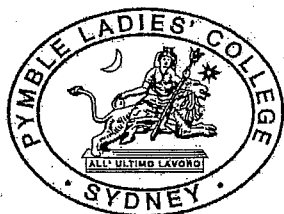


Mr Antonio  
 Mrs Collett  
 Mr Dudley  
 Mrs Kerr  
 Ms Lau  
 Mrs Sear  
 Miss Singlé  
 Mrs Soutar

Name: \_\_\_\_\_  
 Teacher: \_\_\_\_\_



2011  
 TRIAL HIGHER SCHOOL CERTIFICATE  
 EXAMINATION

# Mathematics

## General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided
- All necessary working should be shown in every question
- Start each question in a new booklet
- Marks may be deducted for careless or untidy work

## Total Marks – 120

- Attempt Questions 1-10
- All questions are of equal value

Mark	/120
Rank	/
Highest Mark	/120

Question 1 (12 marks) Use a SEPARATE Writing Booklet.

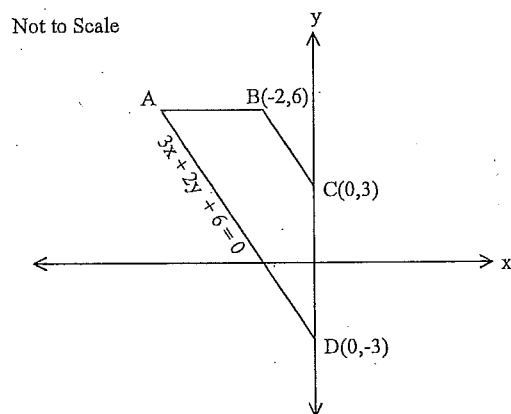
Marks

- (a) Evaluate, correct to two decimal places  $\frac{2.35}{\sqrt{43.7+3.19}}$ . 2
- (b) Simplify  $\frac{2}{x(x-3)} - \frac{1}{x}$ . 2
- (c) Evaluate  $\sum_{n=3}^5 (2n-1)^2$ . 1
- (d) Solve the simultaneous equations. 2
- $$\begin{aligned} 2x+y &= 13 \\ x-y &= -7 \end{aligned}$$
- (e) Solve  $|2x+1|=5$ . 2
- (f) Factorise  $8x^3+27$ . 1
- (g) Rationalise the denominator of the following fraction. 2
- $$\frac{4}{\sqrt{7}-2}$$

Question 2 (12 marks) Use a SEPARATE Writing Booklet.

Marks

(a) In the diagram below,  $ABCD$  is a quadrilateral.



The equation of the line  $AD$  is  $3x + 2y + 6 = 0$ .

- |   |   |
|---|---|
| (i) Show that $ABCD$ is a trapezium by showing that $BC$ is parallel to $AD$ .        | 2 |
| (ii) The line $AB$ is parallel to the $x$ -axis. Find the coordinates of $A$ .        | 1 |
| (iii) Find the equation of $BC$ .   | 1 |
| (iv) Find the length of $BC$ .  | 1 |
| (v) Show that the perpendicular distance from $B$ to $AD$ is $\frac{12}{\sqrt{13}}$ . | 2 |
| (vi) Hence, or otherwise, find the area of the trapezium $ABCD$ .                     | 2 |
- (b) Find the equation of the normal to the curve  $y = x^2 - 5x + 1$  when  $x = -1$ .

Question 3 (12 marks) Use a SEPARATE Writing Booklet.

Marks

(a) Differentiate the following with respect to  $x$ .

- |                              |   |
|------------------------------|---|
| (i) $e^x \ln 2x$ .           | 1 |
| (ii) $4(x^2 + 2)^3$ .        | 1 |
| (iii) $\frac{\tan x}{x^3}$ . | 2 |

(b) Find:

- |   |   |
|---|---|
| (i) $\int \frac{6}{x^2} - \frac{2}{x^3} dx$ . | 2 |
| (ii) $\int 2 \cos 6x dx$ .                    | 2 |

(c) Evaluate:

- |  |   |
|--|---|
| (i) $\int_2^5 \frac{3x}{x^2 + 1} dx$ . | 2 |
| (ii) $\int_0^\pi \sin 2x dx$ .         | 2 |

Question 4 (12 marks) Use a SEPARATE Writing Booklet.

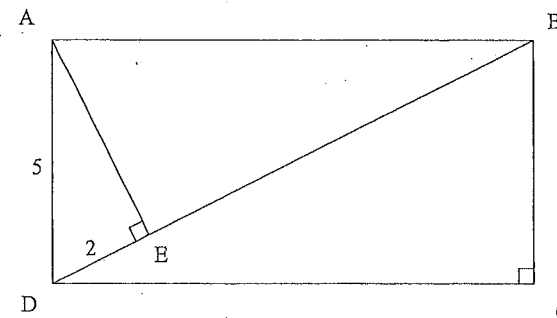
Marks

- (a) The gradient of a curve is given by  $g'(x) = 3x^2 - 4 + \frac{1}{x}$ .  
The curve passes through the point (1, 4). What is the equation of the curve? 3
- (b) Sketch the curve  $y = \cos^2 x + 1$  for  $0 \leq x \leq \pi$ , clearly showing all relevant features. 3
- (c) The region enclosed by the curve  $y = \frac{1}{2}(\sqrt{x-1})$ , the  $x$  and  $y$ -axes and the line  $y = 2$  is rotated about the  $y$ -axis.  
Find the volume of this solid of revolution. 3
- (d) Shade the region in the plane defined by  $x^2 + y^2 \leq 4$  and  $y > x - 1$ .  
Do not find the coordinates of the points of intersection. 3

Question 5 (12 marks) Use a SEPARATE Writing Booklet.

Marks

- (a) Let  $f(x) = 9x(x-2)^2$
- (i) Find the coordinates of the stationary points and determine their nature. 4
- (ii) Find the coordinates of the point(s) where the curve crosses the axes. 1
- (iii) Find the coordinates of the point(s) of inflexion. 2
- (iv) Sketch the graph of  $y = f(x)$ , clearly indicating all relevant features. 2
- (b) ABCD is a rectangle and E is a point on the diagonal BD such that AE is perpendicular to BD.



- (i) Prove that  $\triangle ADE$  is similar to  $\triangle DBC$ . 2
- (ii) If  $AD = 5\text{cm}$  and  $DE = 2\text{cm}$ , find the length of the diagonal  $BD$ . 1

Question 6 (12 marks) Use a SEPARATE Writing Booklet.

