{2x-3}$ and hence sketch the graph of $y = \dfrac{x-1}{2x-3}$. **3**

(ii) Hence, or otherwise, find the values of x for which $\dfrac{x-1}{2x-3} \geq -1$. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Use mathematical induction to prove that $\sum_{r=1}^{n} (r^2+1)r! = n(n+1)!$ **3**

(b) Let $g(x) = 2x^3 + x + 4$.

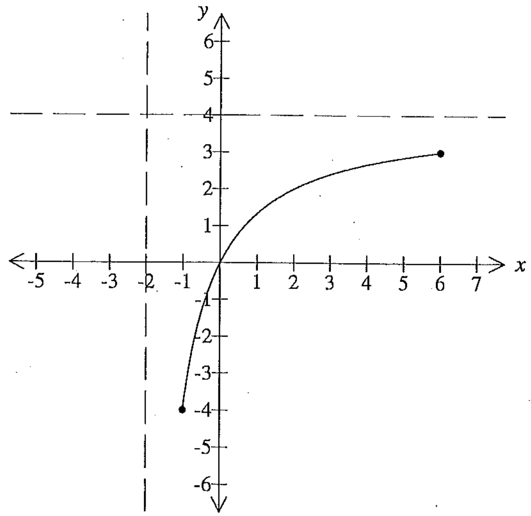
(i) Show that $g(x) = 0$ has a root between the integers -1 and -2. **1**

(ii) Taking $x = -1.5$ as the first approximation to this root, use one application of Newton's method to obtain a better approximation for this root. **2**

Give this approximation correct to 2 significant figures.

(iii) Explain why the function $y = g(x)$ has only one x-intercept. **1**

(c) The diagram below shows a sketch of the graph of $y = f(x)$, where $f(x) = \dfrac{4x}{x+2}$ for $-1 \leq x \leq 6$.



(i) Copy or trace this diagram into your writing booklet. **2**

On the same set of axes, sketch the graph of the inverse function, $y = f^{-1}(x)$.

Clearly show any points of intersection with $y = f(x)$.

(ii) Find an expression for $f^{-1}(x)$ in terms of x. **3**

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) Find the term independent of x in the expansion of $\left(x^2 - \frac{1}{2x}\right)^{12}$. **3**

(b) The velocity, v ms^{-1}, of a particle moving in simple harmonic motion along the x-axis is given by $v^2 = 8 - 2x - x^2$, where x is in metres.

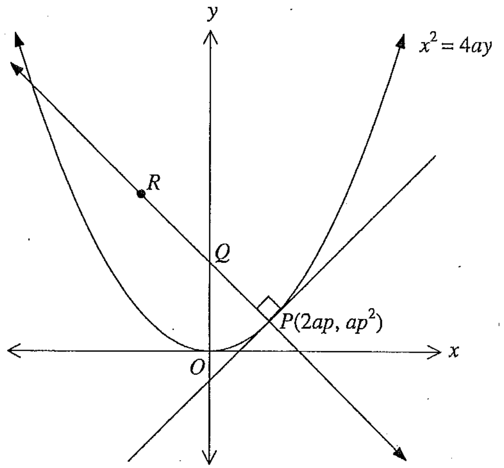
(i) Between which two points is the particle oscillating? **2**

(ii) Find the centre of the motion. **1**

(iii) Find the maximum speed. **1**

(iv) Find an expression for the acceleration of the particle in terms of x. **1**

(c)



The diagram shows a variable point $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$.

The normal to the parabola at P intersects the y-axis at Q. The point Q is the midpoint of PR.

The equation of the normal is $x + py - 2ap - ap^3 = 0$. (Do NOT prove this.)

(i) Find the coordinates of the point Q. **1**

(ii) The locus of the point R is a parabola. **3**

Find the equation of this parabola in Cartesian form and state its vertex.

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Express $\cos x - \sin x$ in the form $A\cos(x + \alpha)$, where $0 \le \alpha \le \frac{\pi}{2}$. **2**

(ii) Hence, or otherwise, solve $\cos x - \sin x = 1$ for $0 \le x \le 2\pi$. **2**

(b) In the binomial expansion of $\left(1 + \frac{x}{k}\right)^n$, the coefficient of x^3 is twice the coefficient of x^2. **2**

Prove that $n = 6k + 2$.

(c) On the way to work, Mathilda passes through a particular intersection. She notices that the traffic lights are red for 15 seconds, amber for 5 seconds and green for 30 seconds. **2**

Mathilda passes through this intersection a total of five times in a week.

