Name: ____________________

Teacher: ____________________

Class: ____________________

FORT STREET HIGH SCHOOL

2011

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics

TIME ALLOWED: 3 HOURS
(PLUS 5 MINUTES READING TIME)

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1	
Manipulates algebraic expressions to solve problems from topic areas such as geometry, co-ordinate geometry, quadratics, trigonometry, probability and logarithms	3,4,7	
Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	2,5,6,8	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	9,10	

Question	1	2	3	4	5	6	7	8	9	10	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/12	/12	/12	/120	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet
- A table of standard integrals is supplied.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

QUESTION 1 (12 marks) Start a NEW booklet. **Marks**

(a) Evaluate $2e^2$ correct to three significant figures **2**

(b) Factorise $2x^2 - x - 6$ **2**

(c) Simplify **2**

$$\frac{2}{x+1} - \frac{3}{x}$$

(d) Solve $|2x - 1| = 9$ **2**

(e) Expand and simplify **2**

$$(\sqrt{5} - 1)(2\sqrt{5} + 3)$$

(f) Find the sum of the first 12 terms of the arithmetic sequence **2**

3+6+9+.......

QUESTION 2 (12 marks) Start a NEW booklet.

(a) Differentiate

(i) $(x^3 + 1)^7$ **2**

(ii) $x^4 \log_e x$ **2**

(iii) **2**

$$\frac{\sin x}{x+1}$$

(b) Find the perpendicular distance from (6, -2) to the line $4x - 3y + 7 = 0$ **2**

(c) (i) Find **1**

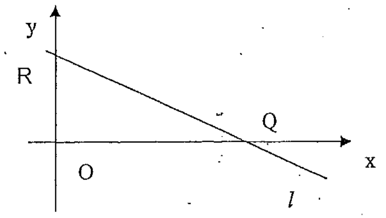
$$\int \frac{1}{x+7} dx$$

(ii) Evaluate **3**

$$\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$$

QUESTION 3 (12 marks) Start a NEW booklet. **Marks**

(a)



The diagram shows the straight line l which cuts the x-axis and y-axis at Q(6,0) and R(0,2) respectively.

(i) Copy the diagram and find the equation of the line l. **2**

(ii) Draw the line k, passing through Q perpendicular to l, and find the equation of the line k. **2**

(iii) Find the coordinates of T, where k cuts the y-axis. **1**

(iv) Find the equation of the circle which has TR as a diameter. Verify that Q lies on this circle. **2**

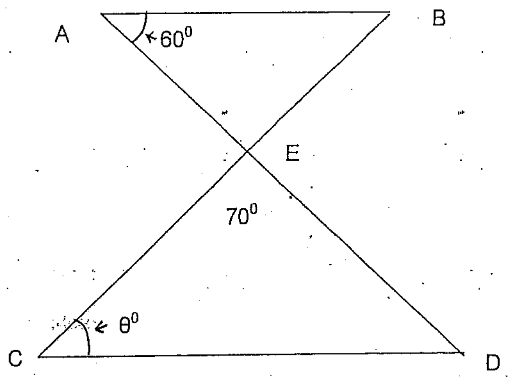
(v) (8,m) also lies on the circle. Find two possible values of m. **2**

(b) Find the equation of the straight line that passes through (1,-2) and the intersection point of the lines $4x + y - 5 = 0$ and $3x - 2y - 12 = 0$ **3**

QUESTION 4 (12 marks) Start a NEW booklet. **Marks**

(a)

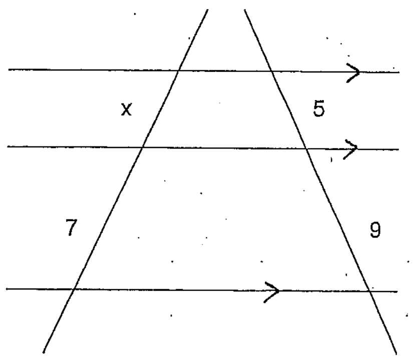
Figure not to scale



In the diagram AB is parallel to CD.
Find the value of θ. Give reasons. **2**

(b)

Figure not to scale

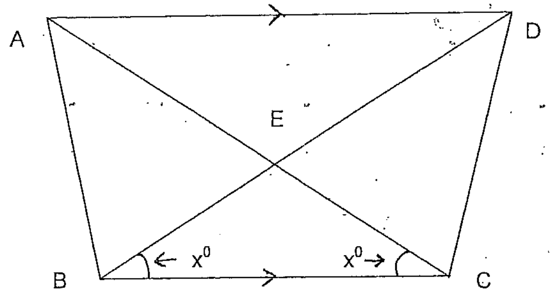


Find the value of x. Give reasons. All measurements are in cm. **2**

QUESTION 4 (continued) **Marks**

(c)

Figure not to scale



In the diagram AD is parallel to BC and angle DBC = angle ACB = x^0.

