CSSA

CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW

2011
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Monday 8 August 2011

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back page
- All necessary working should be shown in every question
- Write your Centre Number and Student Number at the top of this page

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

6200-1

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Total marks – 120
Attempt Questions 1–10
All questions are of equal value.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Solve $\dfrac{7+5x}{6} \geq 2$. **2**

(b) Simplify $\dfrac{a-7}{a^2-49}$. **2**

(c) At the end of 2010, home loan rates increased by 0.4%. The monthly repayment is now $3514. What was the original repayment? **2**

(d) Differentiate $\dfrac{3}{x^2} + 7x^3$. **2**

(e) Solve the pair of simultaneous equations: **2**
$$y = x^2 + 4x$$
$$2x + y = 72$$

(f) Simplify $\sqrt{108} + 2\sqrt{48}$. Leave your answer in exact form. **2**

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) The derivative of a function is given by $\dfrac{dy}{dx} = \dfrac{7}{x}$. **2**
Find the equation of the curve if it passes through $(1, 6)$.

(b) Differentiate:

(i) $\cos(e^x)$ **2**

(ii) $\dfrac{4x-9}{8x+5}$. **2**

(c) (i) Find $\displaystyle\int \sec^2 3x \; dx$. **1**

(ii) Evaluate $\displaystyle\int_0^3 \dfrac{1}{(2x-3)^2} \, dx$. **3**
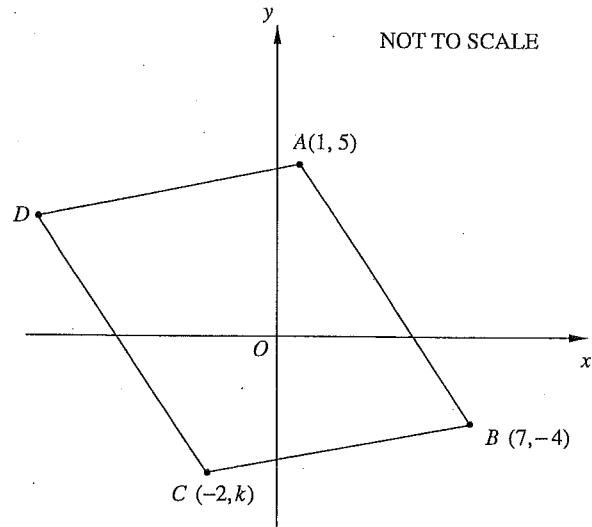
(d)
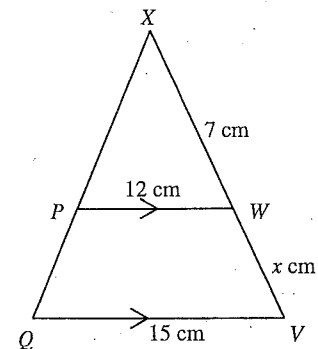
NOT TO SCALE

In the diagram, triangles XPW and XQV are similar. **2**
Find the length of WV.

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) Find the gradient of the normal to the curve $y = xe^x$ at the point where $x = 2$.   **2**

(b) In the diagram A, B and C have coordinates $(1, 5)$, $(7, -4)$ and $(-2, k)$ respectively. C is in the 3rd quadrant and $ABCD$ is a parallelogram.


(i) Find the gradient of AB.   **1**

(ii) Show that the equation of AB is $3x + 2y - 13 = 0$.   **1**

(iii) Write down an expression, in terms of k, for the perpendicular distance from C to the line AB.   **1**

(iv) Find the length of the interval AB.   **1**

(v) Given that the area of $ABCD$ is 90 square units, find the value of k.   **3**

Question 3 (continued)

(c) From a point P above the ground, the angle of elevation to the top of a leaning wall T is $42°$ and the angle of depression to the base of the leaning wall B is $28°$.

A wire 20m long is attached from this point P to the top of the leaning wall T and another 15m wire is attached to the base of the leaning wall B.
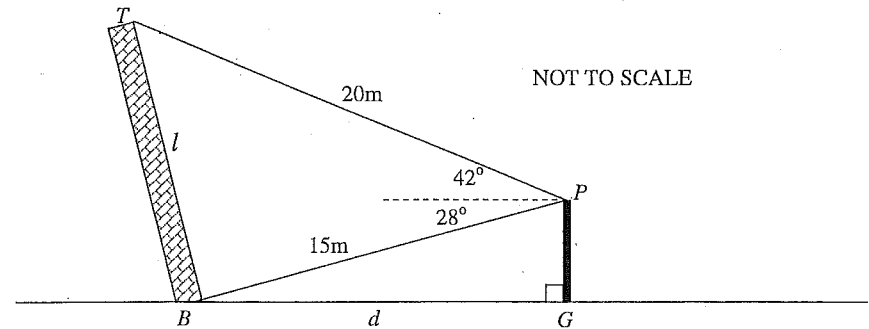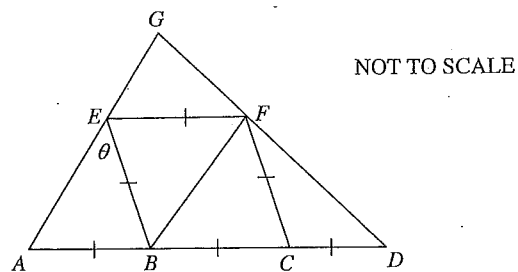

(i) Find the horizontal distance (d) in the diagram above.   **1**

(ii) Calculate the slant height (l) of the leaning wall.   **2**

End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Write down the discriminant of $3x^2 - 2mx + 5m$. **1**

(ii) For what values of m does $3x^2 - 2mx + 5m = 0$ have no real roots? **2**

(b) $ABCD$ is a straight line such that $AB = BC = CD$.
$EFCB$ is a rhombus and $\angle AEB = \theta$.

NOT TO SCALE

(i) Show that $\angle EBC = 2\theta$. **1**

(ii) The diagonal BF is drawn in the rhombus $EFCB$. Show that BF is parallel to AE. **2**

(iii) Hence, or otherwise, find the size of $\angle AGD$. **2**

(c) Vincent travels to school everyday by train. He has found that the probability he misses his train on Mondays is 30% while on Tuesdays it is 20%.