What is the probability that she is stopped by a red light on exactly one occasion?

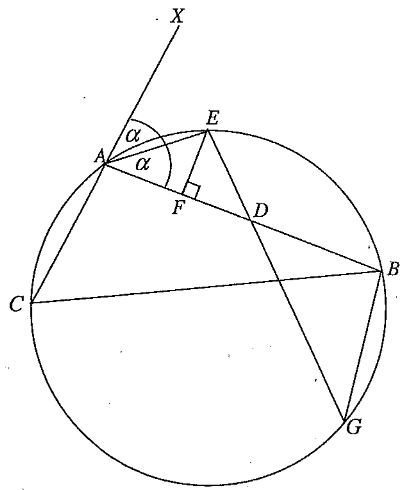
Question 6 continues on page 9

Question 6 (continued)

(d) The diagram shows a circle which passes through the vertices A, B, and C of a triangle. The point E lies on the circle with AE bisecting the exterior angle, $\angle BAX$, of the triangle such that $\angle XAE = \angle BAE = \alpha$.

The point F lies on AB such that $EF \perp AB$.

The chord EG intersects AB at D such that $DF = AF$ and $\Delta EFD \equiv \Delta EFA$.



(i) Explain why $\angle BGE = \angle BAE$. **1**

(ii) Hence, show that $\angle BDG = \angle BGD = \alpha$. **1**

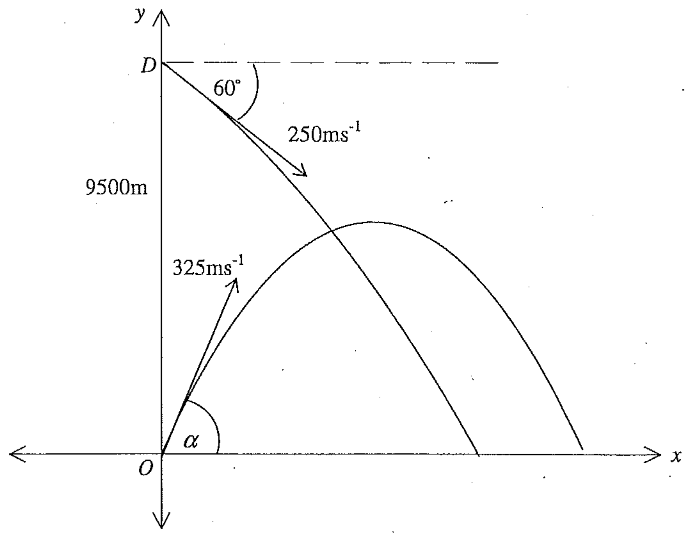
(iii) Show that $\Delta CAB \equiv \Delta GBA$. **2**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) During an army exercise, a surface to air missile is launched from the point O in order to intercept a dummy bomb that is released from a point D.

The point D is 9500 metres directly above O.



The dummy bomb is released at an angle of 60° below the horizontal, with a velocity of 250 ms^{-1}. It can be shown that the equations of motion of the dummy bomb are:

$$x_D = 125t \quad \text{and} \quad y_D = 9500 - 125\sqrt{3}t - 5t^2. \text{ (Do NOT prove this.)}$$

(i) Calculate how long it would take the dummy bomb to reach the ground (correct to the nearest second) and where it would strike the ground (correct to the nearest metre). **2**

Question 7 continues on page 11

Question 7 (continued)

The missile is launched at the same time as the dummy bomb is released. It is launched with an initial velocity of 325 ms^{-1} and its angle of projection *above* the horizontal is α.

The equations of motion of the missile are:

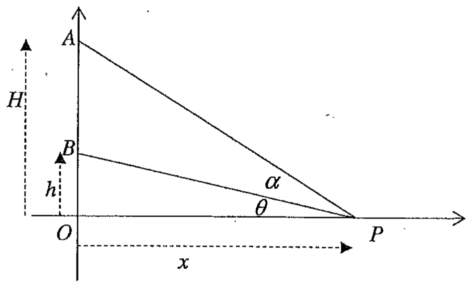
$$x_M = 325t\cos\alpha \text{ and } y_M = 325t\sin\alpha - 5t^2. \text{ (Do NOT prove this.)}$$

(ii) Show that in order for the missile to intercept the dummy bomb it must be launched with an angle of projection $\alpha = \cos^{-1}\left(\frac{5}{13}\right)$. **1**

(iii) How high above the ground, correct to the nearest metre, does the collision occur? **3**

(b) The diagram shows the point P on the horizontal axis, a variable distance x from the origin O.