(i) Show that AE=DE. **2**

(ii) Prove that the triangles ABC and DCB are congruent. **3**

(iii) Deduce angle ABD = angle DCA. **3**

QUESTION 5 (12 marks) Start a NEW booklet. **Marks**

(a) The gradient of a curve is given by

$$\frac{dy}{dx} = \frac{1}{\sqrt{2x+1}}$$ **3**

The curve passes through (4, 5). What is the equation of the curve? (Answer in surd form)

(b) Consider the geometric series

$$1 + 3x + 9x^2 + 27x^3 + \cdots$$

(i) For what values of x does this series have a limiting sum? **2**

(ii) The limiting sum of this series is 100. Find the value of x **2**

(c) A 5kg mass of metal is placed in an acid and it starts to Dissolve. After t hours, the mass M kg of undissolved metal is given by

$$M = 5e^{-kt}.$$

(i) Find k, given M = 4.2 when t = 2 (Answer in exact form) **2**

(ii) After how many hours will half of the initial mass remain undissolved? Note: This known as the half-life of the metal in this acid. (Answer to 2 decimal places) **3**

QUESTION 6 (12 marks) Start a NEW booklet. **Marks**

(a) A particle moves along a straight line so that its distance x, in metres, from a fixed point O is given by

$$x = 1 - 2\sin 2t,$$

where the time t is measured in seconds from t = 0.

(i) Where is the particle initially? **1**

(ii) When and where, does the particle come to rest? **3**

(iii) Where does the particle next come to rest? **2**

(iv) What is the acceleration of the particle when $t = \frac{\pi}{12}$? **2**

(b) (i) Graph y = cos x, for $0 \le x \le 2\pi$ **1**

(ii) On your diagram shade the regions bounded by the curve y = cos x, the x-axis and the lines x = 0 and x = π. Calculate the total area of these regions. **2**

(iii) Solve $\cos x = \frac{\sqrt{3}}{2}$ for $0 \le x \le 2\pi$ (Answer in terms of π) **1**

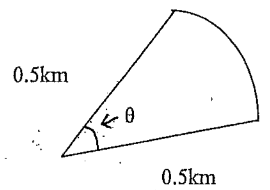
QUESTION 7 (12 marks) Start a NEW booklet **Marks**

(a) Solve **3**

$$\log_e x - \frac{3}{\log_e x} = 2$$

(Answer in exact form)

(b)



A car travels at 45 km/h on a circular curve whose radius is 0.5km.

(i) Find the distance l km, that the car travel in one minute. **1**

(ii) Calculate the size of the angle θ through which the car turns in one minute. Give your answer to the nearest degree. **2**

(c) Pat and Chris each threw a die

(i) Find the probability that they throw the same number. **1**

(ii) Find the probability that the number thrown by Chris is greater than the number thrown by Pat. **1**

(d) The focus of a parabola is S (1, -2) and the directrix is the line y = 5.

(i) Write down the co-ordinates of the vertex. **1**

(ii) Find the focal length. **1**

(iii) Write down the equation of the parabola in the form **2**

$$(x-h)^2 = 4a(y-k)$$

and sketch the parabola

QUESTION 8 (12 marks) Start a NEW booklet **Marks**

(a) Consider the curve given by $y = 3x^2 - x^3$

(i) Find the stationary points and determine their nature. **4**

(ii) Sketch the curve, indicating where it crosses the x-axis and and showing stationary points. **2**

(iii) Find the equation of the tangent to the curve at the point R (-1,4). **2**

(b) (i) Differentiate $\log_e(\cos x)$ with respect to x. **1**

(ii) Hence or otherwise, show **3**

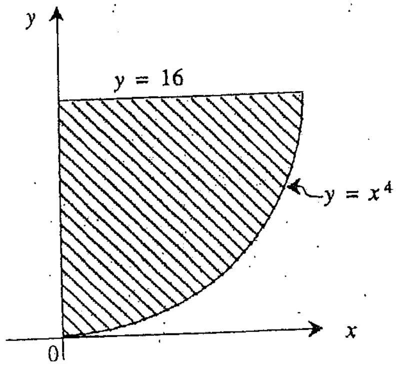
$$\int_0^{\frac{\pi}{4}} \tan x\,dx = \frac{1}{2}\log_e 2$$