(i) Draw a probability tree diagram for Monday & Tuesday. **1**

(ii) Find the probability that Vincent misses the train on ONE of these days. **1**

(d) (i) Show that $g(x) = \sqrt{36 - x^2}$ is an even function. **1**

(ii) Find the range of $g(x)$. **1**

**Question 5** (12 marks) Use a SEPARATE writing booklet.

(a) Solve $2\sin^2 \alpha - \sin \alpha = 0$ for $0 < \alpha < 2\pi$. **3**

(b) The graphs of the functions $y = x^2 - 6x + 8$ and $y = 4x - x^2$ are shown in the diagram below. They intersect at P and $(4,0)$.

NOT TO SCALE
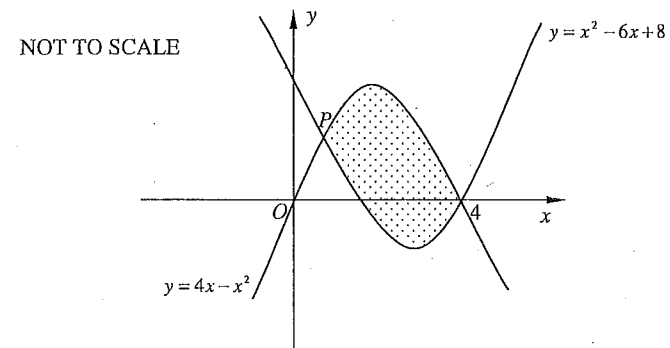
(i) Show that the coordinates of P is $(1, 3)$. **1**

(ii) Calculate the area of the shaded region. **3**

(c) Evaluate w for the limiting sum $5 + 5w^2 + 5w^4 + \ldots = \dfrac{49}{8}$. **3**

(d) Find $\dfrac{d}{dx}\left(\ln(\sin x)\right)$. Leave your answer in simplified form. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a)

NOT TO SCALE

Storm water pipe   Cross-section

8 m

4 m
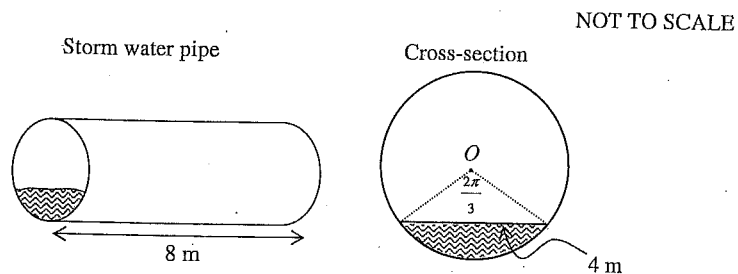
The diagram above shows a stormwater pipe and the circular cross-section of this pipe. The shaded area represents water that flows through the pipe. The surface of the water in this pipe is 4 m wide and subtends an angle of $\dfrac{2\pi}{3}$ as shown in the cross-section.

(i) Show that the radius of the stormwater pipe is $\dfrac{4}{\sqrt{3}}$ m.   **1**

(ii) Find the shaded area of the cross-section correct to ONE decimal place.   **2**

(iii) The pipe is 8 m in length. If the water level remains constant, find the volume of water which passes through the pipe every 10 minutes if the rate of water flow is 500L/min.

Give your answer in kL.   **2**

(b) Consider the curve $y = x^3 + 6x^2 - 36x + 15$.

(i) Find the stationary points and determine their nature.   **3**

(ii) Find the point of inflexion.   **1**

(iii) Sketch the curve.   **2**

(iv) For what values is the curve concave down?   **1**

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) A stained glass window in the local chapel is being constructed. Its steel frame is in the shape of a rectangle and semi-circle as shown in the diagram.

Let the dimensions of the rectangle be $2x$ metres and y metres. The length of steel for the frame is to be 4π metres.


2x

y

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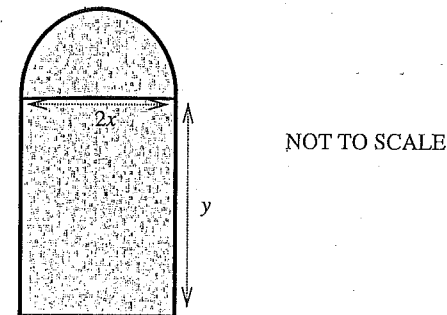
(i) Show that $y = 2\pi - 2x - \dfrac{\pi x}{2}$.   **2**

(ii) Hence show that the area, A, of the window is given by

$$A = 4\pi x - \left(4 + \frac{\pi}{2}\right)x^2 \text{ square metres.}$$ **2**

(iii) Hence, find the radius of the semi-circle which maximises the area of the stained glass window.   **3**

(b) Amanda has a new job in sales. She intends to sell $750 worth of stock in her first week of sales. Amanda also intends to increase her sales by $150 per week until her review at the end of twelve weeks.

(i) How much stock does she sell at the end of the twelfth week in her new job?   **1**

(ii) Amanda will become a permanent employee if her sales exceed $18 000 over the entire twelve week period. Will she keep her new job? Justify your answer with mathematical calculations.   **2**

(iii) To be in line for a promotion, Amanda's entire stock sales needs to total $147 000. If she continues in this manner, how many weeks must pass before Amanda receives her promotion?   **2**

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) The number line graph represents the solution to the inequality: $|x-a| \le b$ **2**

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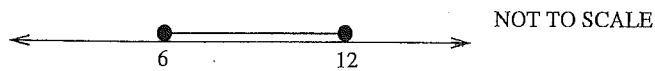
Find the value of a and b.

(b) (i) Copy and complete the following table into your writing booklet,   **2**
correct to three decimal places where necessary, for the curve

$$y = x\cos\left(\frac{x}{2}\right).$$

x	0	0.5	1	1.5	2
y	0				1.081

(ii) Use Simpson's rule with five function values to find an approximation   **2**

to $\displaystyle\int_0^2 x\cos\left(\frac{x}{2}\right) dx$.

(c) A particle starts to move from the origin with a velocity 2 m/s along the
x-axis.

Its acceleration at any time t seconds is given by: $a = \dfrac{6}{3t+4}$.