The points A and B are fixed points on the vertical axis, with distances H and h respectively, from the origin O.



Let $\angle BPO = \theta$ and $\angle APB = \alpha$.

(i) Show that $\alpha = \tan^{-1}\left(\frac{H}{x}\right) - \tan^{-1}\left(\frac{h}{x}\right)$. **1**

(ii) Show that α is a maximum when $x^2 = Hh$. **3**

(There is no need to justify the maximum.)

(iii) Using the expansion of $\tan(A-B)$, or otherwise, show that the maximum value of α occurs when $\tan\alpha = \frac{H-h}{2\sqrt{Hh}}$. **2**

End of paper

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MATHEMATICS EXTENSION 1

Question 1 (12 marks)

(a) (2 marks)

Outcomes assessed: H3

Targeted Performance Bands: E2

Criteria	Marks
• Correct answer (no penalty for incorrect rounding)	2
• Substantial progress towards solution	1

Sample Answer:

$$4^{x-3}=27$$
$$(x-3)\ln 4=\ln 27$$
$$x-3=\frac{\ln 27}{\ln 4}=2.377..$$
$$x=5.38(2dp)$$

(b) (2 marks)

Outcomes assessed: HE4

Targeted Performance Bands: E2

Criteria	Marks
• Correct answer	2
• Chooses correct inverse trig function	1

Sample Answer:

$$\int_0^1 \frac{1}{\sqrt{2-x^2}}\,dx=\left[\sin^{-1}\frac{x}{\sqrt{2}}\right]_0^1$$
$$=\left[\sin^{-1}\frac{1}{\sqrt{2}}-\sin^{-1}0\right]=\frac{\pi}{4}$$

(c) (2 marks)

Outcomes assessed: PE3

Targeted Performance Bands: E2

Criteria	Marks
• Correct solution (no need to simplify)	2
• Finds correct number of permutations with no repetition	1

Sample Answer:

Arrangements of the letters of the word AUSTRALIA $=\frac{9!}{3!}=60480$

(d) (3 marks)

Outcomes assessed: PE2, HE7

Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct answer	3
• Progress towards finding the angle between the line and the curve	2
• Correctly determines the gradient of the curve at the given point	1

Sample Answer:

$$x+2y-1=0\rightarrow m_1=-\frac{1}{2}$$

$$y=1-x^3\rightarrow\frac{dy}{dx}=-3x^2$$
$$\therefore m_2=-3(1)^2=-3\ at\,(1,0)$$

$$\tan\theta=\left|\frac{-3-\left(\frac{-1}{2}\right)}{1+-3\times\frac{-1}{2}}\right|=\left|\frac{-2\frac{1}{2}}{2\frac{1}{2}}\right|=1$$

$$\theta=45^\circ$$

DISCLAIMER
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(e) (3 marks)
Outcomes assessed: HE6
Targeted Performance Bands: E2

Criteria	Marks
• Correct solution ($+c$ not essential)	3
• Finds the correct primitive	2
• Correctly uses substitution to rewrite the integral	1

Sample Answer:

$u = 1+2x$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2}du$

$$\begin{aligned}\int \frac{x}{\sqrt{1+2x}}\,dx &= \frac{1}{2}\int \frac{u-1}{\sqrt{u}} \times \frac{1}{2}du \\ &= \frac{1}{4}\int u^{\frac{1}{2}} - u^{-\frac{1}{2}} du \\ &= \frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right) \\ &= \frac{1}{6}(1+2x)^{\frac{3}{2}} - \frac{1}{2}(1+2x)^{\frac{1}{2}} + c\end{aligned}$$

Question 2 (12 marks)
(a) (i) (3 marks)
Outcomes assessed: HE6
Targeted Performance Bands: E2–E3

Criteria	Marks
• Gives correct answer	3
• Correctly evaluates the integral apart from a minor error	2
• Uses the substitution $\sin^2 3x = \frac{1}{2}(1-\cos 6x)$ OR • Correctly integrates a non-trivial expression used to replace $\sin^2 3x$	1