QUESTION 9 (12 marks) Start a NEW booklet. **Marks**

(a) Solve the equation **2**

$$3^{2x} + 2 \times 3^{x} - 15 = 0$$

(b)



The shaded region in the diagram is bounded by the curve $y = x^4$, the y-axis, and the line $y = 16$ **4**

Calculate the volume of solid of revolution formed when this region is rotated about the y-axis. (Answer in terms of π)

(c) A woman contributes \$1500 each year into a superannuation fund for the first 25 years of her working life. For the next fifteen years (until retirement), she decides to increase this, and invest a total of \$5000 each year. Each contribution is paid at the beginning of the year. **6**

If the investment earns 7% p.a. paid yearly over the whole year period, how much will her investment be upon retiring? (Answer to the nearest dollar)

QUESTION 10 (12 marks) Start a NEW booklet. **Marks**

(a) The speed of a train was recorded at intervals of one minute. The times, in minute, and the corresponding speeds v, in kilometres per hour, are listed in the following table.

time (min)	0	1	2	3	4
v (km/h)	0	25	34	30	40

(i) Explain why the distance x, in km, travelled by the train in these four minutes is given by **2**

$$x = \int_0^{\frac{1}{15}} v\,dt$$

(ii) Estimate x by using Simpson's Rule with five function values. (Answer to 1 decimal place) **3**

(b) A truck is to travel 1000 kilometres at a constant speed of v km/h.

When travelling at v km/h, the truck consumes fuel at the rate of

$\left(6 + \frac{v^2}{50}\right)$ litres per hour.

The truck company pays \$1.50/litre for fuel and pays each of 2 drivers \$36 per hour whilst the truck is travelling.

(i) Let the total cost of fuel and the drivers' wages for the trip be C dollars. Show that **3**

$$C = 30v + \frac{81000}{v}$$

(ii) The truck must take no longer than 12 hours to complete the trip, and speed limits require that $v \le 100$. **4**
At what speed v should the truck travel to minimize the cost C.
(Answer to 2 decimal places)

END OF EXAMINATION

2011 2U Trial HSC Solutions

QUESTION ONE

(a) $2e^2 = 14 \cdot 7781123\ldots$
$= 14 \cdot 8$ (to 3 sig fig)

Some students did not show the initial answer from calculator.

(b) $2x^2 - x - 6$
$= 2x^2 - 4x + 3x - 6$
$= 2x(x-2) + 3(x-2)$
$= (2x+3)(x-2)$

(c) $\dfrac{2}{x+1} - \dfrac{3}{x}$

$= \dfrac{2x}{x(x+1)} - \dfrac{3(x+1)}{x(x+1)}$

$= \dfrac{2x - 3x - 3}{x(x+1)}$

$= \dfrac{-x-3}{x(x+1)}$

Some students made mistakes with the minus sign.

(d) $|2x-1| = 9$

either $2x-1 = 9$ or $2x-1 = -9$
$2x = 10$ $\quad$ $2x = -8$
$x = 5$ $\quad$ $x = -4$

(e) $(\sqrt{5}-1)(2\sqrt{5}+3)$
$= \sqrt{5}(2\sqrt{5}+3) - 1(2\sqrt{5}+3)$
$= 10 + 3\sqrt{5} - 2\sqrt{5} - 3$
$= 7 + \sqrt{5}$

Some students need to revise how to add and subtract surds

(f) $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

$= \dfrac{12}{2}\left[2 \times 3 + (12-1) \times 3\right]$

$= 6 \times 39$

$= 234$

Some students did not remember the sum of AP or did not use it correctly.

QUESTION TWO

(a) (i) $y = (x^3+1)^7$

$y' = 3x^2 \times 7 \times (x^3+1)^6$ ✓

$= 21x^2(x^3+1)^6$ ✓

Students needed to use proper setting out involving y', $\frac{dy}{dx}$ or $\frac{d}{dx}$

(ii) $y = x^4 \log_e x$

$y' = \frac{x^4}{x} + 4x^3 \log_e x$ ✓

$= x^3 + 4x^3 \log_e x$ ✓

(iii) $y = \frac{\sin x}{x+1}$

$y' = \frac{(x+1)\cos x - 1 \times \sin x}{(x+1)^2}$ ✓

$= \frac{(x+1)\cos x - \sin x}{(x+1)^2}$ ✓

(b) perp dist $= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$ ✓

$= \frac{|4 \times 6 - 3 \times -2 + 7|}{\sqrt{4^2+(-3)^2}}$

$= \frac{37}{5}$ ✓

Many students did not know correct formula

(c) (i) $\int \frac{1}{x+7}\,dx$

$= \log_e(x+7) + C$ ✓

Remember constant in all indefinite integrals.