Marks

- (a) Find the values of  $m$  for which the equation  $x^2 + (m-2)x + 4 = 0$  has no real roots. 2

- (b) Consider the parabola  $4y = x^2 - 6x + 1$
- (i) Express in the form  $(x-h)^2 = 4a(y-k)$ . 1
- (ii) Write down the coordinates of the vertex. 1
- (iii) Find the coordinates of the focus. 2

- (c) Use Simpson's rule with five function values 3

to estimate  $\int_0^4 \frac{1}{\sqrt{x^3+1}} dx$ .

Give your answer correct to 2 decimal places.

- (d) (i) Find the limiting sum of the geometric series 2
- $$3 + \frac{3}{1+\sqrt{3}} + \frac{3}{(1+\sqrt{3})^2} + \dots$$

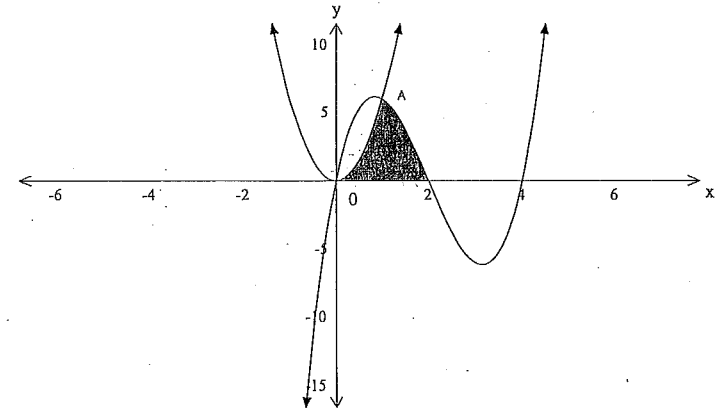
- (ii) Explain why the geometric series 1
- $$3 + \frac{3}{1-\sqrt{3}} + \frac{3}{(1-\sqrt{3})^2} + \dots$$

does NOT have a limiting sum.

Question 7 (12 marks) Use a SEPARATE Writing Booklet.

Marks

- (a) (i) Solve  $e^{x-2} - 1 = 0$ . 1
- (ii) Sketch  $y = e^{x-2} - 1$ , clearly showing all the relevant features. 2
- (b)



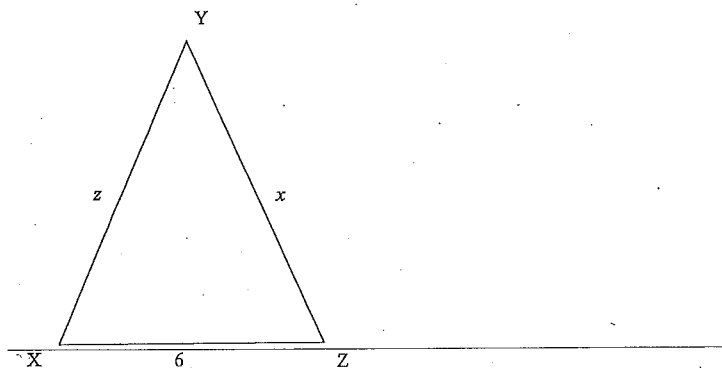
In the diagram, the shaded region is bounded by  $y = 6x^2$ ,  $y = 2x(x-2)(x-4)$  and the  $x$ -axis. The curves intersect in three places, two of which  $(0, 0)$  and  $A$ , are shown on the diagram.

- (i) Find the  $x$  coordinate of  $A$ . 2
- (ii) Find the area of the shaded region bounded by  $y = 6x^2$ ,  $y = 2x(x-2)(x-4)$  and the  $x$ -axis. 3
- (c) A vessel initially contains 100 litres of liquid. It is being emptied and the rate of change of volume is  $\frac{dV}{dt} = -\left(2 + \frac{20}{t+1}\right)$  where  $V$  is the volume in litres after  $t$  minutes.
- (i) At what rate is the vessel emptying initially? 1
- (ii) Find how many litres of liquid remain in the vessel after five minutes. Answer correct to 3 significant figures. 3

Question 8 (12 marks) Use a SEPARATE Writing Booklet.

Marks

- (a) Find the maximum value of the function  $y = -16x^2 + 160x - 256$ . 2
- (b) The triangle  $XYZ$  has  $XZ = 6$ ,  $YZ = x$ ,  $XY = z$  as shown below. The perimeter of triangle  $XYZ$  is 16. All measurements are in centimetres.



- (i) Express  $z$  in terms of  $x$ . 1
- (ii) Using the cosine rule, express  $z^2$  in terms of  $x$  and  $\cos Z$ . 1
- (iii) Hence show that  $\cos Z = \frac{5x-16}{3x}$ . 2
- (iv) Let the area of triangle  $XYZ$  be  $A$ .  
Show that  $A^2 = 9x^2 \sin^2 Z$ . 1
- (v) Hence, show that  $A^2 = -16x^2 + 160x - 256$ . 2
- (vi) What is the maximum area for triangle  $XYZ$ ? 1
- (c) Let  $f(x) = 1 + 10^x$ . 2
- Show that  $f(x) \times f(-x) = f(x) + f(-x)$ .

Question 9 (12 marks) Use a SEPARATE Writing Booklet

Marks

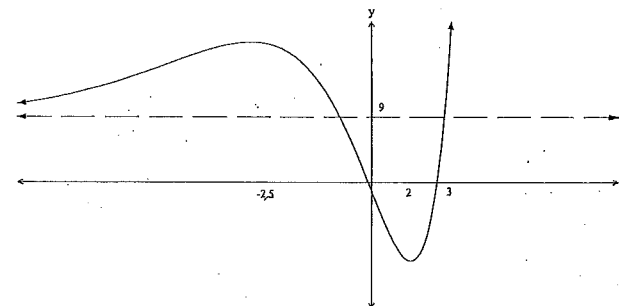
- (a) Assume the population,  $P$ , of Tasmanian devils has been decreasing at a rate proportional to  $P$  due to the infection of individuals by Devil facial tumour disease. That is  $\frac{dP}{dt} = -kP$  where  $k$  is a positive constant. 4

It was estimated that 130 000 individuals were in Tasmania in 1985. Twenty three years later, in 2008, the estimated number stood at just 10 000.

If the population continues to decrease at this rate, how many Tasmanian devils will there be in Tasmania in 2020?

- (b) A particle moves in a straight line such that after  $t$  seconds its acceleration function is  $\ddot{x} = (6t - 2) \text{ ms}^{-2}$ . Initially the velocity of the particle is  $-1 \text{ ms}^{-1}$ .
- (i) Find the particle's velocity after 2 seconds. 2
- (ii) Find the time at which the particle is stationary. 1
- (iii) Find the distance travelled by the particle in the third second of motion. 2

- (c) The diagram shows the graph of  $y = f(x)$ .