Sample Answer:

$$\begin{aligned}\int_0^{\frac{\pi}{6}} \sin^2 3x\,dx &= \frac{1}{2}\int_0^{\frac{\pi}{6}} (1-\cos 6x)\,dx \\ &= \frac{1}{2}\left[x - \frac{1}{6}\sin 6x\right]_0^{\frac{\pi}{6}} \\ &= \frac{1}{2}\left[\left(\frac{\pi}{6} - \frac{1}{6}\sin\pi\right) - \left(0 - \frac{1}{6}\sin 0\right)\right] = \frac{\pi}{12}\end{aligned}$$

(b) (i) (1 mark)
Outcomes assessed: PE3
Targeted Performance Bands: E2

Criteria	Mark
• Correct solution	1

Sample Answer:

$P(x) = 2x^3 + 7x^2 - 46x + 21$

$P(3) = 2(3)^3 + 7(3)^2 - 46(3) + 21 = 0$

$\therefore\ x-3$ is a factor

(b) (ii) (2 marks)
Outcomes assessed: PE3
Targeted Performance Bands: E2

Criteria	Marks
• Correct factorisation	2
• Finds correct quadratic factor or equivalent progress	1

Sample Answer:

$$\begin{aligned}P(x) &= 2x^3 + 7x^2 - 46x + 21 \\ &= (x-3)(2x^2 + 13x - 7) \\ &= (x-3)(x+7)(2x-1)\end{aligned}$$

(c) (i) (1 mark)
Outcomes assessed: PE3
Targeted Performance Bands: E3

Criteria	Mark
• Correct answer	1

Sample Answer:

$$^{10}C_4 = 210$$

(c) (ii) (2 marks)
Outcomes assessed: PE3, HE5
Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct solution	2
• Finds the number of teams including the brother and sister, or equivalent progress	1

Sample Answer:

$$\frac{^5C_1\,^3C_1}{^6C_2\,^4C_2} = \frac{1}{6}$$

(d) (i) (1 mark)
Outcomes assessed: PE3
Targeted Performance Bands: E2

Criteria	Mark
• Correct answer	1

Sample Answer:

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = \frac{5}{2}$$

(d) (ii) (2 marks)
Outcomes assessed: PE3
Targeted Performance Bands: E2

Criteria	Marks
• Correct answer	2
• Significant progress towards the correct answer	1

Sample Answer:

$$\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma} = \frac{2(\alpha\beta + \beta\gamma + \gamma\alpha)}{\alpha\beta\gamma} = \frac{2\left(\frac{5}{2}\right)}{\frac{1}{2}} = 10$$

Question 3 (12 marks)
(a) (2 marks)
Outcomes assessed: HE4
Targeted Performance Bands: E3

Criteria	Marks
• Correct answers	2
• Either correct domain or correct range	1

Sample Answer:

$$y = 3\cos^{-1} 2x$$

Domain: $-1 \le 2x \le 1$, $-\frac{1}{2} \le x \le \frac{1}{2}$

Range: $0 \le \cos^{-1} 2x \le \pi$, $0 \le 3\cos^{-1} 2x \le 3\pi$, $\therefore\ 0 \le y \le 3\pi$

(b) (i) (2 marks)
Outcomes assessed: HE3, HE7
Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct solution	2
• Shows that the given equation satisfies the initial condition or equivalent merit	1

Sample Answer:

$$T = 5 + 95e^{-kt} \rightarrow 95e^{-kt} = T - 5 \qquad t = 0, T = 5 + 95e^0 = 5 + 95 = 100$$

$$\frac{dT}{dt} = -95ke^{-kt} = -k(T-5) \qquad \therefore \text{ satisfies initial condition}$$

(b) (ii) (3 marks)
Outcomes assessed: HE3, HE7
Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct solution	3
• Makes significant progress towards finding t	2
• Finds k or equivalent merit	1

Sample Answer:

$t = 10, T = 60 \rightarrow$

$$60 = 5 + 95e^{-10k}$$

$$\frac{55}{95} = e^{-10k}$$

$$k = -\frac{1}{10}\ln\left(\frac{55}{95}\right)$$

$T = 20, t = ?$

$$20 = 5 + 95e^{-\left[-\frac{1}{10}\ln\left(\frac{55}{95}\right)\right]t}$$

$$\ln\left(\frac{15}{95}\right) = \frac{1}{10}\ln\left(\frac{55}{95}\right)t$$

$$t = \ln\left(\frac{15}{95}\right) \div \frac{1}{10}\ln\left(\frac{55}{95}\right)$$

$\therefore t = 33.7727.. = 34$ minutes (nearest minut

(c) (i) (3 marks)

