Student Number:

St. Catherine's School

Waverley

August 2011

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Complete each question in a separate booklet

- **Total Marks – 120**
- Attempt Questions 1 – 10
- All questions are of equal value

Total marks - 120
Attempt Questions 1-10
All questions are of equal value

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use the Question 1 Writing Booklet.

(a) Write down the equation of a circle with centre $(3,-4)$ and radius 5 units. **1**

(b) If $a=\frac{1}{2}$ and $b=-\frac{1}{3}$, find the value of $\frac{a+b}{a-b}$. **1**

(c) Solve $8x^2=2x$. **2**

(d) Express $\frac{1}{\sqrt{3}}+\frac{1}{2+\sqrt{3}}$ with a rational denominator. **2**

(e) Find $\sum_{n=1}^{100}(4n+3)$ **2**

(f) Find $\lim_{x\to 2}\frac{x-2}{x^3-8}$. **2**

(g) If the limiting sum of a geometric series is 48 and the first term is 4, find the common ratio, r. **2**

Question 2 (12 marks) Use the Question 2 Writing Booklet.

(a) Solve $|2x-3|>5$ and graph your solution on a number line. **3**

(b) Show that $\frac{d}{dx}xe^{\sin x}=e^{\sin x}(1+x\cos x)$. **1**

(c) Find $\int \frac{1}{(5x-7)^3}\,dx$. **1**

(d) Evaluate:

(i) $\int_0^1 e^{\pi x}\,dx$ **2**

(ii) $\int_{-1}^0 \frac{dx}{2x+3}$ **2**

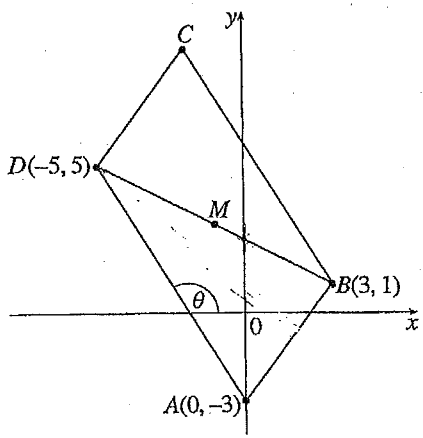
(e) Let $f(x)=\log_e(x-2)$. What is the domain of $f(x)$? **1**

(f) Find the gradient of the tangent to the curve $y=e^{\frac{x}{2}}$ at the point where $x=2$. **2**

Question 3 (12 marks) Use the Question 3 Writing Booklet.

(a) How many sides has a regular polygon if each of its internal angles is $168°$? **2**

(b)



Not to scale

$A(0,-3)$, $B(3,1)$, $C(x,y)$ and $D(-5,5)$ are the vertices of a parallelogram, as shown in the diagram above.

(i) Find θ to the nearest degree. **2**

(ii) Find the coordinates of M, the midpoint of BD. **1**

(iii) Find the coordinates of C. **1**

(iv) Show that the line AB has equation $4x-3y-9=0$. **2**

(v) Find the perpendicular distance between D and the line AB. **2**

(vi) Find the area of the parallelogram $ABCD$. **2**

Question 4 (12 marks) Use the Question 4 Writing Booklet.

(a) (i) The fourth term of a geometric series is -27 and the seventh term is 729. Find the first term and common ratio. **2**

(ii) Find the sum of the first seven terms of the series. **2**

(b) (i) Copy and complete the table below. Give your answers to 2 decimal places. **1**

x	1	2	3	4	5
$\frac{4}{x(x+1)}$					

(ii) Use all the information above and Simpson's rule, to find an approximation to $\int_1^5 \frac{4}{x(x+1)}\,dx$. **2**

(iii) Show that $\frac{4}{x} - \frac{4}{x+1} = \frac{4}{x(x+1)}$. **1**

(iv) Deduce the value of the integral $\int_1^5 \frac{4}{x(x+1)}\,dx$. **2**

(v) Hence find the percentage error given by Simpson's rule. Answer to 2 decimal places. **2**

Question 5 (12 marks) Use the Question 5 Writing Booklet.