- (i) For which values of  $x$  is the derivative,  $f'(x)$  negative? 1
- (ii) Sketch the graph  $y = f'(x)$ . 2

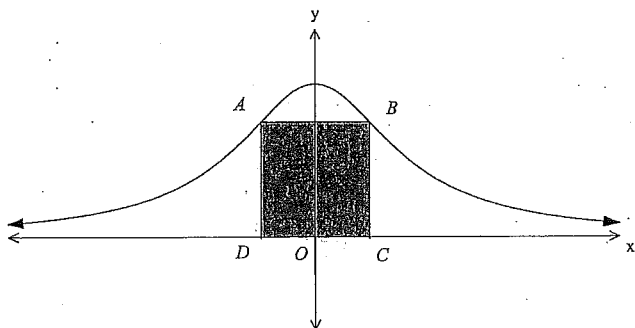
Question 10 (12 marks) Use a SEPARATE Writing Booklet.

Marks

(a) Hercule decides to save for a deposit on a new igloo. The Greater Antarctic Savings Bank is offering an interest rate of 7.2% per annum, compounded monthly. Hercule receives his salary on the first day of the month and at this time puts \$500 into his savings account.

- (i) What will be the value of the first \$500 invested after 2 years? 1
- (ii) Hercule's dream igloo requires a deposit of \$35000. If he continues to invest \$500 each month from his salary, how long will it take him to be able to afford the deposit? 3

(b)



In the diagram above the function  $y = \frac{b}{x^2 + a}$  has a maximum turning point at (0,4) and passes through (3,1).

A rectangle  $ABCD$  is inscribed within the curve as shown.  
 $OY$  is the axis of symmetry.

- (i) Find the value of  $a$  and  $b$ . 2
- (ii) If  $C$  has coordinates  $(p, 0)$ , find the coordinates of  $B$  in terms of  $p$ . 1
- (iii) Show that the area of  $ABCD = \frac{24p}{p^2 + 3}$ . hello : ) 1
- (vi) Hence find the exact maximum area of  $ABCD$ . 4

End of Paper

2011 TRIAL HSC SOLUTIONS

QUESTION ONE

a)  $\frac{2 \cdot 35}{\sqrt{43 \cdot 7} + 3 \cdot 19}$   
 $= 0.24$  to 2 dec. pl. (2)

b)  $\frac{2}{x(x-3)} - \frac{1}{x} = \frac{2}{x(x-3)} - \frac{(x-3)}{x(x-3)}$  (1) correct numerator  
 $= \frac{2-x+3}{x(x-3)}$   
 $= \frac{5-x}{x(x-3)}$  (1) correct answer

c)  $\sum_{n=3}^5 (2n-1)^2 = (2 \times 3 - 1)^2 + (2 \times 4 - 1)^2 + (2 \times 5 - 1)^2$   
 $= 5^2 + 7^2 + 9^2$   
 $= 25 + 49 + 81$   
 $= 155$  (1) r/w

d)  $2x + y = 13$  — (1)  
 $x - y = -7$  — (2)

(1) + (2)

$3x = 6$

$x = 2$  (1)

sub  $x = 2$  in (2)

$2 - y = -7$

$-y = -9$

$y = 9$  (1)

$\therefore x = 2, y = 9$

e)  $|2x+1| = 5$

$2x+1 = 5$        $2x+1 = -5$

$2x = 4$        $2x = -6$

$x = 2$  or  $x = -3$

(1)      (1)

f)  $8x^3 + 27 = (2x+3)(4x^2 - 6x + 9)$  (1) r/w

g)  $\frac{4}{\sqrt{7}-2} = \frac{4}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$  (1) correct multiple

$= \frac{4\sqrt{7}+8}{7-4}$

$= \frac{4\sqrt{7}+8}{3}$  (1) answer simplified

3

QUESTION TWO

i) Find gradient AD

$$3x + 2y + 6 = 0$$

$$2y = -3x - 6$$

$$y = -\frac{3x}{2} - 3$$

∴

$$m_{AD} = -\frac{3}{2}$$

∴

Find gradient BC

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6 - 3}{-2 - 0}$$

$$= \frac{3}{-2}$$

$$= -\frac{3}{2}$$

$$∴$$

$$= -\frac{3}{2}$$

① gradients

① statement justifying

wng.  $BC \parallel AD$

Since  $m_{AD} = m_{BC}$ ,  $AD \parallel BC$  and ABCD is a trapezium.

ii. Since AB is parallel to the x-axis, it has the same y-coordinate as B. That is  $y = 6$

sub  $y = 6$  into  $3x + 2y + 6 = 0$

$$3x + 2(6) + 6 = 0$$

$$3x + 18 = 0$$

$$3x = -18$$

$$x = -6$$

∴ A is  $(-6, 6)$

① r/w

iii. Equation of BC

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{3}{2}(x - 0)$$

∴

$$2y - 6 = -3x$$

$$3x + 2y - 6 = 0$$

① r/w

iv. length BC

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-2 - 0)^2 + (6 - 3)^2}$$

$$= \sqrt{(-2)^2 + (3)^2}$$

$$= \sqrt{4 + 9}$$

$$= \sqrt{13} \text{ units}$$

① r/w

v. perpendicular distance

$$p = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|3(-2) + 2(6) + 6|}{\sqrt{3^2 + 2^2}}$$