Outcomes assessed: PE5, HE7

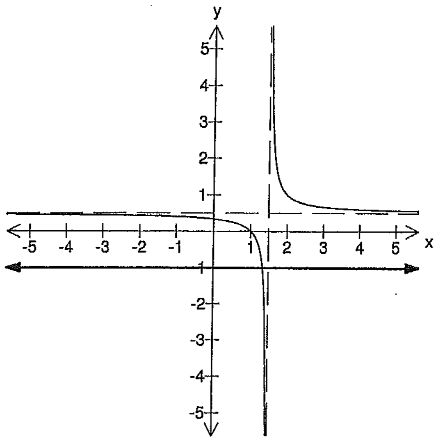
Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct solution	3
• Both asymptotes correct, or equivalent merit	2
• Correct horizontal or vertical asymptote	1

Sample Answer:

$y=\dfrac{x-1}{2x-3}$ Vertical asymptote: $x=\dfrac{3}{2}$ Horizontal asymptote $\lim\limits_{x\to\infty}\dfrac{x-1}{2x-3}=\lim\limits_{x\to\infty}\dfrac{1-\frac{1}{x}}{2-\frac{3}{x}}=\dfrac{1}{2}$

$\therefore y=\dfrac{1}{2}$



(c) (ii) (2 marks)

Outcomes assessed: PE6

Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct solution	2
• Finds the point of intersection of $y=-1$ with curve	1

Sample Answer:

$y=-1$ drawn on above graph

$\dfrac{x-1}{2x-3}=-1\rightarrow x-1=-2x+3$

$3x=4$

$x=\dfrac{4}{3}$

$\dfrac{x-1}{2x-3}\geq -1$

$\therefore x\leq\dfrac{4}{3},\ x>\dfrac{3}{2}$ This is where the curve is above the line, $y=-1$.

Question 4 (12 marks)

(a) (3 marks)

Outcomes assessed: HE2

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	3
• Verifies the case for $n=1$ or equivalent merit and attempts to use the induction assumption with minor errors	2
• Verifies the case for $n=1$ or equivalent merit	1

Sample Answer:

$\sum\limits_{r=1}^{n}\left(r^2+1\right)r!=n(n+1)!$

$\left(1^2+1\right)1!+\left(2^2+1\right)2!+\left(3^2+1\right)3!+\ldots+\left(n^2+1\right)n!=n(n+1)!$

when $n=1$, $LHS=\left(1^2+1\right)1!=2$

$RHS=1(1+1)!=2$ $\therefore$ true for $n=1$

Assume true for $n=k$: $\left(1^2+1\right)1!+\left(2^2+1\right)2!+\left(3^2+1\right)3!+\ldots+\left(k^2+1\right)k!=k(k+1)!$

$S_k=k(k+1)!$

when $n=k+1$, $S_{k+1}=S_k+\left((k+1)^2+1\right)(k+1)!$

$=k(k+1)!+\left(k^2+2k+1+1\right)(k+1)!$

$=(k+1)!\left[k+k^2+2k+1+1\right]$

$=(k+1)!\left[k^2+3k+2\right]$

$=(k+1)!\left[(k+2)(k+1)\right]=(k+2)!(k+1)$

$\therefore$ true for $n=k+1$,

and since true for $n=1$, then by induction true for all integral values of $n\geq 1$.

(b) (i) (1 mark)

Outcomes assessed: HE7

Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	1

Sample Answer:

$g(x)=2x^3+x+4$

$g(-1)=2(-1)^3+(-1)+4=1>0$

$g(-2)=2(-2)^3+(-2)+4=-14<0$

Since there is a sign change and the curve is continuous,

then there is a root between $x=-1$ and $x=-2$

(b) (ii) (2marks)

Outcomes assessed: HE7

Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	2
• Significant progress towards correct approximation	1

Sample Answer:

$g(x)=2x^3+x+4 \qquad \therefore g'(x)=6x^2+1$

For $x_1=-1.5 \quad g(x_1)=2(-1.5)^3+(-1.5)+4=-4.25.$

$g'(x_1)=6(-1.5)^2+1=14.5.$

$$\therefore x_2=x_1-\frac{g(x_1)}{g'(x_1)}$$

$$=-1.5-\frac{-4.25}{14.5}$$

$$=-1.206..$$

$$\doteqdot -1.2 \ (2\ s.f.)$$

(b) (iii) (1 mark)

Outcomes assessed: HE7

Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	1

Sample Answer:

Since $g'(x)=6x^2+1>0$ for all x then $g(x)$ is always increasing and therefore can only intersect the x-axis once.