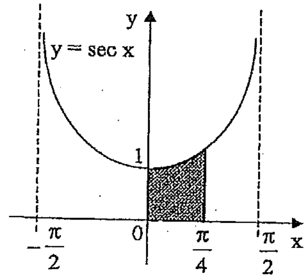
(a) P is the parabola $y^2 + 4y = x + 3$.

(i) Express P in the form $(y-k)^2 = 4a(x-h)$. **2**

(ii) Find the equation of the directrix. **1**

(iii) Find the coordinates of the focus. **1**

(b)



Not to scale

The shaded region which lies between the x axis and the curve $y = \sec x$ from $x = 0$ to $x = \frac{\pi}{4}$ is rotated about the x axis to form a solid.

Find the volume of the solid. **3**

(c) A pool is being drained and the number of litres of water, L, in the pool at time t minutes is given by the equation $L = 120(40-t)^2$.

(i) At what rate is the water draining out of the pool when $t = 6$ minutes? **2**

(ii) How long will it take for the pool to completely empty? **1**

(d) Consider the function whose derivative is given by $\frac{dy}{dx} = x^3(x-2)(x+3)$. **2**

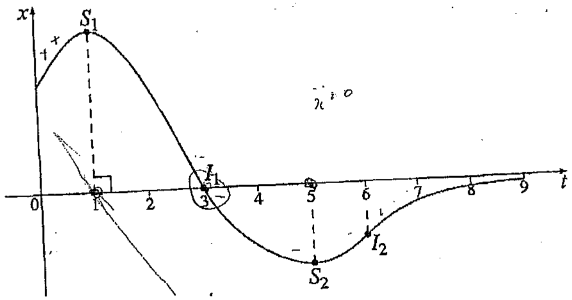
At what value of x will a maximum turning point exist on the graph of the function? Give a reason for your answer.

Question 6 (12 marks) Use the Question 6 Writing Booklet.

(a) Solve the equation $e^{4x} + 2e^{2x} = 8$. **3**

(b)

at pt. of inflexion $\ddot{x} = 0$



The graph shows the position of a particle, moving on a straight line, for the first nine seconds of the motion. S_1 and S_2 are stationary points, I_1 and I_2 are points of inflexion.

State the times, or periods of time, for which:

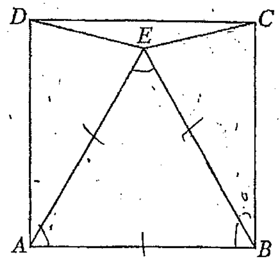
(i) the particle is stationary **1**

(ii) the velocity is negative **1**

(iii) the acceleration is positive **1**

Question 6 continues on page 9

(c)



$ABCD$ is a square and ΔABE is an equilateral triangle.

(i) Prove $\angle EBC = 30^0$. **1**

(ii) Prove triangles EBC and EAD are congruent. **2**

(iii) If the lengths of the sides of the square are d cm, prove that the area of triangle CDE is $\dfrac{d^2(2-\sqrt{3})}{4}\ cm^2$. **3**

End of Question 6

Question 7 (12 marks) Use the Question 7 Writing Booklet.

(a) A particle is moving in straight line so that at time t seconds its displacement from the origin is x metres. Initially the particle is 1 metre to the left of the origin.

The velocity of the particle is given by $v = 2\cos t - 1$.

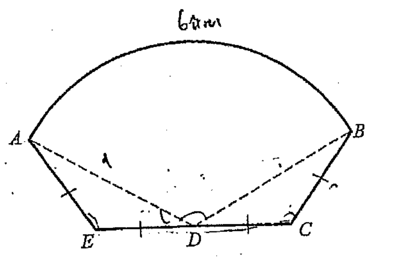
(i) Express the displacement x as a function of t. **2**

(ii) At what time is the particle first at rest? **2**

(iii) Find the position of the particle at this instant. **1**

(iv) Draw the graph of $v = 2\cos t - 1$ for $0 \le t \le 2\pi$, showing clearly all intercepts with the axes. **2**

(b) The Smiths are building an unusually shaped pool in their backyard.