(ii) $\int_0^{\frac{\pi}{8}} \sec^2 2x\,dx$

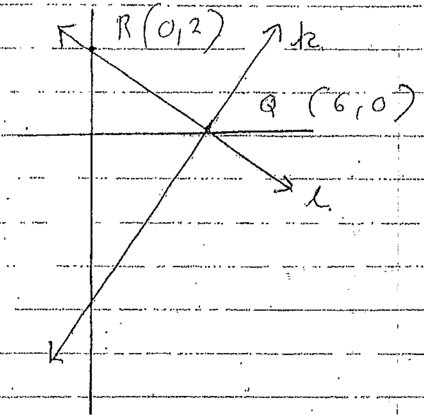
$= \left[\frac{1}{2}\tan 2x\right]_0^{\frac{\pi}{8}}$ ✓

$= \frac{1}{2}\tan\frac{\pi}{4}$ ✓

$= \frac{1}{2}$ ✓

QUESTION THREE

(a) (i)



Some had the lines confused

gradient of $l = \frac{rise}{run} = -\frac{2}{6} = -\frac{1}{3}$ ✓

equation of l: $y = mx + b$

$y = -\frac{1}{3}x + 2$ ✓

(b = y-intercept = 2)

Should have been obvious that line l has a y int of 2! Yet some silly answers were given.

ii) See diagram for drawing of k

$m_l = -\frac{1}{3} \Rightarrow m_k = 3$ ✓ (neg reciprocal)

equation of k: $y - y_1 = m(x - x_1)$

$y - 0 = 3(x - 6)$ ✓

$\therefore y = 3x - 18$

(iii) k cuts the y-axis when $x = 0$

ie when $y = 3(0) - 18$
$= -18$

$\therefore T(0, -18)$ ✓

Need to learn to read the question. Many did not give the co-ordinates of T.

(iv) midpoint $TR = \left(\frac{0+0}{2}, \frac{2-18}{2}\right)$

$= (0, -8)$

$\therefore$ circle has centre $(0, -8)$
radius: 10

$\therefore$ eqn is $x^2 + (y+8)^2 = 100$ ✓

Quite a few 'errors carried' here + hence many had wrong equation.

sub Q (6, 0) into

$x^2 + (y+8)^2 = 100$

LHS $= 6^2 + (0+8)^2$
$= 36 + 64$
$= 100$ ✓
$=$ RHS $\therefore$ Q (6, 0) lies on circle

For a 'show' question should start with LHS & work through to show it is equal to RHS. Those who had wrong circle eqn. obviously had a problem $\therefore$ should have realised they had made an error. (No mark awarded)

(v) $x^2 + (y+8)^2 = 100$

sub $(8, m) \Rightarrow$

$8^2 + (m+8)^2 = 100$
$(m+8)^2 = 36$
$m + 8 = \pm 6$ ✓
$m = \pm 6 - 8$

$\therefore m = -2$ or $m = -14$ ✓

(v) Some used 'y' instead of 'm' – need 2 values of m

(b) Using k method

$4x + y - 5 + k(3x - 2y - 12) = 0$ ✓

sub $(1, -2)$

$4 - 2 - 5 + k(3 + 4 - 12) = 0$

$-3 + -5k = 0$ ✓

$k = -\frac{3}{5}$

$\therefore 4x + y - 5 - \frac{3}{5}(3x - 2y - 12) = 0$

$20x + 5y - 25 - 9x + 6y + 36 = 0$

$11x + 11y + 11 = 0$ ✓

$x + y + 1 = 0$

$y = -x - 1$

If this method was used some errors with 'k'

OR

$4x + y - 5 = 0$ (1)

$3x - 2y - 12 = 0$ (2)

$8x + 2y - 10 = 0$ (3) = (1) $\times 2$

$11x - 22 = 0$ (2) + (3)

$x = 2 \quad y = -3$ ✓

$\therefore$ pt. of intersection of (1) & (2) is $(2, -3)$

grad from $(2, -3)$ to $(1, -2)$ ✓

$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 + 2}{2 - 1} = -1$

$\therefore$ required eqn: $y - y_1 = m(x - x_1)$

$y + 2 = -1(x - 1)$

$y + 2 = -x + 1$

$y = -x - 1$ ✓

Most used this method with reasonable success

QUESTION FOUR

(a) $\angle EDC = \angle BAD$ (alternate angles $AB \parallel CD$) ✓

$= 60°$

$\therefore \angle ECD = \theta$

$= 180° - 70° - 60°$ (angle sum of $\triangle ECD$ is $180°$) ✓

$= 50°$

well done

(b) $\frac{x}{7} = \frac{5}{9}$ (ratio of intercepts) ✓

$x = \frac{7 \times 5}{9}$

$= \frac{35}{9}$

$= 3\frac{8}{9}$ ✓

Some students did not provide a reason.