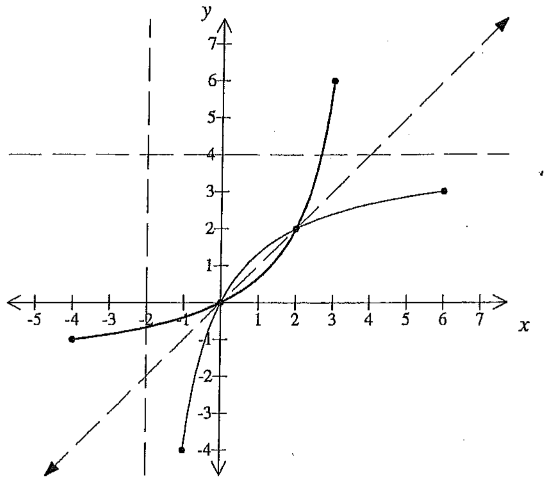
(c) (i) (2 marks)

Outcomes assessed: HE4

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct graph with clearly identifiable points of intersection (no penalty for incorrect domain)	2
• Attempts to reflect the graph in the line $y=x$	1

Sample Answer:



(c) (ii) (3 marks)

Outcomes assessed: HE4

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct expression for $f^{-1}(x)$ with correct domain	3
• Correct expression for $f^{-1}(x)$	2
• Obtains $x=\dfrac{4y}{y+2}$, or equivalent merit	1

Sample Answer:

$$x=\frac{4y}{y+2}$$

$$xy+2x=4y$$

$$y(4-x)=2x$$

$$y=\frac{2x}{4-x}, \qquad -4\le x\le 3$$

Question 5 (12 marks)
(a) (3 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E3–E4

Criteria	Marks
• Finds the correct term	3
• Establishes the value of n which gives the correct term	2
• Writes a correct expression for T_{n+1}	1

Sample Answer:

$$T_{n+1} = \binom{12}{n}(x^2)^{12-n}\left(\frac{-1}{2x}\right)^n = \binom{12}{n}x^{24-2n}\left(\frac{-1}{2}\right)^n x^{-n}$$

$$= \binom{12}{n}\left(\frac{-1}{2}\right)^n x^{24-3n}$$

For the term independent of x: $24-3n=0 \quad \therefore n=8,$

$\therefore$ The term independent of x is $\binom{12}{8}\left(\frac{-1}{2}\right)^8 = \frac{495}{256}$

(b) (i) (2 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	2
• Attempts to find when $v^2=0$ or equivalent merit	1

Sample Answer:

$8-2x-x^2=0$

$(4+x)(2-x)=0$ The particle is oscillating between $x=-4$ and $x=2$

$x=-4$ or $x=2$

(b) (ii) (1 mark)
Outcomes assessed: HE3
Targeted Performance Bands: E3–E4

Criteria	Mark
• Correct answer	1

Sample Answer:

The centre of motion is $x=-1$.

(b) (iii) (1 mark)
Outcomes assessed: HE3
Targeted Performance Bands: E3–E4

Criteria	Mark
• Correct answer	1

Sample Answer:

The maximum speed occurs at the centre of motion.

$v^2=8-2(-1)-(-1)^2=9 \qquad \therefore Max\ speed\ v=3$

(b) (iv) (1 mark)
Outcomes assessed: HE3
Targeted Performance Bands: E3–E4

Criteria	Mark
• Correct solution	1

Sample Answer:

$$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\left(4-x-\frac{x^2}{2}\right)$$
$$= -1-x$$

(c) (i) (1 mark)
Outcomes assessed: P4
Targeted Performance Bands: E2

Criteria	Mark
• Correct answer	1

Sample Answer:

$x+py-2ap-ap^3=0 \rightarrow Q(0,?) \qquad \therefore py-2ap-ap^3=0$

$py=2ap+ap^3$

$y=2a+ap^2 \rightarrow Q(0,\ 2a+ap^2)$

(c) (ii) (3marks)

Outcomes assessed: PE3

Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct solution	3
• Finds the Cartesian equation of the locus of R or equivalent merit	2
• Finds coordinates of R or equivalent merit	1

Sample Answer:

$$R=(x,y)\to \quad 0=\frac{x+2ap}{2} \qquad 2a+ap^2=\frac{y+ap^2}{2}$$

$$0=x+2ap \qquad 4a+2ap^2=y+ap^2$$

$$x=-2ap \qquad y=4a+ap^2$$

$$\therefore p=\frac{-x}{2a} \qquad y=4a+a\left(\frac{-x}{2a}\right)^2$$

$$y=4a+\frac{x^2}{4a} \quad \to x^2=4a(y-4a)$$

$$Vertex(0,4a)$$

Question 6 (12 marks)