① correct substitution

$$= \frac{|-6 + 12 + 6|}{\sqrt{9 + 4}}$$

$$= \frac{|12|}{\sqrt{13}}$$

$$= \frac{12}{\sqrt{13}}$$

$$= \frac{12}{\sqrt{13}} \text{ units}$$

① Evaluation

vi. length AD

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-6 - 0)^2 + (6 - 3)^2}$$

$$= \sqrt{36 + 9}$$

$$= \sqrt{45}$$

$$= 3\sqrt{5} \text{ units}$$

① length

Area trapezium ABCD

$$A = \frac{1}{2} \times h \times (a + b)$$

$$= \frac{1}{2} \times 12 \times (\sqrt{17} + \sqrt{13})$$

$$= \frac{6\sqrt{17} + 6\sqrt{13}}{\sqrt{13}}$$

$$= 24 \text{ units}^2$$

① Answer



QUESTION THREE

a) i. Differentiate  $e^x \ln 2x$

$$u = e^x \quad v = \ln 2x$$

$$u' = e^x \quad v' = \frac{2}{2x} = \frac{1}{x}$$

$$y' = vu' + uv'$$

$$= \ln 2x \times e^x + e^x \times \frac{1}{x}$$

$$= e^x (\ln 2x + \frac{1}{x})$$

① r/w

ii. Differentiate  $4(x^2+2)^3$

$$y' = 4 \times 3 \times 2x \times (x^2+2)^2$$

$$= 24x(x^2+2)^2$$

① r/w

iii. Differentiate  $\frac{\tan x}{x^3}$

$$u = \tan x \quad v = x^3$$

$$u' = \sec^2 x \quad v' = 3x^2$$

$$y' = \frac{vu' - uv'}{v^2}$$

$$= \frac{x^3 \times \sec^2 x - \tan x \times 3x^2}{(x^3)^2}$$

$$= \frac{x^3 \sec^2 x - 3x^2 \tan x}{x^6}$$

$$= \frac{x^2 (x \sec^2 x - 3 \tan x)}{x^6}$$

$$= \frac{x \sec^2 x - 3 \tan x}{x^4}$$

① correct subs into quotient rule

① correct simplification

b)  $y = x^2 - 5x + 1$

$$\frac{dy}{dx} = 2x - 5$$

when  $x = -1$ ,

$$\frac{dy}{dx} = 2(-1) - 5$$

$\frac{dy}{dx}$

$$= -2 - 5$$

$$= -7$$

① correct calc of

so the tangent at  $x = -1$  has gradient  $-7$ , gradient

Since  $m_1 = -7$  and  $m_1 \times m_2 = -1$  when they are at point

perpendicular,

$$-7 \times m_2 = -1$$

$$m_2 = \frac{1}{7}$$

① correct calculation of normal gradient

so the normal has gradient  $\frac{1}{7}$

when  $x = -1$

$$y = (-1)^2 - 5(-1) + 1$$

$$= 1 + 5 + 1$$

$$= 7$$

using point gradient formula,

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{1}{7}(x - (-1))$$

$$7y - 49 = x + 1$$

$$x - 7y + 50 = 0$$

— eqn of normal

① correct equation from pt gradient formula

b) i)  $\int \frac{6x^2 - 2}{3x^3 + x} dx = \int (6x^2 - 2x^{-2}) dx$   
 $= \frac{6x^3}{3} - \frac{2x^{-1}}{-1} + C$  (i) integration  
 $= 2x^3 + x^{-1} + C$   
 $= 2x^3 + \frac{1}{x} + C$  (ii) returning to original format

ii)  $\int 2 \cos 6x dx = 2 \int \cos 6x dx$   
 $= 2 \left[ \frac{1}{6} \sin 6x \right] + C$  (i)  $\frac{1}{3}$   
 $= \frac{2}{3} \sin 6x + C$  (ii)  $\sin 6x$  and  $+ C$

c) i)  $\int_2^5 \frac{3x}{x^2+1} dx = \frac{3}{2} \int_2^5 \frac{2x}{x^2+1} dx$   
 $= \frac{3}{2} \left[ \log_e(x^2+1) \right]_2^5$  (i)  
 $= \frac{3}{2} \left[ \log_e(5^2+1) - \log_e(2^2+1) \right]$   
 $= \frac{3}{2} (\log_e 26 - \log_e 5)$  (ii)

ii)  $\int_0^{\pi} \sin 2x dx$   
 $= \left[ \frac{-\cos 2x}{2} \right]_0^{\pi}$  (i) integral  
 $= \frac{-\cos 2\pi}{2} - \left( \frac{-\cos 0}{2} \right)$   
 $= \frac{-1}{2} + \frac{1}{2}$

QUESTION FOUR

a)  $g'(x) = 3x^2 - 4x + \frac{1}{x^2}$   
 $g(x) = 3x^3 - 4x^2 + \frac{x^{-1}}{-1} + C$   
 $= 3x^3 - 4x^2 - \frac{1}{x} + C$  (i) integration

when  $x=1$ ,  $g(x)=4$   
 $4 = (1)^3 - 4(1) - \frac{1}{1} + C$   
 $4 = 1 - 4 - 1 + C$  (ii) correct c value  
 $C = 8$

$g(x) = 3x^3 - 4x^2 - \frac{1}{x} + 8$  (iii) correct eq<sup>n</sup>

b)  $y = \cos^2 x + 1$  for  $0 \leq x \leq \pi$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	$\pi$
y	2	$\frac{5}{2}$	1	$\frac{5}{2}$	2



- (i) correct domain
- (ii) correct range
- (iii) correct shape

$$c) y = \frac{1}{2}(\sqrt{x-1})$$

$$2y = \sqrt{x-1}$$

$$4y^2 = x-1$$

$$x = 4y^2 + 1$$

$$V = \pi \int_a^b x^2 dy$$

$$= \pi \int_0^2 (4y^2 + 1)^2 dy$$

$$= \pi \int_0^2 (16y^4 + 8y^2 + 1) dy$$

$$= \pi \left[ \frac{16y^5}{5} + \frac{8y^3}{3} + y \right]_0^2$$

$$= \pi \left\{ \frac{16}{5} \times 2^5 + \frac{8}{3} \times 2^3 + 2 - \left( \frac{16}{5} \times 0^5 + \frac{8}{3} \times 0^3 + 0 \right) \right\}$$

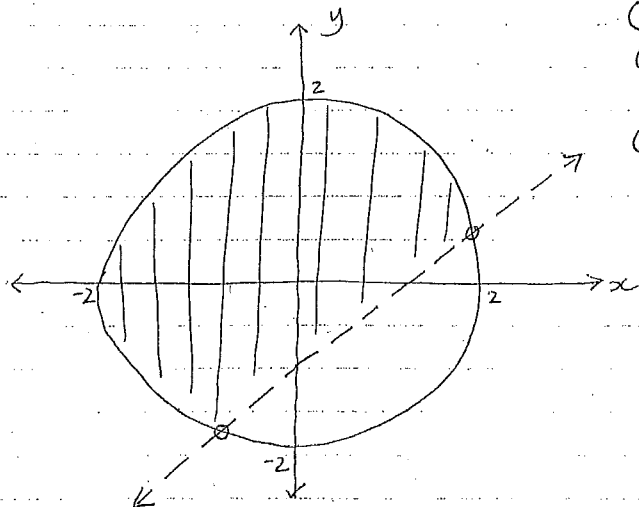
$$= \frac{1886\pi}{15} \text{ units}^3$$