In the diagram, $ABCDE$ represents the shape of the surface of the pool.

The sector ABD has centre D and $\angle ADB = \frac{2\pi}{3}$. The points C, D and E lie on a straight line. The arc AB has a length of 6π metres. $AE = ED = DC = CB$.

(i) Show that $AD = 9$ metres. **1**

(ii) Find the exact length of BC. **2**

(iii) Find the exact area of the pool's surface. **2**

Question 8 (12 marks) Use the Question 8 Writing Booklet.

(a) For what values of k does the quadratic equation $(5k-3)x^2 - 4kx + k + 1 = 0$ have no real roots? **3**

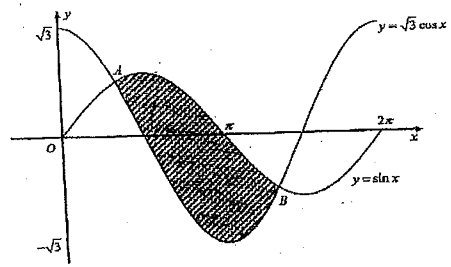
(b) In a netball competition between the Green team and the Blue team, on average the Green team has won three out of the four of their games.

(i) Find the probability that the Green team wins the next two games. **2**

(ii) In the next three games, what is the probability that the Green team wins more games than the Blue team? **2**

(c) The diagram shows the graph of $y = \sin x$ and $y = \sqrt{3}\cos x$, $0 \le x \le 2\pi$.

The graphs intersect at points A and B.



Not to scale

(i) Show that point A has co-ordinates $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$. **1**

(ii) Find the coordinates of point B. **1**

(iii) Find the shaded area enclosed by the two graphs. **3**

Question 9 (12 marks) Use the Question 9 Writing Booklet.

(a) Mark borrows \$20 000 from the *ABC* bank. This loan plus interest is to be repaid in equal monthly instalments of \$399 over five years. Interest of 7.2% p.a is compounded monthly on the balance owing at the start of each month.

Let $\$A_n$ be the amount owing after n months.

(i) Over the five year repayment period, how much interest is charged? **1**

(ii) Show that $A_1 = 19721$. **1**

(iii) Clearly show that $A_2 = 20000 \times 1.006^2 - 399(1 + 1.006)$. **2**

(iv) Deduce then that $A_n = 66500 - 46500 \times 1.006^n$. **2**

(v) After two years of repayments Mark decides on the very next day to repay the loan in one full payment. How much will this one payment be? **1**

(b) The amount Q grams of a carbon isotope in a dead tree trunk is given by $Q = Q_o e^{-kt}$ where Q_o and k are positive constants and time t is measured in years from the death of the tree.

(i) Show that Q satisfies the equation $\dfrac{dQ}{dt} = -kQ$. **1**

(ii) Show that if the half life of the carbon isotope is 5500 years (i.e. if it takes 5500 years for the carbon isotope to reduce to half its mass), then $k = \dfrac{\ln 2}{5500}$. **2**

(iii) For a particular dead tree trunk, the amount of carbon isotope is only 15% of the original amount in the living tree. How long ago did the tree die? Give your answer to the nearest 1000 years. **2**

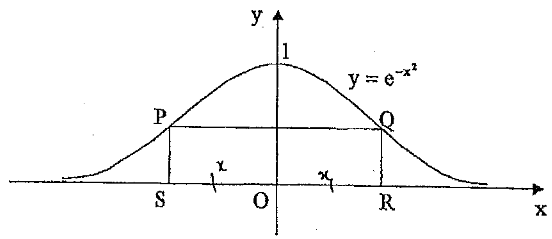
Question 10 (12 marks) Use the Question 10 Writing Booklet.