(c) (i) $\angle ADE = \angle CBE$, $AD \parallel BC$

$= x°$, alternate $\angle s$ ✓

Similarly $\angle DAE = x°$

$AE = DE$ (sides opposite equal $\angle s$ ADE, DAE) ✓

(ii) In $\triangle BEC$

$EB = EC$ (opposite equal $\angle s$ EBC, ECB)

But $AC = AE + EC$

and $DB = DE + EB$

$\therefore AC = DB$ ✓

In $\triangle s$ ABC, DCB ✓

$AC = DB$ (as above)

BC is common

Some students need to provide clearer sequential geometrical proofs.

$\angle ACB = \angle DBC$ (given)
$\therefore \triangle ABC \equiv \triangle DCB$ (SAS)

(iii) $\because \angle ABC = \angle DCB$
(Corresponding $\angle s$ in congruent $\triangle s$)

i.e. ~~$\angle ABD + \angle DBC = \angle DCA + \angle ACB$~~
But $\angle DBC = \angle ACB$ (given)
$\therefore \angle ABD = \angle DCA$

To prove $\angle ABD = \angle DCA$ some students also proved ~~$\triangle AEB \equiv \triangle DEC$~~ This was also acceptable

QUESTION FIVE

(a) $\frac{dy}{dx} = \frac{1}{\sqrt{2x+1}}$

$\frac{dy}{dx} = (2x+1)^{-\frac{1}{2}}$

$\therefore y = \frac{(2x+1)^{\frac{1}{2}}}{\frac{1}{2} \times 2} + C$

$y = \sqrt{2x+1} + C$

sub $(4,5) \Rightarrow 5 = \sqrt{2(4)+1} + C$
$5 = 3 + C$
$C = 2$

$\therefore$ equation is $y = \sqrt{2x+1} + 2$

Well done

(b) (i) For limiting sum
$-1 < 3x < 1$
$-\frac{1}{3} < x < \frac{1}{3}$

(ii) $S_\infty = \frac{a}{1-r}$

$100 = \frac{1}{1-3x}$

$1 - 3x = \frac{1}{100}$

$-3x = \frac{-99}{100}$

$x = \frac{33}{100}$

Confusion in solving this simple equation

(c) $M = 5e^{-kt}$

(i) when $M = 4.2$, $t = 3$

$\therefore 4.2 = 5e^{-3k}$

$e^{-3k} = 0.84$

$\therefore \ln(0.84) = -3k$

$\therefore k = \frac{\ln(0.84)}{-3}$

$= 0.0581176693$ (Calc)

(ii) we want to find t when $M = \frac{1}{2} \times 5 = 2.5$

$2.5 = 5e^{-0.0581176693t}$

$0.5 = e^{-0.0581176693t}$

$\ln(0.5) = -0.0581176693t$

$t = \frac{\ln(0.5)}{-0.0581176693}$

$= 11.93$ hours (to 2 dec pls)

well done

QUESTION SIX

(a) (i) $x = 1 - 2\sin 2x$

at $t = 0$, $x = 1 - 2\sin 0 = 1$

ie 1 metre on right-hand side of O

(ii) $v = \frac{dx}{dt} = -4\cos 2t$

'comes to rest' means '$v = 0$'

$-4\cos 2t = 0$

$\cos 2t = 0$

$2t = \frac{\pi}{2}$ (1st time)

$t = \frac{\pi}{4}$

$t = \frac{\pi}{4}$ $x = 1 - 2\sin 2\frac{\pi}{4}$

$= 1 - 2\sin\frac{\pi}{2}$

$= 1 - 2$

$= -1$

ie first comes to rest 1 metre on the left-hand side of O

Some students answered when a particle come to rest, not where.