① Correct expression

① Correct integral

① Correct answer

$$d) x^2 + y^2 \leq 4, y > x - 1$$



① Circle correct

① line correct  
and broken

① correct shading

## QUESTION FIVE

$$a) i. f(x) = 9x(x-2)^2$$

$$u = 9x \quad v = (x-2)^2$$

$$u' = 9 \quad v' = 2(x-2)$$

$$f'(x) = vu' + uv'$$

$$= (x-2)^2 \times 9 + 9x \times 2(x-2)$$

$$= 9(x-2)^2 + 18x(x-2)$$

$$= 9(x-2)(x-2+2x)$$

$$= 9(x-2)(3x-2)$$

① correct  $f'(x)$

Stat. points occur when  $f'(x) = 0$

$$9(x-2)(3x-2) = 0$$

$$(x-2)(3x-2) = 0$$

$$x-2 = 0 \quad 3x-2 = 0$$

$$x = 2 \quad 3x = 2$$

$$x = \frac{2}{3}$$

① correct  $x$

values of st. pts

when  $x = 2$ ,

$$f(2) = 9(2)(2-2)^2$$

$$= 0$$

when  $x = \frac{2}{3}$

$$f\left(\frac{2}{3}\right) = 9\left(\frac{2}{3}\right)\left(\frac{2}{3}-2\right)^2$$

$$= \frac{32}{3}$$

$$= 10\frac{2}{3}$$

① correct  $y$  values

$$f(x) = 9x(x-2)^2$$

$$f'(x) = 9(x-2)(3x-2)$$

$$u = 9(x-2) \quad v = 3x-2$$

$$u' = 9 \quad v' = 3$$

$$f''(x) = vu' + uv'$$

$$= (3x-2) \times 9 + 9(x-2) \times 3$$

$$= 9(3x-2) + 27(x-2)$$

$$= 27x - 18 + 27x - 54$$

$$= 54x - 72$$

when  $x=2$ ,

$$f''(2) = 54(2) - 72$$

$$= 36$$

$$> 0$$

$\therefore$  concave up with minimum turning point

when  $x = \frac{2}{3}$

$$f''\left(\frac{2}{3}\right) = 54\left(\frac{2}{3}\right) - 72$$

$$= -36$$

$$< 0$$

$\therefore$  concave down with maximum turning point

$\therefore$  Stat point at  $(2, 0)$  is a minimum (1) correct

Stat point at  $\left(\frac{2}{3}, 10\frac{2}{3}\right)$  is a maximum. testing of nature

ii.  $f(x) = 9x(x-2)^2$

when  $x=0$ ,

$$f(0) = 9(0)(0-2)^2$$

$$= 0$$

when  $f(x)=0$

$$9x(x-2)^2 = 0$$

$$9x = 0 \quad (x-2)^2 = 0$$

$$x = 0 \quad x - 2 = 0$$

$$x = 2$$

Curve crosses at  $(0, 0)$ ,  $(2, 0)$ . (1) r/w

iii.  $f''(x) = 54x - 72$

when  $f''(x) = 0$ ,

$$54x - 72 = 0$$

$$54x = 72$$

$$x = \frac{1}{3}$$

There is a possible point of inflexion at  $x = \frac{1}{3}$

Test concavity changes:

$x$	1	$\frac{1}{3}$	$\frac{2}{3}$
$f''(x)$	-18	0	18

Since concavity changes, there is a point of inflexion at  $x = \frac{1}{3}$ .

when  $x = \frac{1}{3}$ ,

$$f\left(\frac{1}{3}\right) = 9\left(\frac{1}{3}\right)\left(\frac{1}{3}-2\right)^2$$

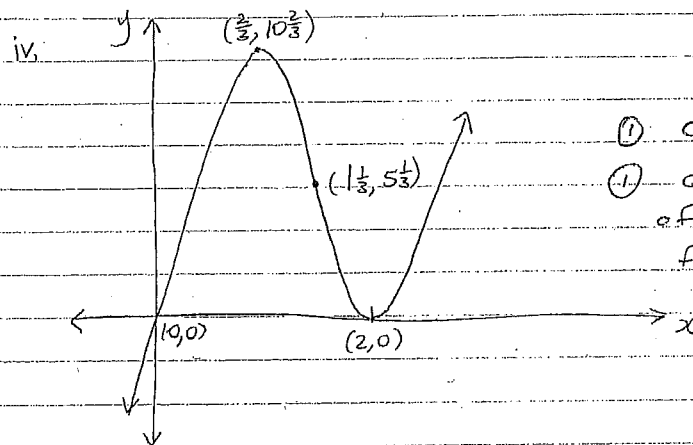
$$= \frac{16}{3}$$

$$= 5\frac{1}{3}$$

(1) Testing

(1) correct point  $(x, y)$

$\therefore$  Point of inflexion at  $\left(\frac{1}{3}, 5\frac{1}{3}\right)$



(1) Correct shape

(1) correct labelling of relevant features

b) i. Since ABCD is a rectangle,  
 $AD \parallel BC$  and  $AB \parallel DC$  (opposite sides of a rectangle are parallel) (1)

In  $\triangle ADE$  and  $\triangle DBC$

$$\angle AED = \angle DCB \text{ (given)}$$

$\angle ADE = \angle DBC$  (alternate angles on parallel lines are equal,  $AD \parallel BC$ ) (1)

$\therefore \triangle ADE \parallel \triangle DBC$  (equiangular)

ii.  $\frac{BD}{DA} = \frac{BC}{DE}$  (corresponding sides in similar triangles are in the same ratio)

$$\frac{BD}{5} = \frac{2}{2}$$

$$BD = \frac{2 \times 5}{2}$$

$$BD = 5$$

① r/w

$$= 12.5$$

① r/w

### QUESTION SIX

a)  $x^2 + (m-2)x + 4 = 0$  has no real roots when  $\Delta < 0$

$$\Delta = b^2 - 4ac$$

$$= (m-2)^2 - 4 \times 1 \times 4$$

$$= m^2 - 4m + 4 - 16$$

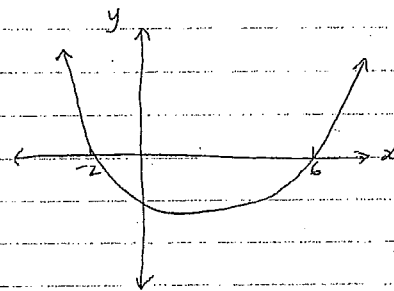
$$= m^2 - 4m - 12$$

① Evaluation of  $\Delta$

$$\Delta < 0$$

$$m^2 - 4m - 12 < 0$$

$$(m-6)(m+2) < 0$$



From the sketch,

$$-2 < m < 6$$

① Collect answer

b) i.  $4y = x^2 - 6x + 1$

$$4y - 1 = x^2 - 6x$$

$$4y - 1 + 9 = x^2 - 6x + 9$$

$$4y + 8 = (x-3)^2$$

$$4(y+2) = (x-3)^2$$

$$(x-3)^2 = 4(y+2)$$

① r/w

ii. Vertex:  $(3, -2)$

① r/w

iii. Focal length:  $4a = 4$

$$a = 1$$

① Focal length

Focus:  $(3, -1)$

① Answer

c)  $\int_0^4 \frac{1}{\sqrt{x^3+1}} dx$

$x$	0	1	2	4	5
$f(x)$	1	$1/\sqrt{2}$	$1/3$	$1/\sqrt{28}$	$1/\sqrt{65}$

① calculation of values

$\int_a^b f(x) dx \approx \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3) + 2(y_2)]$  ① correct  
 $h = \frac{b-a}{n}$   
 Subs into