(a) The sum of the first 10 terms of the series,

$$\log_2\left(\frac{1}{x}\right) + \log_2\left(\frac{1}{x^2}\right) + \log_2\left(\frac{1}{x^3}\right) + \ldots\ldots\ldots (x > 0) \quad \text{is } 440.$$

Find the value of x. **3**

(b) The diagram shows a rectangle $PQRS$ where P and Q are on the curve and R and S are on the x axis. The point O is the origin and the lengths OS and OR are equal.



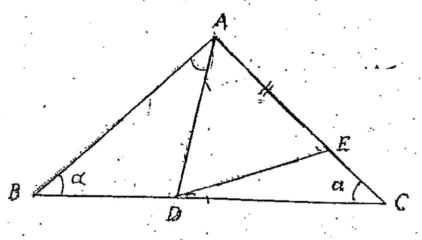
Let the length $OR = x$.

(i) Show that the area of the rectangle is represented by the expression $A = 2xe^{-x^2}$. **1**

(ii) Find the value of x for which $PQRS$ has a maximum area. **3**

Question 10 continues on page 14

Question 10 (continued)



In the isosceles triangle ABC, $\angle ABC = \angle ACB = \alpha$. The points D and E lie on BC and AC, so that $AD = AE$, as shown in the diagram. Let $\angle BAD = \beta$.

(i) Explain why $\angle ADC = \alpha + \beta$. **1**

(ii) Find $\angle DAC$ in terms of α and β. **2**

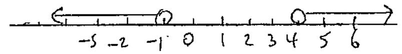
(iii) Hence, or otherwise, find $\angle EDC$ in terms of β. **2**

End of Question 10

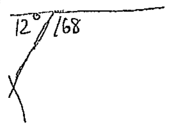
End of paper

TRIAL HSC MATHEMATICS 2011 SOLUTIONS

Qn	Solutions	Marks	Comments; Criteria
1 a)	$(x-3)^2+(y+4)^2=25$	1	
b)	$\dfrac{a+b}{a-b} = \dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}$ $= \dfrac{\frac{1}{6}}{\frac{5}{6}}$ $= \dfrac{1}{5}$	1	
c)	$8x^2-2x=0$ $2x(4x-1)=0$ $2x=0$ or $4x-1=0$ $\therefore x=0, \frac{1}{4}$	2	
d)	$\dfrac{\sqrt{3}}{3}+2-\sqrt{3} = \dfrac{\sqrt{3}+6-3\sqrt{3}}{3}$ $= \dfrac{6-2\sqrt{3}}{3}$	2	
e)	$\sum_{n=1}^{100}(4n+3) = 7+11+15+\dots+403$ $S_{100} = \frac{100}{2}(7+403)$ $= 20\,500$	2	
f)	$\lim\limits_{x\to 2} \dfrac{\cancel{x-2}}{\cancel{(x-2)}(x^2+2x+4)} = \lim\limits_{x\to 2}\dfrac{1}{x^2+2x+4}$ $= \dfrac{1}{12}$	2	
g)	$S_\infty = \dfrac{a}{1-r}$ $48 = \dfrac{4}{1-r}$ $48-48r=4$ $-48r=-44$ $\therefore r=\frac{11}{12}$	2	