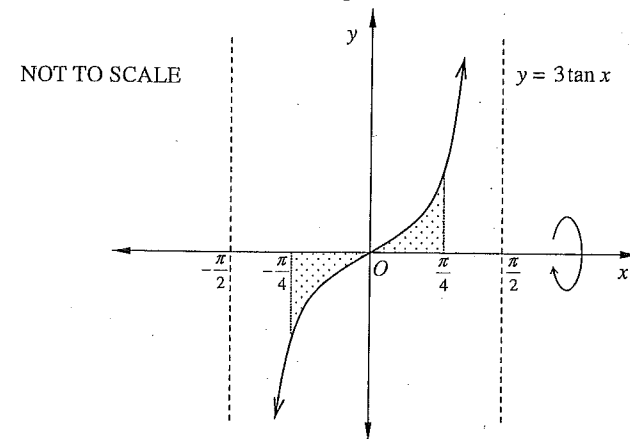(i) Show that the velocity, v, of the particle at time t is: **2**

$$v = 2\ln\left(\frac{3}{4}t +1\right) + 2 \quad \text{m/s}.$$

(ii) Find the time taken by the particle to reach a velocity 4 m/s.   **2**

(iii) Does the particle ever return to the origin? Give reasons.   **2**

Question 9 (12 marks) Use a SEPARATE writing booklet

(a) The area enclosed by the curve $y = 3\tan x$, the lines $x = \dfrac{\pi}{4}$, $x = -\dfrac{\pi}{4}$ and the
x-axis is shaded as shown in the diagram below.

NOT TO SCALE

(i) Show that the volume, V, of the solid formed when this shaded region   **3**
is rotated about the x-axis is given by:

$$V = 18\pi \int_0^{\frac{\pi}{4}} \left(\sec^2 x - 1\right) dx.$$

(ii) Hence, or otherwise, calculate the volume, V. Leave your answer in   **2**
exact form.

(b) The University of Gauss offers scholarships to young Mathematicians.
The fund is set up with a single investment of $70 000. The fund earns
interest at 8% p.a. compounded yearly. A scholarship, valued at $10 000, is
awarded each year by the university. The first scholarship is awarded 1 year
after the investment is made.

Let F_n be the value of the fund after n years.

(i) Show that the value of the fund after 3 years is   **2**

$$F_3 = 70\,000(1.08)^3 - 10\,000(1.08)^2 - 10\,000(1.08) - 10\,000.$$

(ii) Deduce that $F_n = 125\,000 - 55\,000(1.08)^n$.   **2**

(iii) Calculate the number of years the full value of the scholarship can be   **3**
awarded by the university.

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a) The light intensity, I units, passing through y metres of water is given by the equation:

$$I = I_0 \, e^{-ky} \qquad y \geq 0$$

where I_0 units is the light intensity at the surface and k is a constant called the absorption coefficient.
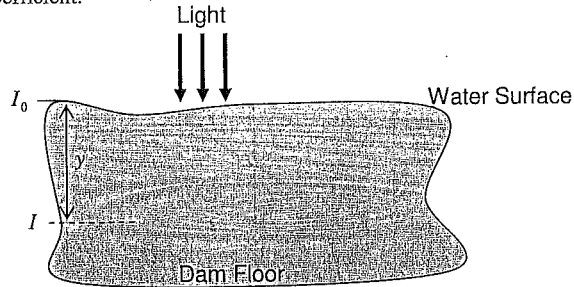
Above is the cross-section of Warragamba Dam and in the table below are the readings of two light intensity measurements from the dam.

y metres	I Units
2	1.2
8	0.9

(i) Using the table show that $k = \dfrac{1}{6}\log_e\left(\dfrac{4}{3}\right)$, and hence find I_0.　　3

(ii) Sketch the graph $I = I_0 e^{-ky}$ indicating the vertical intercept.　　2

(iii) The fish that live in this dam require light in order to survive. One kind of fish requires light of an intensity that is no less than 35% of the intensity at the surface.　　2

Determine the maximum depth, correct to the nearest metre, at which this kind of fish can survive.

Question 10 (continued)

(b) A normal die is biased so that the probability of rolling a one is $\dfrac{x}{3}$, rolling a six is $\dfrac{1-x}{3}$ and rolling a two, three, four or five is each the same.

(i) Show that the probability of rolling a 2, 3, 4, or 5 is $\dfrac{1}{6}$.　　1

(ii) The die is rolled twice and the sum of the two upper most faces is recorded.　　2

Show that if T is the probability of the sum being seven then
$$T = \frac{2(x - x^2) + 1}{9}.$$

(iii) Hence find the value x can attain to maximise the probability of a sum of seven as a result of rolling the die twice.　　2

End of Paper

CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW

2011 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS – SUGGESTED SOLUTIONS

These marking guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

Question 1 (12 marks)
(a) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Gives the correct solution	2
• Progresses towards solution	1

Sample answer:

$$\frac{7 + 5x}{6} \geq 2$$
$$7 + 5x \geq 12$$
$$5x \geq 5$$
$$x \geq 1$$

(b) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Gives the correct expression	2
• Factorises denominator correctly	1

Sample answer:

$$\frac{a - 7}{a^2 - 49} = \frac{a - 7}{(a - 7)(a + 7)}$$
$$= \frac{1}{a + 7}$$

(c) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct amount	2
• Progress towards answer	1

Sample answer:

Old payment + 0.4% Old Payment = \$3514
$$1.004x = 3514$$
$$x = \$3500$$
Old payment was \$3500

(d) (2 marks)
Outcomes Assessed: P7
Targeted Performance Bands: 2-3

Criteria	Marks
• Gives the correct answer	2
• Differentiates ONE term correctly	1

Sample answer:

$$\frac{3}{x^2} + 7x^3$$
$$\frac{d}{dx}\left(\frac{3}{x^2} + 7x^3\right) = \frac{d}{dx}\left(3x^{-2} + 7x^3\right)$$
$$= -6x^{-3} + 21x^2$$
$$= -\frac{6}{x^3} + 21x^2$$

(e) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 3-4

Criteria	Marks
• Gives correct solution for x and y	2
• Progress towards solution	1

Sample answer:

$y = x^2 + 4x \ \ ...①$

$2x + y = 72 \ , \ \ y = 72 - 2x \ ...②$

Substitute ① into equation ②

$x^2 + 4x = 72 - 2x$

$x^2 + 6x - 72 = 0$

$(x + 12)(x - 6) = 0$

$\therefore x = -12, 6$

For $x = -12, \ y = 96$

and $x = 6, \ y = 60$

(f) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Gives the correct solution	2
• Simplifies ONE surd correctly	1