a version of Simpson's rule

$n = \frac{4-0}{4}$

$\int_0^4 \frac{1}{\sqrt{x^3+1}} dx \approx \frac{1}{3} \left[ \left(1 + \frac{1}{\sqrt{65}}\right) + 4 \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{28}}\right) + 2 \left(\frac{1}{3}\right) \right]$   
 $\approx 1.791685824$   
 $\approx 1.79$  (to 2 dec. pl.) ① Answer

d) i.  $3 + \frac{3}{1+\sqrt{3}} + \frac{3}{(1+\sqrt{3})^2} + \dots$

$a = 3$   
 $r = \frac{1}{1+\sqrt{3}}$

$S_{\infty} = \frac{a}{1-r}$   
 $= \frac{3}{1 - \frac{1}{1+\sqrt{3}}}$

① correct  $S_{\infty}$  and substitution

$= \frac{3(1+\sqrt{3})}{1+\sqrt{3}-1}$

$= \sqrt{3}(1+\sqrt{3})$

① correct answer nothing for decimal approximation

ii.  $r = \frac{1}{1+\sqrt{3}} \approx -1.366025404$

① r/w

Since we don't have  $|r| < 1$ , we do not have a limit...

QUESTION SEVEN

a) i.  $e^{x-2} - 1 = 0$

$e^{x-2} = 1$

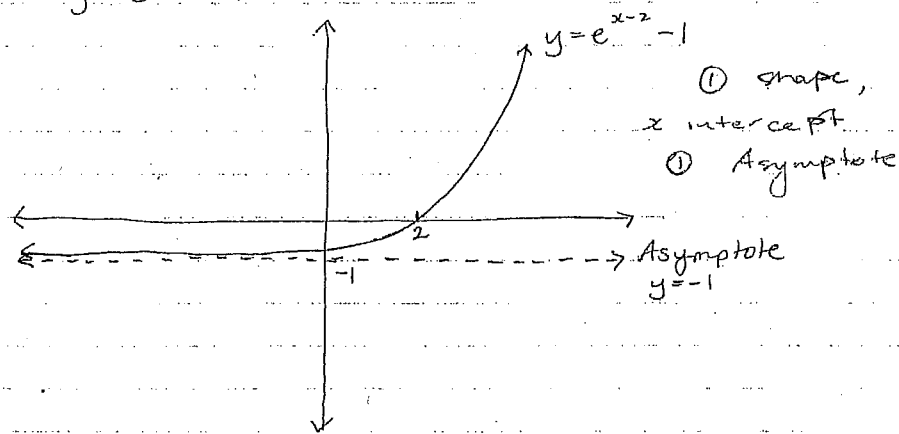
$\log_e e^{x-2} = \log_e 1$

$x-2 = 0$

$x = 2$

① r/w

ii.  $y = e^{x-2} - 1$



b) i.  $y = 6x^2$  — ①

$y = 2x(x-2)(x-4)$  — ②

$0 = ②$

$6x^2 - 2x(x-2)(x-4)$

① solving simultaneous

$6x^2 = 2x(x^2 - 6x + 8)$

to achieve a correct

$6x^2 = 2x^3 - 12x^2 + 16x$

eq<sup>n</sup>

$0 = 2x^3 - 18x^2 + 16x$

$0 = x^3 - 9x^2 + 8x$

$0 = x(x^2 - 9x + 8)$

$0 = x(x-8)(x-1)$

$x = 0 \quad x - 8 = 0 \quad x - 1 = 0$

$x = 8 \quad x = 1$

$\therefore$  x coordinate of A is  $x = 1$  ① Answer

$$\begin{aligned} \text{ii. } & \int_0^1 6x^2 dx + \int_1^2 2x(x-2)(x-4) dx \\ & = \int_0^1 (6x^2) dx + \int_1^2 (2x^3 - 12x^2 + 16x) dx \quad \textcircled{1} \text{ correct statement} \\ & = \left[ \frac{6x^3}{3} \right]_0^1 + \left[ \frac{2x^4}{4} - \frac{12x^3}{3} + \frac{16x^2}{2} \right]_1^2 \quad \textcircled{1} \text{ integral} \\ & = \left[ 2x^3 \right]_0^1 + \left[ \frac{x^4}{2} - 4x^3 + 8x^2 \right]_1^2 \\ & = \left[ 2(1)^3 - 0 \right] + \left[ \left( \frac{2^4}{2} - 4(2)^3 + 8(2)^2 \right) - \left( \frac{1^4}{2} - 4(1)^3 + 8(1)^2 \right) \right] \\ & = 2 + \left[ (8 - 32 + 32) - \left( \frac{1}{2} - 4 + 8 \right) \right] \\ & = 2 + (8 - 4\frac{1}{2}) \\ & = 5\frac{1}{2} \text{ units}^2 \quad \textcircled{1} \text{ substitution and evaluation} \end{aligned}$$

$$\begin{aligned} \text{c) i. } & \frac{dV}{dt} = -\left( \frac{2+20}{t+1} \right) \\ & \text{when } t=0 \\ & \frac{dV}{dt} = -\left( \frac{2+20}{0+1} \right) \\ & = -22 \end{aligned}$$

Emptying at 22 L/min  $\textcircled{1}$  r/w

$$\begin{aligned} \text{ii. } & \frac{dV}{dt} = -2 - \frac{20}{t+1} \quad V = \int -\left( 2 + \frac{20}{t+1} \right) dt \quad \textcircled{1} \\ & V = -2t - 20 \log_e(t+1) + C \quad \text{Correct integral and correct integration} \\ & \text{When } t=0, V=100 \\ & 100 = -2(0) - 20 \log_e(0+1) + C \\ & 100 = C \\ & V = -2t - 20 \log_e(t+1) + 100 \quad \textcircled{1} \text{ calculation of "c"} \\ & \text{When } t=5, \\ & V = -2(5) - 20 \log_e(5+1) + 100 \\ & = 54.16481062 \\ & = 54.2 \text{ (to 2 s.f.)} \quad \textcircled{1} \text{ calculation of} \end{aligned}$$