Qn	Solutions	Marks	Comments; Criteria
2 a)	$\lvert 2x-3\rvert > 5$ $2x-3>5$ or $2x-3<-5$ $2x>8$ $2x<-2$ $x>4$ $x<-1$ $\therefore x<-1$ or $x>4$ 	3	
b)	$\frac{d}{dx}\, x e^{\sin x} = 1e^{\sin x} + x.\cos x\, e^{\sin x}$ $= e^{\sin x}(1+x\cos x)$, as required	1	
c)	$\displaystyle\int \frac{1}{(5x-7)^3}\,dx = \int (5x-7)^{-3}\,dx$ $= \dfrac{(5x-7)^{-2}}{-2\times 5}+c$ $= \dfrac{(5x-7)^{-2}}{-10}+c$ $\left(= \dfrac{-1}{10(5x-7)^2}+c\right)$	1	
d) (i)	$\displaystyle\int_0^1 e^{\pi x}\,dx = \left[\frac{1}{\pi}e^{\pi x}\right]_0^1$ $= \frac{1}{\pi}e^{\pi} - \frac{1}{\pi}e^0$ $= \frac{1}{\pi}(e^{\pi}-1)$	2	
(ii)	$\displaystyle\int_{-1}^{0} \frac{dx}{2x+3} = \frac{1}{2}\int_{-1}^{0}\frac{2}{2x+3}\,dx$ $= \frac{1}{2}\left[\ln(2x+3)\right]_{-1}^{0}$ $= \frac{1}{2}\left[\ln 3 - \cancel{\ln 1}\right]$ $= \frac{1}{2}\ln 3$	2	

Qn	Solutions	Marks	Comments: Criteria
	Q2 continued		
e)	$x-2>0$ $\therefore x>2$ is the domain of $f(x)$.	1	
f)	$y=e^{\frac{x}{2}}$ $y'=\frac{1}{2}e^{\frac{x}{2}}$ Gradient of tangent at $x=2$ is $\frac{1}{2}e^{1}=\frac{1}{2}e$	2	

Qn	Solutions	Marks	Comments: Criteria
3 a)	 $180-168=12^\circ$ $360\div12=30$ $\therefore$ Polygon has 30 sides.	2	
b)	(i) $m_{AD}=\frac{5+3}{-5-0}$ $=\frac{-8}{5}$ $\therefore \tan\theta=-\frac{8}{5}$ $\theta=180-(58^\circ\ldots)$ $\therefore \theta=122^\circ$ (nearest degree)	2	
	(ii) $M=\left(\frac{-5+3}{2},\ \frac{5+1}{2}\right)$ $=(-1,3)$	1	
	(iii) $C=(-2,9)$	1	
	(iv) $m_{AB}=\frac{1+3}{3}$ $\therefore m_{AB}=\frac{4}{3}$. $(y+3)=\frac{4}{3}(x)$ $3y+9=4x$ $\therefore 4x-3y-9=0$, as required.	2	
	(v) $\perp d=\frac{\lvert 4(-5)-3(5)-9\rvert}{\sqrt{16+9}}$ $=\frac{\lvert -20-15-9\rvert}{5}$ $=8\frac{4}{5}$ units	2	

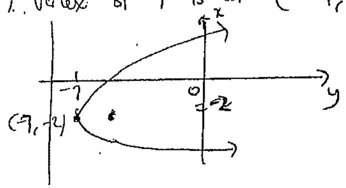
Qn	Solutions	Marks	Comments; Criteria
	Q3 continued		
	(vi) $A = bh$.		
	$d_{AB} = \sqrt{3^2 + (1+3)^2}$ $= \sqrt{25}$ $= 5$ units	2	
	Area of parallelogram $= 5 \times \frac{44}{5}$ $= 44u^2$		

Qn	Solutions	Marks	Comments; Criteria
4 a)	(i). $T_4 = -27$ $T_7 = 729$		
	$\therefore ar^3 = -27$ ① ; $ar^6 = 729$ ②		
	② ÷ ① $r^3 = -27$ $\therefore r = -3$		
	Sub $r = -3$ into ①: $-27a = -27$ $\therefore a = 1$	2	
	$\therefore a = 1,\ r = -3$.		
	(ii) $S_n = \frac{a(1-r^n)}{1-r}$		
	$\therefore S_7 = \frac{(1-(-3)^7)}{4}$ $= \frac{2188}{4}$ $= 547$	2	
b)	(i) see table below	1	
	(ii) $h = 1$ $\therefore A = \frac{1}{3}\left[(2 + 0.13) + 4(0.67 + 0.2) + 2(0.33)\right]$ $= 2.09$	2	
	(iii) $\frac{4}{x} - \frac{4}{x+1} = \frac{4(x+1) - 4x}{x(x+1)}$ $= \frac{\cancel{4x} + 4 - \cancel{4x}}{x(x+1)}$ $= \frac{4}{x(x+1)}$, as required	1	