(a) (i) (2 marks)

Outcomes assessed: HE3

Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct answer	2
• Correctly obtains A or α, or equivalent progress	1

Sample Answer:

$$A\cos(x+\alpha)=A\cos x\cos\alpha-A\sin x\sin\alpha$$

$$\therefore \left.\begin{array}{l}A\cos\alpha=1\\ A\sin\alpha=1\end{array}\right\}\tan\alpha=1\to\alpha=\frac{\pi}{4} \qquad A=\sqrt{1^2+1^2}=\sqrt{2}$$

$$\therefore \cos x-\sin x=\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)$$

(a) (ii) (2 marks)

Outcomes assessed: HE3

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	2
• Obtains one correct result for $x+\alpha$	1

Sample Answer:

$$\cos x-\sin x=1$$

$$\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)=1$$

$$\cos\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}} \qquad \therefore\ x+\frac{\pi}{4}=\frac{\pi}{4},\frac{7\pi}{4},\frac{9\pi}{4}$$

$$x=0,\frac{3\pi}{2},2\pi$$

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(b) (2 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	2
• Sets up a correct identity, or equivalent merit	1

Sample Answer:

$$\left(1+\frac{x}{k}\right)^n \to \textit{coefficient of } x^3 = 2\times \textit{coefficient of } x^2$$

$$\binom{n}{3}\left(\frac{1}{k}\right)^3 = 2\times\binom{n}{2}\left(\frac{1}{k}\right)^2$$

$$\frac{n!}{3!(n-3)!k^3} = \frac{2(n!)}{2!(n-2)!k^2} \to \frac{n(n-1)(n-2)}{6k^3} = \frac{n(n-1)}{k^2}$$

$$n-2=6k \qquad \therefore n = 6k+2$$

(c) (2 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct answer	2
• Correctly sets up binomial probability, or equivalent merit	1

Sample Answer:

P (red) = $\frac{15}{50} = \frac{3}{10}$ P(not red) = $\frac{7}{10}$

P (exactly one red in 5 times) = ${}^5C_1\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)^4 = \frac{7203}{20000} = 0.36015$

(d) (i) (1 mark)
Outcomes assessed: PE3
Targeted Performance Bands: E2

Criteria	Mark
• Correct reason	1

Sample Answer:

Angles at the circumference subtended by the same arc *EB* are equal.

(d) (ii) (1 mark)
Outcomes assessed: PE3
Targeted Performance Bands: E2

Criteria	Mark
• Correct solution	1

Sample Answer:

$\angle EAF = \angle EDF = \alpha$ (*corresponding angles of congruent triangles* $\Delta EFD \equiv \Delta EFA$)

$\angle EDF = \angle BDG = \alpha$ (*vertically opposite angles*)

$\therefore \angle BDG = \alpha = \angle BGD$ (*proven in (i)*)

(d) (iii) (2 marks)
Outcomes assessed: HE5
Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct solution	2
• Significant progress towards correct solution	1

Sample Answer:

$\angle CAB = 180^\circ - 2\alpha$ (*adj angles on a straight angle*)

$\angle GBA = 180^\circ - 2\alpha$ (*angle sum of* ΔDBG)

$\angle CAB = \angle GBA$

$\angle ACB = \angle BGA$ (*angles subtended by the same arc AB*)

$AB = BA$ (*common side*)

$\therefore \Delta CAB \equiv \Delta GBA$ (*AAS*)

Question 7 (12 marks)
(a) (i) (2 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	2
• Finds when the dummy bomb hits the ground, or uses incorrect time to find where it strikes the ground	1

Sample Answer:

Hits the ground $y_D = 0$

$$0 = 9500 - 125\sqrt{3}t - 5t^2$$
$$t^2 + 25\sqrt{3}t - 1900 = 0$$
$$t = \frac{-25\sqrt{3} \pm \sqrt{\left(25\sqrt{3}\right)^2 - 4\times -1900}}{2}$$
$$= \frac{-25\sqrt{3} \pm \sqrt{9475}}{2}$$
$$t = 27.01917\ldots, -70.3204..$$

Reaches the ground after 27 seconds (nearest second)