(iii) $v = 0$ $\cos 2t = 0$

$2t = \frac{\pi}{2}, \frac{3\pi}{2}$

$\therefore t = \frac{\pi}{4}, \frac{3\pi}{4}$

For $t = \frac{3\pi}{4}$

$x = 1 - 2\sin(2 \times \frac{3\pi}{4})$

$= 1 - 2\sin\frac{3\pi}{2}$

$= 1 - 2 \times -1$

$= 1 + 2 = 3$

ie 3 metres on right-hand of O

Some students answered when a particle come to a rest next, not where.

(iv) $a = \frac{dv}{dt} = \frac{d}{dt}(-4\cos 2t)$

$= 8 \sin 2t$ ✓

$t = \frac{\pi}{12}$: $a = 8 \sin\left(2 \times \frac{\pi}{12}\right)$

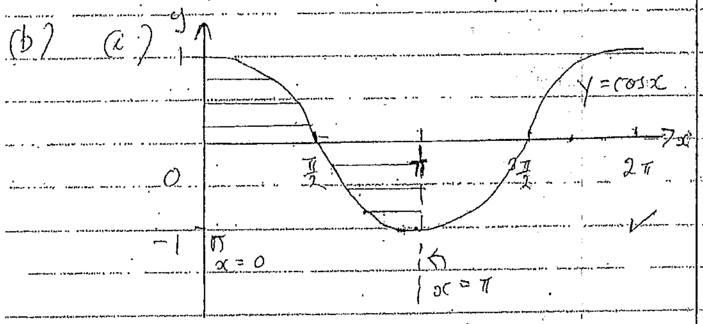
$= 8 \sin \frac{\pi}{6}$

$= 8 \times \frac{1}{2}$

$= 4$

ie acceleration is $4\ ms^{-2}$ ✓

(b) (i)



(ii) See above, for shaded regions

$A = \int_0^{\frac{\pi}{2}} \cos x\, dx + \left|\int_{\frac{\pi}{2}}^{\pi} \cos x\, dx\right|$

$= \left[\sin x\right]_0^{\frac{\pi}{2}} + \left|\left[\sin x\right]_{\frac{\pi}{2}}^{\pi}\right|$

$= \sin\frac{\pi}{2} - \sin 0 + \left|\sin \pi - \sin\frac{\pi}{2}\right|$

$= 1 - 0 + |0 - 1|$

$= 1 + 1$

$\therefore$ Area $= 2$ units^2 ✓

Some students did not take the absolute of the area below the x-axis

OR $A = 2\int_0^{\frac{\pi}{2}} \cos x\, dx$ ✓

$= 2(\sin x)_0^{\frac{\pi}{2}}$

$= 2\left(\sin\frac{\pi}{2} - \sin 0\right)$

$= 2(1 - 0)$ ✓

Area $= 2$ units^2

(iii) $\cos x = \frac{\sqrt{3}}{2}$, $0 \le x \le 2\pi$

$x = \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$ ✓

Well done

QUESTION SEVEN

(a) $\log_e x - \frac{3}{\log_e x} = 2$

let $u = \log_e x$

$u - \frac{3}{u} = 2$

$u^2 - 3 = 2u$

$u^2 - 2u - 3 = 0$ ✓

$(u-3)(u-1) = 0$

$u = 3$ or $u = -1$ ✓

$\log_e x = 3$ $\quad \log_e x = -1$

$x = e^3$ $\quad x = e^{-1}$ ✓

(b) (i) 45 km in 1 hour

= 45 km in 60 minutes.

= $\frac{3}{4}$ km in 1 min ✓

(ii) $l = r\theta$

$\frac{3}{4} = 0.5 \times \theta$ ✓

$\theta = \frac{3}{4} \times 2$

= 1.5 radians

= $1.5 \times \frac{180^0}{\pi}$

$\theta = 85.94366927$

$= 86^0$ (nearest deg) ✓

(c) (i) Probability

= P [(1,1) or (2,2) or (3,3) or (4,4) or (5,5) or (6,6)]

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$

$= \frac{6}{36} = \frac{1}{6}$ ✓

or Probability (Chris's throw same as Pat's)

$= \frac{1}{6}$ ✓

(ii) Let Pat be first, Chris be second relevant pairs are (1,2) (1,3) (1,4) (1,5) (1,6)

(2,3) (2,4) (2,5) (2,6)

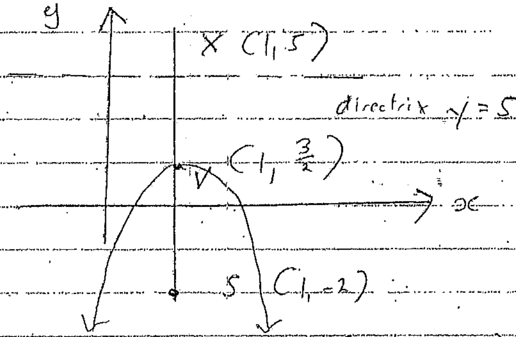
(3,4) (3,5) (3,6) ✓

(4,5) (4,6)

(5,6)

P (Chris' no > Pat's no) $= \frac{15}{36} = \frac{5}{12}$ ✓

(d)



(i) Vertex = midpt SX $= \left(\frac{1+1}{2}, \frac{5-2}{2}\right)$

$= \left(1, \frac{3}{2}\right)$ ✓

Confusion with the focal length and vertex.