Table for b)(i):

x	1	2	3	4	5
$\frac{4}{x(x+1)}$	2	0.67	0.33	0.2	0.13

Qn	Solutions	Marks	Comments: Criteria
	Question 4 continued.		
	(iv) $\int_1^5 \frac{4}{x(x+1)}\,dx = \int_1^5 \left(\frac{4}{x} - \frac{4}{x+1}\right)dx$		
	$= \left[4\ln x - 4\ln(x+1)\right]_1^5$		
	$= (4\ln 5 - 4\ln 6) - (\cancel{4\ln 1} - 4\ln 2)$	2	
	$= 4(\ln 5 - \ln 6 + \ln 2)$		
	(v) Exact value $= 4(\ln 5 - \ln 6 + \ln 2)$		
	$\therefore$ % Error $= \frac{2.09 - 4(\ln 5 - \ln 6 + \ln 2)}{4(\ln 5 - \ln 6 + \ln 2)} \times 100$		
	$= 2.29\%$ (2dp)	2	

Qn	Solutions	Marks	Comments: Criteria
5	(a) $y^2 + 4y = x + 3$		
	(i) $y^2 + 4y + 4 = x + 7$		
	$(y+2)^2 = x + 7$	2	
	(ii) $\therefore$ Vertex of P is at $(-7, -2)$		
			
	$4a = 1$ $\therefore a = \frac{1}{4}$		
	$\therefore$ Equation of directrix is $x = -\frac{29}{4}$	1	
	(iii) Focus $= (-6\frac{3}{4}, -2)$	1	
	b) $V = \pi\int_a^b y^2\,dx$		
	$= \pi\int_0^{\frac{\pi}{4}} \sec^2 x\,dx$		
	$= \pi\left[\tan x\right]_0^{\frac{\pi}{4}}$	3	
	$= \pi\left[\tan\frac{\pi}{4} - \tan 0\right]$		
	$= \pi\cdot(1 - 0)$		
	$\therefore V = \pi\ u^3$		

Qn	Solutions	Marks	Comments: Criteria

Question 5 continued.

c) $L = 120(40-t)^2$

(i) $\frac{dL}{dt} = 240(40-t).-1$

$= -240(40-t)$.

$\therefore$ when $t=6$, $\frac{dL}{dt} = -240(40-6)$

$= -8160 \text{ L/min}$ — 2

$\therefore$ Water is draining out of the pool at 8160 L/min.

(ii) $0 = 120(40-t)^2$

$\therefore (40-t)^2 = 0$

$\therefore t = 40$

$\therefore$ Pool will completely empty after 40 minutes. — 1

d) $\frac{dy}{dx} = x^3(x-2)(x+3)$

Let $x^3(x-2)(x+3) = 0$

$\therefore x = 0, 2, -3$.

At $x=0$

x	-1	0	1
y'	+ve	—	-ve

$\therefore$ At $x=0$, a maximum turning pt exists. — 2

At $x=2$

x	1	2	3
y'	-ve	—	+ve

×

At $x=-3$

x	-4	-3	-2
y'	-ve	—	+ve

×

Qn	Solutions	Marks	Comments: Criteria

6 a) let $u = e^{2x}$

$\therefore (e^{2x})^2 + 2e^{2x} - 8 = 0$

$u^2 + 2u - 8 = 0$.