Sample answer:

$\sqrt{108} + 2\sqrt{48} = 6\sqrt{3} + 2 \times 4\sqrt{3}$

$= 14\sqrt{3}$

Question 2 (12 marks)
(a) (2 marks)
Outcomes Assessed: P6, H5, H8
Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct equation	2
• Finds the correct primitive	1

Sample answer:

$\dfrac{dy}{dx} = \dfrac{7}{x}$

$y = 7\ln x + C$

When $x = 1, \ y = 6$

$6 = 7\ln 1 + C$

$\therefore C = 6$

Therefore $y = 7\ln x + 6$

(b) (i) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct derivative	2
• Progress towards solution	1

Sample answer

Let $y = \cos(e^x)$

$\dfrac{dy}{dx} = -e^x \sin(e^x)$

3

4

(b) (ii) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Bands: 2-3

Criteria	Marks
• Gives the correct derivative	2
• Progress towards solution	1

Sample answer:

Let $y = \dfrac{4x-9}{8x+5}$ using quotient rule $y' = \dfrac{vu'-uv'}{v^2}$,

$u = 4x-9, \quad v = 8x+5$

$u' = 4, \quad\quad v' = 8$

$$y' = \frac{(8x+5)\times 4 - (4x-9)\times 8}{(8x+5)^2}$$

$$= \frac{32x+20-32x+72}{(8x+5)^2}$$

$$= \frac{92}{(8x+5)^2}$$

(c) (i) (1 mark)
Outcomes Assessed: H8
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct answer	1

Sample answer:

$$\int \sec^2 3x \ dx = \frac{1}{3}\tan 3x + C$$

(c) (ii) (3 marks)
Outcomes Assessed: P7, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct solution	3
• Substitutes into primitive	2
• Progress towards correct primitive	1

Sample answer:

$$\int_0^3 \frac{1}{(2x-3)^2}\ dx = \int_0^3 (2x-3)^{-2}\ dx$$

$$= \left[\frac{(2x-3)^{-1}}{-1\times 2}\right]_0^3$$

$$= \left[-\frac{1}{2(2x-3)}\right]_0^3$$

$$= -\left[\frac{1}{6} - \frac{1}{-6}\right]$$

$$= -\frac{1}{3}$$

(d) (2 marks)
Outcomes Assessed: P2, P3, P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Gives the correct value for *VW*	2
• Gives the correct proportion statement	1

Sample answer:

$\triangle XPW$ is similar to $\triangle XQV$ Therefore sides are in proportion.

$$\frac{PW}{QV} = \frac{XW}{XV}$$

$$\frac{12}{15} = \frac{7}{7+x}$$

$$84 + 12x = 105$$

$$x = 1\frac{3}{4}$$

Question 3 (12 marks)
(a) (2 marks)
Outcomes Assessed: P7, P8, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct gradient	2
• Applies the product rule to differentiate $y = xe^x$	1

Sample answer:

$$y = xe^x$$

$$\frac{dy}{dx} = xe^x + e^x$$

$$= e^x(x+1)$$

at $x = 2$, $\frac{dy}{dx} = e^2(2+1)$

$$= 3e^2$$

$\therefore$ Gradient of normal $= \dfrac{-1}{3e^2}$

(b) (i) (1 mark)
Outcomes Assessed: P3, P4, H5
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct gradient	1

Sample answer:

Gradient: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{5 - (-4)}{1 - 7}$$

$$= -\frac{3}{2}$$

(b) (ii) (1 mark)
Outcomes Assessed: P3, P4, H5
Targeted Performance Bands: 2-3

Criteria	Mark
• Shows the correct substitution	1

Sample answer:

Equation of line: $y - y_1 = m(x - x_1)$

$$y - 5 = -\frac{3}{2}(x - 1)$$
$$2y - 10 = -3x + 3$$
$$3x + 2y - 13 = 0$$

(b) (iii) (1 mark)
Outcomes Assessed: P3, P4, H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Gives a correct expression in k	1

Sample answer:

Perpendicular distance $= \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$$= \frac{|3(-2) + 2k - 13|}{\sqrt{3^2 + 2^2}}$$
$$= \frac{|2k - 19|}{\sqrt{13}} \quad \text{units}$$

(b) (iv) (1 mark)
Outcomes Assessed: P3, P4, H5
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct distance	1

Sample answer:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(1 - 7)^2 + (5 - (-4))^2}$$
$$= \sqrt{36 + 81}$$
$$= \sqrt{117}$$
$$= 3\sqrt{13} \quad \text{units}$$

(b) (v) (3 marks)
Outcomes Assessed: P3, P4, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct value of k	3
• Solves ONE of the equations	2
• Writes correct substitution for area of parallelogram	1

Sample answer:

Area of parallelogram = base ×perpendicular height.   Length of base = $3\sqrt{13}$.

$$90 = 3\sqrt{13} \times \frac{|2k - 19|}{\sqrt{13}}$$

Perpendicular height $= \dfrac{|2k - 19|}{\sqrt{13}}$

$$90 = 3 \times |2k - 19|$$
$$30 = |2k - 19|$$

So:

$$30 = 2k - 19 \qquad 30 = -(2k - 19)$$
$$49 = 2k \qquad \text{OR} \qquad 11 = -2k$$
$$k = 24.5 \qquad \qquad k = -5.5$$

Point C is in 3rd quadrant $\therefore k = -5.5$ only

9 quadrant

9

10

(c) (i) (1 mark)
Outcomes Assessed: P4, H5
Targeted Performance Bands: 2-3

Criteria	Mark
• Gives the correct distance	1

Sample answer:

$\angle PBG = 28°$ alternate angles

$\cos 28° = \dfrac{d}{15}$

$d = 15\cos 28°$

$d = 13.24$ metres

(c) (ii) (2 marks)
Outcomes Assessed: P4, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct length l	2
• Shows correct substitution into cosine rule	1

Sample answer:

$l^2 = 20^2 + 15^2 - 2 \times 20 \times 15 \times \cos 70°$

$l^2 = 419.787914$

$l = 20.488$

The wall is approximately 20.5 metres.

angle $TPB = 42° + 28°$
$= 70°$

Question 4 (12 marks)
(a) (i) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 3-4

Criteria	Mark
• Gives the correct answer	1

Sample answer:

$\Delta = (-2m)^2 - 4(5m)(3)$
$= 4m^2 - 60m$

(a) (ii) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly identifies that for no real roots $\Delta < 0$ and writes the correct inequality	2
• Progress towards the solution	1

Sample answer:

For no real roots, $\Delta < 0$
$4m^2 - 60m < 0$
$4m(m-15) < 0$
$\therefore 0 < m < 15$

(b) (i) (1 mark)
Outcomes Assessed: H2, H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correct reasoning shown	1

Sample answer:

$\angle BAE = \theta$ (base $\angle$'s of isosceles $\triangle$ ABE)
$\angle EBC = 2\theta$ (Exterior angle of a triangle is equal to the sum of the two opposite interior angles.)