When $t = 27.01917\ldots$, $x_D = 125(27.01917..) = 3377.396..$
Strikes the ground at 3377 m (or 3375 m if $t = 27$ is used)

(a) (ii) (1 mark)
Outcomes assessed: HE3
Targeted Performance Bands: E3–E4

Criteria	Mark
• Correct solution	1

Sample Answer:

$$x_D = x_M \rightarrow \quad 125t = 325t\cos\alpha$$
$$\cos\alpha = \frac{125t}{325t} = \frac{5}{13}$$
$$\therefore \alpha = \cos^{-1}\left(\frac{5}{13}\right)$$

(a) (iii) (3 marks)
Outcomes assessed: HE3
Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	3
• Correctly finds the value of t	2
• Equates $y_D = y_M$ and attempts to find t or correctly finds the value of $\sin\alpha$	1

Sample Answer:

Collision occurs when

$$y_D = y_M \rightarrow \quad 9500 - 125\sqrt{3}t - 5t^2 = 325t\sin\alpha - 5t^2$$
$$9500 - 125\sqrt{3}t = 325t\sin\alpha \qquad \sin\alpha = \frac{12}{13}\left(from\,(ii)\right)$$
$$9500 - 125\sqrt{3}t = 325t\left(\frac{12}{13}\right)$$
$$9500 = 300t + 125\sqrt{3}t$$
$$t = \frac{9500}{300 + 125\sqrt{3}} = 18.3928\ldots$$

$$y_M = 325(18.3928..)\frac{12}{13} - 5(18.3928..)^2$$
$$= 3826.3649..$$
$$= 3826\ m$$

(b) (i) (1 mark)
Outcomes assessed: HE4
Targeted Performance Bands: E3–E4

Criteria	Mark
• Correct solution	1

Sample Answer:

In ΔAPO, $\tan(\theta + \alpha) = \frac{H}{x} \rightarrow \tan^{-1}\left(\frac{H}{x}\right) = \theta + \alpha$

In ΔBPO, $\tan\theta = \frac{h}{x} \rightarrow \tan^{-1}\left(\frac{h}{x}\right) = \theta$

$$\therefore \alpha = (\theta + \alpha) - \theta = \tan^{-1}\left(\frac{H}{x}\right) - \tan^{-1}\left(\frac{h}{x}\right)$$

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(b) (ii) (3 marks)

Outcomes assessed: HE4

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	3
• Finds an expression for $\frac{d\alpha}{dx}$	2
• Correctly differentiates either $\tan^{-1}\left(\frac{H}{x}\right)$ or $\tan^{-1}\left(\frac{h}{x}\right)$	1

Sample Answer:

For max $\alpha, \frac{d\alpha}{dx}=0$

$$\frac{d\alpha}{dx}=\frac{\frac{-H}{x^2}}{1+\left(\frac{H}{x}\right)^2}-\frac{\frac{-h}{x^2}}{1+\left(\frac{h}{x}\right)^2}$$

$$=\frac{-H}{x^2+H^2}+\frac{h}{x^2+h^2}$$

$$\therefore \quad \frac{d\alpha}{dx}=0 \quad \text{when} \quad \frac{H}{x^2+H^2}=\frac{h}{x^2+h^2}$$

$$Hx^2+Hh^2=hx^2+H^2h$$

$$x^2(H-h)=Hh(H-h)$$

$$x^2=Hh \text{ (since } H\neq h \text{ for maximum } \alpha)$$

(b) (iii) (2 marks)

Outcomes assessed: HE4

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	2
• Applies the expansion of $\tan(A-B)$, or equivalent merit	1

Sample Answer:

α is a max when $x=\sqrt{Hh}$ (taking $x>0$)

$$\alpha=\tan^{-1}\left(\frac{H}{\sqrt{Hh}}\right)-\tan^{-1}\left(\frac{h}{\sqrt{Hh}}\right) \text{ (using (i))}$$

$$\therefore \tan\alpha=\tan\left[\tan^{-1}\left(\frac{H}{\sqrt{Hh}}\right)-\tan^{-1}\left(\frac{h}{\sqrt{Hh}}\right)\right]$$

$$=\frac{\frac{H}{\sqrt{Hh}}-\frac{h}{\sqrt{Hh}}}{1+\frac{H}{\sqrt{Hh}}\times\frac{h}{\sqrt{Hh}}}$$

$$=\frac{\frac{H-h}{\sqrt{Hh}}}{2}$$

$$=\frac{H-h}{2\sqrt{Hh}}$$

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