### QUESTION EIGHT

$$\text{a) } y = -16x^2 + 160x - 256$$

axis of symmetry

$$\begin{aligned} x &= \frac{-b}{2a} \\ &= \frac{-160}{2 \times -16} \\ &= 5 \end{aligned}$$

$\textcircled{1}$  For  $x$  value

$$\begin{aligned} \text{When } x=5, \\ y &= -16(5)^2 + 160(5) - 256 \\ &= 144 \end{aligned}$$

Maximum value is 144  $\textcircled{1}$  Maximum value

b) i. Perimeter:

$$\begin{aligned} P &= 6 + x + z \\ 16 &= 6 + x + z \\ 10 &= x + z \\ z &= 10 - x \end{aligned}$$

$\textcircled{1}$  r/w

$$\text{ii. } c^2 = a^2 + b^2 - 2ab \cos C$$

$$z^2 = x^2 + 6^2 - 2 \times x \times 6 \times \cos Z$$

$$z^2 = x^2 + 36 - 12x \cos Z \quad \textcircled{1} \text{ r/w}$$

$$\text{iii. } (10-x)^2 = x^2 + 36 - 12x \cos Z \quad \textcircled{1} \text{ substitution}$$

$$100 - 20x + x^2 = x^2 + 36 - 12x \cos Z \quad \text{into cosine rule}$$

$$12x \cos Z = 20x - 64$$

$$\cos Z = \frac{20x - 64}{12x}$$

$$\cos Z = \frac{5x - 16}{3x}$$

$\textcircled{1}$  Correct rearrangement to achieve "show"

$$\begin{aligned} \text{iv. } A &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times 6 \times x \sin Z \\ &= 3x \sin Z \\ A^2 &= 9x^2 \sin^2 Z \end{aligned}$$

① r/w

$$\begin{aligned} \text{v. } A^2 &= 9x^2 \sin^2 Z \\ &= 9x^2 (1 - \cos^2 Z) \\ &= 9x^2 \left( 1 - \left( \frac{5x-16}{3x} \right)^2 \right) \\ &= 9x^2 \left( 1 - \frac{(25x^2 - 160x + 256)}{9x^2} \right) \\ &= 9x^2 \left( \frac{9x^2 - 25x^2 + 160x - 256}{9x^2} \right) \\ &= -16x^2 + 160x - 256 \end{aligned}$$

① correct combination of earlier parts

① correct expansion and simplification to achieve "show"

vi. from a) Maximum value for  $A^2 = 144$   
 $\therefore$  Maximum area of  $\Delta XYZ$  is  $\sqrt{144}$   
 $= 12 \text{ units}^2$

① r/w

$$\begin{aligned} \text{c) } f(x) &= 1 + 10^x \\ f(-x) &= 1 + 10^{-x} \end{aligned}$$

$$\begin{aligned} f(x) \times f(-x) &= (1 + 10^x)(1 + 10^{-x}) \\ &= 1 + 10^x + 10^{-x} + 10^x \cdot 10^{-x} \\ &= 1 + 10^x + 10^{-x} + 10^0 \\ &= 1 + 10^x + 10^{-x} + 1 \\ &= 10^x + 10^{-x} + 2 \end{aligned}$$

① LHS

$$\begin{aligned} f(x) + f(-x) &= 1 + 10^x + 1 + 10^{-x} \\ &= 10^x + 10^{-x} + 2 \end{aligned}$$

① RHS

$$\therefore f(x) \times f(-x) = f(x) + f(-x)$$

### QUESTION NINE

$$\text{a) } \frac{dP}{dt} = -kP$$

$$P = P_0 e^{kt}$$

① correct diff Eq<sup>n</sup>

$$\text{when } t=0 (1985), P=130000$$

$$130000 = P_0 e^{k \times 0}$$

$$\therefore P_0 = 130000$$

① calc  $P_0$

$$\text{when } t=23 (2008), P=10000$$

$$10000 = 130000 e^{k \times 23}$$

$$1 = e^{23k}$$

13

$$\log_e \left( \frac{1}{13} \right) = \log_e e^{23k}$$

$$23k = \log_e \left( \frac{1}{13} \right)$$

$$k = \frac{1}{23} \log_e \left( \frac{1}{13} \right)$$

① correct "k"

$$= -0.1115195373$$

$$\therefore P = 130000 e^{-0.1115195373t}$$

$$\text{when } t=35$$

$$P = 130000 e^{-0.1115195373 \times 35}$$

$$= 2623.083792$$

① Answer

$$\approx 2623 \text{ (to nearest whole number)}$$

$$\text{b) i. } \ddot{x} = 6t - 2$$

$$\dot{x} = 6t^2 - 2t + C$$

2

$$\dot{x} = 3t^2 - 2t + C$$

$$\text{when } t=0, \dot{x}=-1$$

$$-1 = 3(0)^2 - 2(0) + C$$

$$C = -1$$

$$\dot{x} = 3t^2 - 2t - 1$$

① correct  $\dot{x}$

$$\text{when } t=2$$

$$\dot{x} = 3(2)^2 - 2(2) - 1$$

$$= 12 - 4 - 1$$

①  $\dot{x}$  when  $t=2$



ii. The particle is stationary when  $\dot{x} = 0$ .

$$\dot{x} = 3t^2 - 2t - 1$$

$$0 = 3t^2 - 2t - 1$$

$$0 = 3t^2 - 3t + t - 1$$

$$0 = 3t(t-1) + (t-1)$$

$$0 = (3t+1)(t-1)$$

$$3t+1=0 \quad t-1=0$$

$$3t = -1 \quad t = 1$$

$$t = \frac{-1}{3}$$

3

since  $t > 0$ ,  $t = 1$  s.