Left column

(b) (ii) (2 marks)
Outcomes Assessed: H2, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Shows $BF \parallel AE$, with appropriate reasoning	2
• Reasons that $\angle EBF = \angle CBF$	1

Sample answer:

$\angle EBF = \angle CBF$ (diagonals of rhombus bisects angle)

$\therefore BF \parallel AE$ (corresponding angles are equal, $\angle CBF = \angle BAE$)

(b) (iii) (2 marks)
Outcomes Assessed: H2, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Finds the size of $\angle AGD$ with appropriate reasoning	2
• Correctly deduces that EC and BF are perpendicular	1

Sample answer:

Similarly, $EC \parallel FD$.

EC and BF meet at right angles (diagonals of a rhombus are perpendicular)

$\therefore AG \perp GD$

$\therefore \angle AGD = 90°$

(c) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Draws tree diagram correctly	1

Sample answer:

Right column
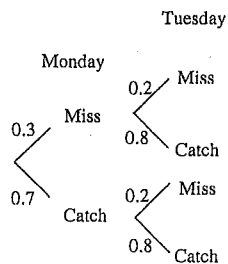
(c) (ii) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Calculates the probability correctly	1

Sample answer:

$P(\text{Miss, Catch}) + P(\text{Catch, Miss})$

$= \dfrac{30}{100} \times \dfrac{80}{100} + \dfrac{70}{100} \times \dfrac{20}{100}$

$= \dfrac{24}{100} + \dfrac{14}{100}$

$= \dfrac{38}{100} = \dfrac{19}{50} = 0.38$

(d) (i) (1 mark)
Outcomes Assessed: P4, P5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly shows that $g(x) = g(-x)$	1

Sample answer:

For an even function $g(x) = g(-x)$

LHS $= \sqrt{36 - x^2}$

RHS $= \sqrt{36 - (-x)^2}$

$= \sqrt{36 - x^2}$

$=$ LHS

$\therefore g(x) = g(-x)$

$\therefore g(x)$ is an even function

(d) (ii) (1 mark)
Outcomes Assessed: P4, P5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly states the range of the function	1

Sample answer:

$0 \le y \le 6$

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Question 5 (12 marks)
(a) (3 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly applies ASTC to determine all solutions in the given domain	3
• Finds the acute angle solutions correctly	2
• Factorises the equation correctly	1

Sample answer:

$$2\sin^2 \alpha - \sin \alpha = 0$$

$$\sin \alpha \left(2\sin \alpha - 1\right) = 0$$

$$\sin \alpha = 0 \text{ or } \sin \alpha = \frac{1}{2}$$

$$\alpha = \sin^{-1}(0) \text{ or } \alpha = \sin^{-1}\left(\frac{1}{2}\right)$$

Acute $\alpha = 0, \dfrac{\pi}{6}$

$$\therefore \alpha = \pi, \frac{\pi}{6}, \frac{5\pi}{6}$$

(b) (i) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 3-4

Criteria	Mark
• Substitutes correctly into both equations to give correct coordinate	1

Sample answer:

On substitution into $y = x^2 - 6x + 8$, when $x = 1$,

$$y = 1^2 - 6 + 8 = 3$$

On substitution into $y = 4x - x^2$, when $x = 1$,

$$y = 4 - 1 = 3$$

$\therefore$ The coordinate of P is $(1, 3)$

(b) (ii) (3 marks)
Outcomes Assessed: H8
Targeted Performance Bands: 4-5

Criteria	Marks
• Substitutes correctly to find the exact value of the area	3
• Correctly integrates the area formula	2
• Gives the correct integral required to find the area	1

Sample answer:

$$\text{Area} = \int_1^4 \left(\left(4x - x^2\right) - \left(x^2 - 6x + 8\right)\right)dx$$

$$= \int_1^4 \left(-2x^2 + 10x - 8\right)dx$$

$$= -2\int_1^4 \left(x^2 - 5x + 4\right)dx$$

$$= -2\left[\frac{x^3}{3} - \frac{5x^2}{2} + 4x\right]_1^4$$

$$= -2\left[\left(\frac{64}{3} - \frac{80}{2} + 16\right) - \left(\frac{1}{3} - \frac{5}{2} + 4\right)\right]$$

$$= -2\left[-\frac{8}{3} - \frac{11}{6}\right]$$

$$= -2 \times -\frac{9}{2}$$

$$= 9\text{u}^2$$

(c) (3 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 4-5

Criteria	Marks
• Correctly evaluates both values of w	3
• Substitutes correctly into the limiting sum formula	2
• States the values of a and r	1

Sample answer:

For the sequence, $5 + 5w^2 + 5w^4 + \ldots = \dfrac{49}{8}$

$a = 5$, $r = w^2$ and $S_\infty = \dfrac{49}{8}$

$\dfrac{5}{1 - w^2} = \dfrac{49}{8}$

$\dfrac{1 - w^2}{5} = \dfrac{8}{49}$

$1 - w^2 = \dfrac{40}{49}$

$w^2 = \dfrac{9}{49}$

$w = \pm\dfrac{3}{7}$

(d) (2 marks)
Outcomes Assessed: H3, H5
Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct answer	2
• Progress towards answer	1

Sample answer:

$\dfrac{d}{dx}(\ln(\sin x)) = \dfrac{\cos x}{\sin x}$

$\qquad\qquad\qquad = \cot x$

Question 6 (12 marks)
(a) (i) (1 mark)
Outcomes Assessed: H3, H4
Targeted Performance Bands: 4-5

Criteria	Mark
• Shows that the radius of the stormwater pipe is $\dfrac{4}{\sqrt{3}}$ m	1