$(u+4)(u-2) = 0$

$\therefore u = -4$ or 2

$\therefore e^{2x} = -4$ or $e^{2x} = 2$

No solution

$2x \ln e = \ln 2$

$\therefore x = \frac{\ln 2}{2}$ — 3

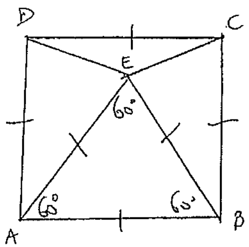
$\therefore$ Solution is $x = \frac{\ln 2}{2}$.

b) (i) Particle is stationary at $t = 1, 5$ — 1

(ii) Velocity is negative when $1 < t < 5$ — 1

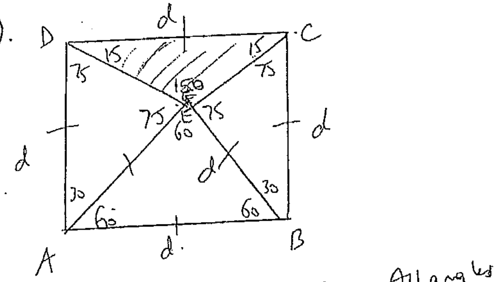
(iii) Acceleration is positive when $3 < t < 6$ — 1

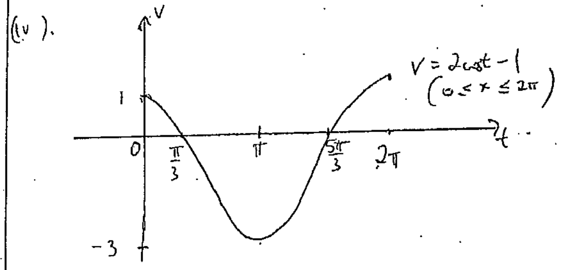
c) (i)



Since ΔABE is equilateral and ABCD is a square, then $\angle EBC = 90 - 60$

$= 30°$, as required — 1

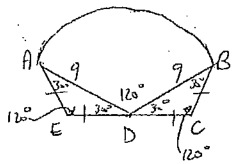
Qn	Solutions	Marks	Comments: Criteria
	Q6 continued		
	(ii) In Δ's EBC, EAD; $\angle EBC = \angle EAD$ $= 30^\circ$ (ΔEAB equilateral and ABCD square) $EB = EA$ (equal sides of equilateral ΔABE) $BC = AD$ (equal sides of square) $\therefore \Delta EBC \equiv \Delta EAD$ (SAS)	2	
	(iii).  ΔAED and ΔBEC isosceles. All angles are marked on diagram. In ΔBCE; $EC^2 = d^2 + d^2 - 2d^2 \cos 30$ $= 2d^2 - \cancel{2}d^2 \cdot \frac{\sqrt{3}}{\cancel{2}}$ $= 2d^2 - \sqrt{3}d^2$. ΔCDE is isosceles, $EC = ED$. $\therefore$ Area of $\Delta CDE = \frac{1}{2} \cdot EC \cdot ED \sin 150$ $= \frac{1}{2} EC^2 \sin 30$ $= \frac{1}{2}(2d^2 - \sqrt{3}d^2) \cdot \frac{1}{2}$ $= \frac{d^2(2-\sqrt{3})}{4}$ cm^2, as required.	3	

Qn	Solutions	Marks	Comments: Criteria
7	$v = 2\cos t - 1$		
	(i) $x = \int (2\cos t - 1)\,dt$ $x = 2\sin t - t + c$ At $t = 0$, $x = -1$ $\therefore -1 = \cancel{2\sin 0 - 0} + c$ $\therefore c = -1$ $\therefore x = 2\sin t - t - 1$	2	
	(ii) When $v = 0$, $2\cos t - 1 = 0$ $2\cos t = 1$ $\cos t = \frac{1}{2}$ $\therefore t = \frac{\pi}{3}$ s. $\therefore$ Particle is first at rest when $t = \frac{\pi}{3}$ s.	2	
	(iii). When $t = \frac{\pi}{3}$, $x = 2\sin\frac{\pi}{3} - \frac{\pi}{3} - 1$ $= 2 \cdot \frac{\sqrt{3}}{2} - \frac{\pi}{3} - 1$ $= \sqrt{3} - \frac{\pi}{3} - 1$ m.	1	
	(iv). 	2	