① r/w

iii.  $\dot{x} = 3t^2 - 2t - 1$

$$x = 3t^3 - 2t^2 - t + k$$

$$x = t^3 - t^2 - t + k$$

when  $t = 2$

$$x = (2)^3 - (2)^2 - (2) + k$$

$$= 8 - 4 - 2 + k$$

$$= 2 + k$$

when  $t = 3$

$$x = (3)^3 - (3)^2 - (3) + k$$

$$= 27 - 9 - 3 + k$$

$$= 15 + k$$

$$\text{Distance travelled} = (15+k) - (2+k)$$

$$= 13 \text{ m}$$

①

① Distance =  $\int_2^3 (3t^2 - 2t - 1) dt$

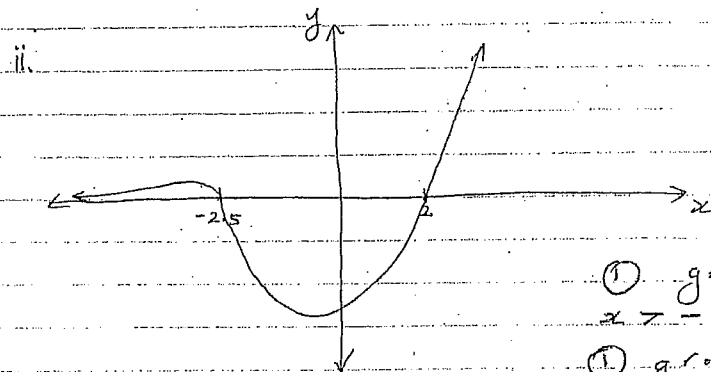
$$① = \left[ \frac{3t^3}{3} - \frac{2t^2}{2} - t \right]_2^3$$

$$= 3^3 - 3^2 - 3 - (2^3 - 2^2 - 2)$$

$$= 13 \text{ m} \quad ①$$

c) i.  $f'(x) < 0$  when  $-2.5 < x < 2$

① r/w



① graph  
 $x > -2.5$

① graph  
 $x < -2.5$

QUESTION TEN

a) i.  $r = 0.072$  p.a

$= 0.006$  p.m

$P = \$500$

$n = 24$

$A = P(1+r)^n$

$= 500(1+0.006)^{24}$

$= \$577.1936461$

$= \$577.19$  (to 2 dec. pl) ① r/w

ii. Last amount:

$A = 500(1+0.006)^1$

Second last amount:

$A = 500(1+0.006)^2$

Third last amount:

$A = 500(1+0.006)^3$

↓

First amount:

$A = 500(1+0.006)^n$

Total amount:

$A = 500(1.006) + 500(1.006)^2 + 500(1.006)^3 + \dots + 500(1.006)^n$  ① Correct

Creation

$= 500 [ 1.006 + 1.006^2 + 1.006^3 + \dots + 1.006^n ]$  of series

When is  $A > \$35000$

$35000 < 500(1.006 + 1.006^2 + 1.006^3 + \dots + 1.006^n)$

$1.006 + 1.006^2 + 1.006^3 + \dots + 1.006^n$  is a GP

with  $a = 1.006$ ,  $r = 1.006$ ,  $n = ?$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$r - 1$

$= \frac{1.006(1.006^n - 1)}{1.006 - 1}$

$1.006 - 1$

$35000 < 500 \frac{(1.006)^n - 1}{1.006 - 1}$

① Correct Series sum

$210 < 503(1.006^n - 1)$

$0.4174950298 < 1.006^n - 1$

$1.4174950298 < 1.006^n$

$\log_e 1.4174950298 < \log_e 1.006^n$

$\log_e 1.4174950298 < n \log_e 1.006$

$\log_e 1.4174950298 < n$

$\log_e 1.006$

$n > 58.32281342$

He will be able to afford the deposit after 59 months.

① Answer

b) i.  $y = \frac{b}{x^2 + a}$

When  $x = 0$ ,  $y = 4$

$4 = \frac{b}{0^2 + a}$

$4 = \frac{b}{a}$  ①

$4a = b$

$a$

When  $x = 3$ ,  $y = 1$

$1 = \frac{b}{3^2 + a}$

$1 = \frac{b}{a + 9}$  ②

$1(a + 9) = b$

$a + 9$

From ①,  $b = 4a$

From ②,  $b = a + 9$

$4a = a + 9$

$3a = 9$

$a = 3$

sub in ①

① "a"  
② "b"

$$4 = b$$

$$a$$

$$4 = b$$

$$3$$

$$b = 12$$

$$\therefore a = 3, b = 12$$

ii. C:  $(p, 0)$

sub  $x = p$  into  $y = \frac{b}{x^2 + a}$

$$y = \frac{b}{p^2 + a}$$

$$y = \frac{12}{p^2 + 3}$$

$$B: \left(p, \frac{12}{p^2 + 3}\right)$$

① r/w

iii. Due to the symmetry:

D has coordinates  $(-p, 0)$

A has coordinates  $(-p, \frac{12}{p^2 + 3})$

$$\text{length DC} = p + p = 2p$$

$$\text{Area ABCD} = 2p \times \frac{12}{p^2 + 3} = \frac{24p}{p^2 + 3}$$

① r/w

iv.  $A = \frac{24p}{p^2 + 3}$

$$u = 24p \quad v = p^2 + 3$$

$$u' = 24 \quad v' = 2p$$

$$A' = \frac{vu' - uv'}{v^2}$$

$$= \frac{(p^2 + 3) \times 24 - (24p)(2p)}{(p^2 + 3)^2}$$

$$= \frac{24p^2 + 72 - 48p^2}{(p^2 + 3)^2}$$

$$= \frac{72 - 24p^2}{(p^2 + 3)^2}$$

① derivative

$A' = 0$  at stat. pts.

$$0 = \frac{72 - 24p^2}{(p^2 + 3)^2}$$

$$24p^2 = 72$$

$$p^2 = 3$$

$$p = \pm\sqrt{3}$$

① "p" values

$$A'' = \frac{72 - 24p^2}{(p^2 + 3)^2}$$

$$u = 72 - 24p^2 \quad v = (p^2 + 3)^2$$

$$u' = -48p \quad v' = 4p(p^2 + 3)$$

① testing

$$A'' = \frac{vu' - uv'}{v^2}$$

$$= \frac{(p^2 + 3)^2 \times -48p - (72 - 24p^2)(4p)(p^2 + 3)}{(p^2 + 3)^4}$$

$$= \frac{-48p(p^2 + 3)^2 - 4p(72 - 24p^2)(p^2 + 3)}{(p^2 + 3)^4}$$

} or use of 1st derivativ test

When  $p = \sqrt{3}$

$$A'' = \frac{-48\sqrt{3}(\sqrt{3}^2 + 3)^2 - 4\sqrt{3}(72 - 24\sqrt{3}^2)(\sqrt{3}^2 + 3)}{(\sqrt{3}^2 + 3)^4}$$

$$= \frac{-48\sqrt{3} \times 36 - 4\sqrt{3} \times 0 \times 6}{6^4}$$

$$= -1728\sqrt{3}$$

$$1296$$

$$= -2.309401077$$

$\therefore$  maximum turning point.

When  $p = \sqrt{3}$

$$A = \frac{24p}{p^2 + 3}$$

$$= \frac{24\sqrt{3}}{(\sqrt{3})^2 + 3}$$

$$= \frac{24\sqrt{3}}{6}$$

$$= 4\sqrt{3} \text{ units}^2$$

① calculation of Area.