Sample answer:

The triangle is isosceles, so base angles $= \dfrac{\pi}{6}$

$\dfrac{r}{\sin\dfrac{\pi}{6}} = \dfrac{4}{\sin\dfrac{2\pi}{3}}$

$r = 4 \times \dfrac{1}{2} \div \left(\dfrac{\sqrt{3}}{2}\right)$

$\quad = \dfrac{4}{\sqrt{3}}$

(a) (ii) (2 marks)
Outcomes Assessed: H3, H4
Targeted Performance Bands: 4-5

Criteria	Marks
• Finds the area correct to ONE decimal place	2
• Progress towards the solution	1

Sample answer:

$A = \dfrac{1}{2} \times \left(\dfrac{4}{\sqrt{3}}\right)^2 \left(\dfrac{2\pi}{3} - \sin\dfrac{2\pi}{3}\right)$

$\quad \cong 3.275\ldots$

$\quad = 3.3\ \text{m}^2\ (\text{1d.p.})$

(a) (iii) (2 marks)
Outcomes Assessed: H3, H4
Targeted Performance Bands: 4-5

Criteria	Marks
• Finds the volume correctly in kL	2
• Progress towards the solution	1

Sample answer:

$V = 3 \cdot 3 \times 8$

$\quad = 26 \cdot 4 \ \text{m}^3$

Water that passes through the pipe in 10 minutes

$= 26 \cdot 4 \times 10 \times 0 \cdot 5$

$= 132 \ \text{kL}$

(b) (i) (3 marks)
Outcomes Assessed: H6, H9
Targeted Performance Bands: 4-5

Criteria	Marks
• Correctly determines the stationary points and their nature	3
• Correctly determines the stationary points	2
• Differentiates correctly and equates to zero	1

Sample answer:

$y = x^3 + 6x^2 - 36x + 15$

$\dfrac{dy}{dx} = 3x^2 + 12x - 36, \ \dfrac{d^2y}{dx^2} = 6x + 12$

stationary at $\dfrac{dy}{dx} = 0$

x	-6	2
$\dfrac{d^2y}{dx^2}$	-24	24
Concavity	Down	Up
	MAX	MIN

$3x^2 + 12x - 36 = 0$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$x = -6, \ x = 2$

when $x = -6, \ y = -6^3 + 6 \times -6^2 - 36 \times -6 + 15$

$\qquad\qquad = 231$

$\therefore (-6, 231)$ is a maximum turning point.

when $x = 2, \ y = 2^3 + 6 \times 2^2 - 36 \times 2 + 15$

$\qquad\qquad = -25$

$\therefore (2, -25)$ is a minimum turning point.

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(b) (ii) (1 mark)
Outcomes Assessed: H6, H9
Targeted Performance Bands: 4-5

Criteria	Mark
• Correctly determines the point of inflexion	1

Sample answer:

Possible point of inflexion at $\dfrac{d^2y}{dx^2} = 0$

$6x + 12 = 0$

$x = -2$

x	-3	-2	-1
$\dfrac{d^2y}{dx^2}$	-24	0	6
Concavity	Down		Up

Concavity changes, therefore $(-2, 103)$ is a point of inflexion.

(b) (iii) (2 marks)
Outcomes Assessed: H6, H9
Targeted Performance Bands: 4-5

Criteria	Marks
• Correctly graphs the curve, indicating stationary points and point of inflexion	2
• Correct shape	1

Sample answer:

MAX
(−6, 231)

(−2, 103)
POI

15

O

(2, −25)
MIN

20

(b) (iv) (1 mark)
Outcomes Assessed: H6, H7, H9
Targeted Performance Bands: 4-5

Criteria	Mark
• Gives the correct answer	1

Sample answer:

concave down when $\dfrac{d^2y}{dx^2} < 0$

$6x + 12 < 0$

$x < -2$

Question 7 (12 marks)
(a) (i) (2 marks)
Outcomes Assessed: P8, H5, H9
Targeted Performance Band: 4-5

Criteria	Marks
• Gives the correct answer	2
• Progress towards the answer	1

Sample Answer:

$$P = \frac{2\pi x}{2} + 2x + 2x + 2y \qquad \therefore 4\pi = \pi x + 4x + 2y$$

$$2y = 4\pi - 4x - \pi x \qquad \therefore y = 2\pi - 2x - \frac{\pi x}{2}$$

(a) (ii) (2 marks)
Outcomes Assessed: P8, H5, H9
Targeted Performance Band: 4-5

Criteria	Marks
• Gives the correct answer	2
• Progress towards the answer	1

Sample Answer:

$$A = \frac{\pi x^2}{2} + 2xy = \frac{\pi x^2}{2} + 2x\left(2\pi - 2x - \frac{\pi x}{2}\right)$$

$$A = \frac{\pi x^2}{2} + 4x\pi - 4x^2 - \pi x^2 = 4\pi x - \left(4 + \frac{\pi}{2}\right)x^2$$

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(a) (iii) (3 marks)
Outcomes Assessed: P8, H5, H9
Targeted Performance Band: 4-5

Criteria	Marks
• Correctly shows that the radius is a maximum when $x = 1.128$	3
• Correctly finds the radius $x = 1.128$	2
• Equates $\dfrac{dA}{dx} = 0$	1

Sample Answer:

$$\frac{dA}{dx} = 4\pi - 2\left(4 + \frac{\pi}{2}\right)x = 0 \text{ for stationary points}$$

$$\left(4 + \frac{\pi}{2}\right)x = 2\pi \qquad \therefore x = \frac{4\pi}{8 + \pi} \approx 1.128$$

$$\frac{d^2A}{dx^2} = -2\left(4 + \frac{\pi}{2}\right) < 0 \qquad \therefore \text{MAXIMUM}$$

(b) (i) (1 mark)
Outcomes Assessed: H4, H5
Targeted Performance Band: 4-5

Criteria	Mark
• Gives the correct answer	1

Sample Answer:

$$a = 750 \quad d = 150 \quad \therefore T_{12} = 750 + (12 - 1)150 = \$2400$$

(b) (ii) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Band: 4-5

Criteria	Marks
• Gives the correct conclusion	2
• Finds S_{12}	1

Sample Answer:

$$S_{12} = \frac{12}{2}(750 + 2400) = \$18\ 900$$

This is greater than \$18 000 so she will keep her job

(b) (iii) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Band: 4-5

Criteria	Marks
• Gives the correct answer	2
• Arrives at the a quadratic $150n^2 + 1350n - 294\ 000 = 0$	1