(ii) Focal length = dist XV

$= 5 - \frac{3}{2} = \frac{7}{2}$ ✓

(iii) $(x-h)^2 = 4a(y-k)$

$(x-1)^2 = -4 \times \frac{7}{2}\left(y - \frac{3}{2}\right)$

$(x-1)^2 = -14\left(y - \frac{3}{2}\right)$

$x^2 - 2x + 1 = -14y + 21$

$x^2 - 2x + 14y - 20 = 0$ ✓

See above for sketch

QUESTION EIGHT

(a) (i) $y = 3x^2 - x^3$ ✓

Well done and well set out

$$\frac{dy}{dx} = 6x - 3x^2 = 0$$

for stationary points

$$2x - x^2 = 0$$

$$x(2-x) = 0$$

$$\therefore x = 0 \text{ or } 2 \quad ✓$$

$$\frac{d^2y}{dx^2} = 6 - 6x$$

at $x = 0$ $\quad \frac{d^2y}{dx^2} > 0$

$\therefore$ minimum at $(0, 0)$ ✓

at $x = 2$ $\quad \frac{d^2y}{dx^2} < 0$

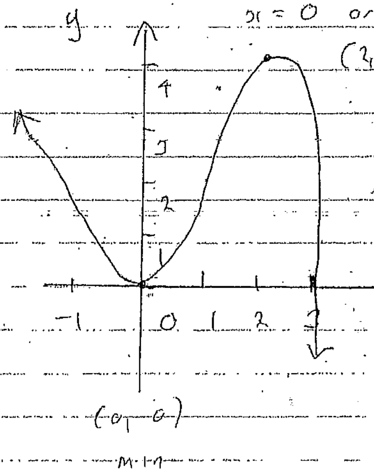
$\therefore$ maximum at $(2, 4)$ ✓

(ii) x intercepts at $y = 0$

$$3x^2 - x^3 = 0$$

$$x^2(3-x) = 0$$

$$x = 0 \text{ or } 3$$



✓ shape

✓ x-int & stationary pts

(iii) $\frac{dy}{dx} = 6x - 3x^2$

at $x = -1$ $\quad \frac{dy}{dx} = -6 - 3 = -9$ ✓

eqn of tangent

$$y - 4 = -9(x+1)$$

$$y - 4 = -9x - 9$$

$$y = -9x - 5$$

$$9x + y + 5 = 0 \quad ✓$$

(b) (i) $y = \log_e(\cos x)$

$$= -\frac{\sin x}{\cos x}$$

$$= -\tan x$$

(ii) $\int_0^{\frac{\pi}{4}} \tan x = -\left[\log_e \cos x\right]_0^{\frac{\pi}{4}}$ ✓

Students need log laws rather than a calculator.

$$= -\left[\log_e \frac{1}{\sqrt{2}} - \log_e 1\right]$$

$$= -\log_e \frac{1}{\sqrt{2}} \quad ✓$$

$$= -\log_e 2^{-\frac{1}{2}} \quad ✓$$

$$= \frac{1}{2}\log_e 2$$

QUESTION NINE

(a) $3^{2x} + 2\times 3^{x} - 15 = 0$ — well done

Let $a = 3^{x}$, $a^2 = 3^{2x}$

Then $a^2 + 2a - 15 = 0$

ie $(a+5)(a-3) = 0$ ✓

$\therefore a = -5$ or $a = 3$

$\therefore 3^x = -5$ or $3^x = 3$

$\therefore$ no soln $\quad x = 1$ ✓

(b) $V = \pi \int_0^{16} x^2\, dy$ — well done

$= \pi \int_0^{16} y^{\frac{1}{2}}\, dy$ ✓

$(y = x^4$ so $y^{\frac{1}{2}} = x^2)$

$V = \pi \left[\frac{2y^{\frac{3}{2}}}{3}\right]_0^{16}$ ✓

$= \pi \left[\frac{2\times 16^{\frac{3}{2}}}{3} - \frac{2\times 0^{\frac{3}{2}}}{3}\right]$ ✓