Sample Answer:

$$\frac{n}{2}\left(2(750) + (n-1)150\right) = 147\ 000$$

$$1500n + 150n^2 - 150n = 294\ 000$$

$$150n^2 + 1350n - 294\ 000 = 0$$

$$n = \frac{-1350 \pm \sqrt{(-1350)^2 - 4(150)(-294\ 000)}}{2(150)}$$

$$n = -49 \text{ or } 40$$

$$\therefore 40 \text{ weeks must pass.}$$

Question 8 (12 marks)
(a) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Band: 4-5

Criteria	Marks
• Correctly solves simultaneously to find a and b	2
• Establishes equations ① and ②	1

Sample Answer:

$|x - a| \le b$

$\therefore x - a \le b \quad \therefore x \le a + b$ and since $x \le 12 \quad \therefore a + b = 12 \quad \ldots ①$

$x - a \ge -b \quad \therefore x \ge a - b$ and since $x \ge 6 \quad \therefore a - b = 6 \quad \ldots ②$

$\therefore$ from equations ① and ② $\quad a = 9$ and $b = 3$

(a) (i) (2 marks)
Outcomes Assessed: H5
Targeted Performance Band: 3-4

Criteria	Marks
• Completes the table correctly	2
• Completes table with ONE mistake	1

Sample Answer:

x	0	0.5	1	1.5	2
y	0	0.484	0.878	1.098	1.081

(b) (ii) (2 marks)
Outcomes Assessed: H5, H8
Targeted Performance Band: 4-5

Criteria	Marks
• Gives the correct answer	2
• Substitutes into Simpson's rule	1

Sample Answer:

Integral $= \dfrac{h}{3}(y_0 + y_4 + 4(y_1 + y_3) + 2y_2), \quad h = 0.5$

$= \dfrac{0.5}{3}[0 + 1.081 + 4(0.484 + 1.098) + 2(0.878)] = \dfrac{0.5}{3}[9.165]$

$= 1.5275$

(c) (i) (2 marks)
Outcomes Assessed: H3, H5
Targeted Performance Band: 5-6

Criteria	Marks
• Gives the correct answer	2
• Correctly finds the expression $v = 2\ln(3t+4) + 2 - 2\ln 4$	1

Sample Answer:

$a = \dfrac{6}{3t+4} \qquad \therefore v = \int \dfrac{6}{3t+4}\, dt = 2\int \dfrac{3}{3t+4}\, dt = 2\ln(3t+4) + c$

when $t = 0$, $v = 2 \qquad \therefore 2 = 2\ln 4 + c \qquad \therefore c = 2 - 2\ln 4$

$\therefore v = 2\ln(3t+4) + 2 - 2\ln 4$

$= 2\ln\left(\dfrac{3t+4}{4}\right) + 2$

$= 2\ln\left(\dfrac{3}{4}t + 1\right) + 2$

(c) (ii) (2 marks)
Outcomes Assessed: H3, H5
Targeted Performance Band: 5-6

Criteria	Marks
• Finds t in exact or decimal form	2
• Correctly substitutes $v = 4$ and progress towards solution	1

Sample Answer:

Substitute $v = 4$

$\therefore 4 = 2\ln\left(\dfrac{3}{4}t + 1\right) + 2 \qquad \therefore 2 = 2\ln\left(\dfrac{3}{4}t + 1\right) \qquad \therefore e = \dfrac{3}{4}t + 1$

$\therefore t = \dfrac{4}{3}(e - 1) = 2.29 \sec$

(c) (iii) (2 marks)
Outcomes Assessed: H3, H4, H5
Targeted Performance Band: 5-6

Criteria	Marks
• Gives the correct answer	2
• Makes a correct statement about v or a	1

Sample Answer:
Since the acceleration is always positive for $t \geq 0$, then its velocity will increase in the positive direction. Hence it will never return to the origin.

Question 9 (12 marks)
(a) (i) (3 marks)
Outcomes Assessed: H4, H8, H9
Targeted Performance Band: 4-6

Criteria	Marks
• Shows the volume is given by $18\pi \int_{0}^{\frac{\pi}{4}} \sec^2 x - 1 \, dx$	3
• Correctly substitutes for $\tan^2 x$ with progress towards the answer	2
• Correctly applying the volume formula	1

Sample Answer:

$$V = \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} (3\tan x)^2 \, dx = \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} 9\tan^2 x \, dx = 2\pi \int_{0}^{\frac{\pi}{4}} 9(\sec^2 x - 1) \, dx$$

$$= 18\pi \int_{0}^{\frac{\pi}{4}} \sec^2 x - 1 \, dx$$

(a) (ii) (2 marks)
Outcomes Assessed: H4, H8, H9
Targeted Performance Band: 4-5

Criteria	Marks
• Substitutes to give the correct answer in exact form	2
• Correctly integrates V from (i)	1

Sample Answer:

$$V = 18\pi[\tan x - x]_{0}^{\frac{\pi}{4}} = 18\pi\left[\left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - (\tan 0 - 0)\right]$$

$$= 18\pi\left[1 - \frac{\pi}{4}\right] \text{units}^3$$

(b) (i) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Band: 4-5

Criteria	Marks
• Shows that $F_3 = 70\,000(1.08)^3 - 10\,000(1.08)^2 - 10\,000(1.08) - 10\,000$.	2
• Progress towards F_2	1

Sample Answer:

$F_1 = 70\,000(1.08) - 10\,000$
$F_2 = [70\,000(1.08) - 10\,000](1.08) - 10\,000$
$\quad = 70\,000(1.08)^2 - 10\,000(1.08) - 10\,000$
$F_3 = [70\,000(1.08^2) - 10\,000(1.08) - 10\,000](1.08) - 10\,000$
$\quad = 70\,000(1.08)^3 - 10\,000(1.08)^2 - 10\,000(1.08) - 10\,000$

(b) (ii) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Band: 4-6

Criteria	Marks
• Shows that $F_n = 125\,0000 - 55\,000(1.08)^n$	2
• Progress towards F_n	1

Sample Answer:

$$F_n = 70\,000(1.08)^n - 10\,000(1 + 1.08 + 1.08^2 + ... + 1.08^{n-1})$$

$$F_n = 70\,000(1.08)^n - 10\,000\left[\frac{(1.08)^n - 1}{1.08 - 1}\right]$$

$$F_n = 70\,000(1.08)^n - 10\,000\left[\frac{(1.08)^n - 1}{0.08}\right]$$

$$F_n = 70\,000(1.08)^n - 125\,000\left[(1.08)^n - 1\right]$$

$$F_n = 70\,000(1.08)^n - 125\,000(1.08)^n + 125\,000$$

$$F_n = 125\,000 - 55\,000(1.08)^n$$

(b) (iii) (3 marks)
Outcomes Assessed: H3, H4, H5
Targeted Performance Band: 4-6

Criteria	Marks
• Gives the correct answer	3
• Progress towards finding n	2
• $125\,000 - 55\,000(1.08)^n \geq 0$	1

Sample Answer:

$$125\,000 - 55\,000(1.08)^n \geq 0$$

$$125\,000 \geq 55\,000(1.08)^n$$

$$\therefore 55\,000(1.08)^n \leq 125\,000$$

$$\therefore (1.08)^n \leq \frac{25}{11}$$

$$\therefore n\ln 1.08 \leq \ln\left(\frac{25}{11}\right) \qquad \therefore n \leq 10.66$$

∴ The number of years the full value of the scholarship can be awarded is 10 years.

Question 10 (12 marks)
(a) (i) (3 marks)
Outcomes Assessed: H2, H3
Targeted Performance Band: 3-5

Criteria	Marks
• Finds I_0 and k correctly	3
• Progress towards solving both equations simultaneously to find k	2
• Find the correct expressions using the table given	1

Sample Answer:

$$I = I_0\, e^{-ky}$$

∴ from the table $\qquad 1.2 = I_0\, e^{-2k} \quad ...\textcircled{1} \quad$ and $\qquad 0.9 = I_0\, e^{-8k} \quad ...\textcircled{2}$

$$\therefore \textcircled{1} \div \textcircled{2} \qquad \frac{1.2}{0.9} = \frac{e^{-2k}}{e^{-8k}} \quad \therefore \frac{4}{3} = e^{6k} \qquad \log_e \frac{4}{3} = 6k$$

$$\therefore k = \frac{1}{6}\log_e\left(\frac{4}{3}\right) = 0.047947$$

$$1.2 = I_0 e^{-2 \times \frac{1}{6}\log_e\left(\frac{4}{3}\right)}$$

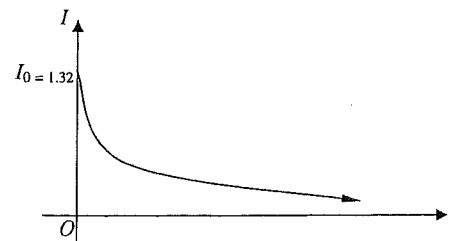$$\therefore 1.2 = I_0 \times 0.90856$$

$$I_0 = 1.3207709 = 1.32$$

(a) (ii) (2 marks)
Outcomes Assessed: H3, H4, H9
Targeted Performance Band: 3-5

Criteria	Marks
• Correctly draws $I = I_0 e^{-ky}$ on graph	2
• Shows the vertical intercept by labelling I_0	1

Sample Answer :

(a) (iii) (2 marks)
Outcomes Assessed: H3, H4, H9
Targeted Performance Band: 4-6

Criteria	Marks
• Finds the correct value of y and hence the maximum depth 21 metres	2
• Correctly substitute $I \geq 35\% \, I_0$ to show $I_0 \, e^{-0.04795y} \geq 0.35 \, I_0$	1

Sample Answer:

If $I > 35\% \, I_0$ $\quad \therefore I_0 \, e^{-0.04795y} \geq 0.35 \, I_0$

$\therefore -0.04795 \, y \geq \ln 0.35$

$\therefore y \leq \dfrac{\ln 0.35}{-0.04795}$ $\quad$ i.e. $y \leq 21.8$

$\therefore$ The maximum depth at which this kind of fish can survive is 21 metres.

(b) (i) (1 mark)
Outcomes Assessed: H4, H5
Targeted Performance Band: 4-6

Criteria	Mark
• Gives the correct answer	1

Sample Answer:

$P(2,3,4 \text{ or } 5) = \left(1 - \left(\dfrac{x}{3} + \dfrac{1-x}{3}\right)\right) \div 4 = \dfrac{1}{6}$

(b) (ii) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Band: 4-6

Criteria	Marks
• Finds the correct expression for T	2
• Progress towards finding T	1

Sample Answer:

$$\begin{aligned}
T(\text{Sum being } 7) &= 2\left[P(1) \times P(6)\right] + 2\left[P(2) \times P(5)\right] + 2\left[P(3) \times P(4)\right] \\
&= 2\left[\frac{x}{3} \times \frac{(1-x)}{3}\right] + 2\left[\frac{1}{6} \times \frac{1}{6}\right] + 2\left[\frac{1}{6} \times \frac{1}{6}\right] \\
&= \frac{2x(1-x)}{9} + \frac{1}{18} + \frac{1}{18} \\
&= \frac{2x(1-x)}{9} + \frac{1}{18} + \frac{1}{18} \\
&= \frac{2x(1-x)}{9} + \frac{1}{9} = \frac{2x(1-x)+1}{9} \\
&= \frac{2(x-x^2)+1}{9}
\end{aligned}$$

(b) (iii) (2 marks)
Outcomes Assessed: P7, H5, H9
Targeted Performance Band: 3-5

Criteria	Marks
• Correctly shows a maximum when $x = \dfrac{1}{2}$	2
• Correctly determine $\dfrac{dT}{dx} = \dfrac{2}{9}(1-2x)$	1

Sample Answer:

$T = \dfrac{2(x-x^2)+1}{9}$

$\therefore \dfrac{dT}{dx} = \dfrac{2}{9}(1-2x) \qquad \therefore 0 = \dfrac{2}{9}(1-2x) \qquad \therefore x = \dfrac{1}{2}$

$\dfrac{d^2T}{dx^2} = -\dfrac{4}{9} < 0 \qquad$ i.e. maximum

$\therefore$ To maximise the probability of a sum of 7, $x = \dfrac{1}{2}$

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