$= \pi \times \frac{128}{3}$

Volume $= \frac{128}{3}\pi$ units^3 ✓

(c) Treat the investment in 2 parts

Part 1 $1500 invested each year for 40 years

1st $1500 $= \$1500 \times 1.07^{40}$

2nd $1500 $= \$1500 \times 1.07^{39}$

↓

last $1500 $= \$1500 \times 1.07^{1}$ ✓

Part 2 : For the last 15 years, the extra $3500 (to make up the $5000 to be invested each year for 15 years)

1st $3500 $= \$3500 \times 1.07^{15}$

2nd $3500 $= \$3500 \times 1.07^{14}$

↓

last $3500 $= \$3500 \times 1.07^{1}$ ✓

Total $= 1500(1.07 + 1.07^2 + \ldots + 1.07^{40})$ plus $3500(1.07 + 1.07^2 + \ldots + 1.07^{15})$ ✓

$= 1500 \times 1.07 \frac{(1.07^{40} - 1)}{1.07 - 1} + 3500 \times 1.07 \frac{(1.07^{15} - 1)}{1.07 - 1}$ ✓

$=$

$\$320414.35 + \$94108.19$ ✓

$= \$414523$ (nearest dollar) ✓

(3 marks for correctly calculating one part only)

Some students had difficulty establishing the G.P for the two separate investments

QUESTION TEN

(a) (i) $v = \frac{dx}{dt}$

$\therefore x = \int_a^b v\,dt$ ✓

Since the initial time is 0, $a = 0$

The unit for velocity was km/h

The final time was 4 mins or $\frac{1}{15}$ h

$\therefore b = \frac{1}{15}$ ✓

$\therefore x = \int_0^{\frac{1}{15}} v\,dt$

> Very leniently marked. Should have been stating v is the rate of change of displacement w.r.t. time $\therefore$ to get position we need to integrate.
>
> generally well explained that velocity was in km/h $\therefore$ time needs to be given in hours. 4 mins $= \frac{1}{15}$ h

(ii) Using the formula:

$\int_0^{\frac{1}{15}} f(t)\,dt$

$= \frac{h}{3}\left(y_0 + y_4 + 4\times(y_1 + y_3) + 2y_2\right)$

where $h = \frac{1}{4}\left(\frac{1}{15} - 0\right)$ i.e. $\frac{1}{60}$ ✓

$x = \frac{1}{3} \times \frac{1}{60}\left(0 + 40 + 4\times(25+30) + 2\times 34\right)$ ✓

$= \frac{1}{180}(328)$

$= 1.8$ km (to 1 dp) ✓

> Some had difficulty calculating h. Some ~~had~~ students still getting formula incorrect.

(b) (i) Using speed $= \frac{\text{distance}}{\text{time}}$

$\therefore \text{time} = \frac{\text{distance}}{\text{speed}}$

$\therefore \text{time} = \frac{1000}{v}$ ✓

But cost per hour

= driver cost/h + fuel cost/h

$= 2\times 36 + \left(6 + \frac{v^2}{50}\right)\times 1.5$

$= 72 + 9 + \frac{1.5v^2}{50}$

$= 81 + \frac{3v^2}{100}$ ✓

$\therefore$ Total Cost $= \left(81 + \frac{3v^2}{100}\right)\times\frac{1000}{v}$ ✓

$= 30v + \frac{81000}{v}$

> Some had difficulty showing C.

(ii) $\frac{dC}{dv} = -\frac{81000}{v^2} + 30 = 0$ ✓

$30v^2 = 81000$

$v^2 = \frac{81000}{30}$

$= 2700$

$v = \sqrt{2700}$

$= 51.96$ km/h ✓ (to 2 decpl)

$\frac{d^2C}{dv^2} = \frac{162000}{v^3} > 0$ ✓

$\therefore$ min at $v = 51.96$

> Many did not test using the 2nd derivative or alternative.

however the trip time must be less than 12 hours

$\frac{1000}{v} \le 12$ ✓

$v \ge \frac{1000}{12} \ge 83\frac{1}{3}$

Hence the soln found to $\frac{dC}{dv} = 0$ does not satisfy the time constraint. Since Cost increases the further you are away from 51.96 km/h, the speed to minimise cost is $83\frac{1}{3}$ km/h $\doteqdot 83.33$ km/h (to 2 dp).

> Some ignored time & speed constraints.
>
> Some ignored using calculus at all — penalised!