Qn	Solutions	Marks	Comments: Criteria

Question 7 continued

(i) $l = r\theta$



$6\pi = r \cdot \frac{2\pi}{3}$

$r = \frac{18\pi}{2\pi}$

$\therefore r = 9$ m (as required) — **1**

(ii) $\frac{BC}{\sin 30} = \frac{9}{\sin 120}$

$BC = \frac{9 \sin 30}{\sin 60}$

$= \frac{9 \cdot \frac{1}{2}}{\frac{\sqrt{3}}{2}}$ — **2**

$= \frac{9}{2} \times \frac{2}{\sqrt{3}}$

$= \frac{9\sqrt{3}}{3}$

$\therefore BC = 3\sqrt{3}$ units

Note: $\sin 120 = \sin 60$

(iii). Area of pool $= \frac{1}{2} r^2\theta + 2\Delta$'s.

$= \frac{1}{2} \cdot 81 \frac{2\pi}{3} + 2 \times \frac{1}{2} (3\sqrt{3})^2 \sin 60$ — **2**

$= 27\pi + 27 \cdot \frac{\sqrt{3}}{2}$ cm^2

$= 27\left(\pi + \frac{\sqrt{3}}{2}\right)$ cm^2

Qn	Solutions	Marks	Comments: Criteria

8 a) $b^2 - 4ac < 0$

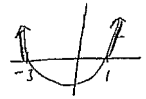
$(-4k)^2 - 4(5k-3)(k+1) < 0$

$16k^2 - 4(5k^2+2k-3) < 0$

$16k^2 - 20k^2 - 8k + 12 < 0$

$-4k^2 - 8k + 12 < 0$

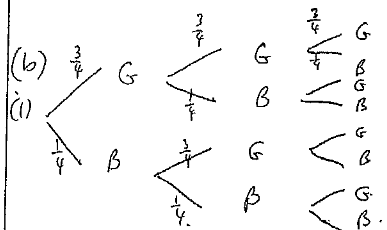
$k^2 + 2k - 3 > 0$ — **3**



$(k-1)(k+3) > 0$

$\therefore k < -3$ or $k > 1$

(b) (i)



2

(i) $P(GG) = \frac{3}{4} \times \frac{3}{4}$

$= \frac{9}{16}$

(ii). $P(GGG$ or GGB or GBG or $BGG)$ — **2**

$= \left(\frac{3}{4}\right)^3 + 3 \times \left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)$

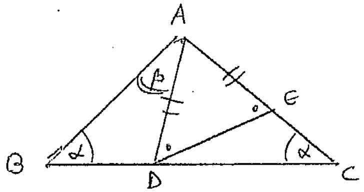
$= \frac{27}{32}$

Qn	Solutions	Marks	Comments; Criteria
	Question 8 continued.		
	c).		
	(i) $y = \sin x$ ① $y = \sqrt{3}\cos x$ ②		
	① ÷ ② $1 = \frac{\sin x}{\sqrt{3}\cos x}$		
	$\therefore \tan x = \sqrt{3}$ $\therefore x = \frac{\pi}{3}, \pi + \frac{\pi}{3}$. $= \frac{\pi}{3}, \frac{4\pi}{3}$	1	
	When $x = \frac{\pi}{3}$, $y = \sin\frac{\pi}{3}$ $= \frac{\sqrt{3}}{2}$		
	$\therefore$ Point $A = \left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$, as required.		
	(ii). When $x = \frac{4\pi}{3}$, $y = \sin\frac{4\pi}{3}$. $y = -\frac{\sqrt{3}}{2}$	1	
	$\therefore$ Point $B = \left(\frac{4\pi}{3}, -\frac{\sqrt{3}}{2}\right)$		
	(iii). Area $= \int_{\pi/3}^{4\pi/3} (\sin x - \sqrt{3}\cos x)\, dx$		
	$= \left[-\cos x - \sqrt{3}\sin x\right]_{\frac{\pi}{3}}^{\frac{4\pi}{3}}$		
	$= \left(-\cos\frac{4\pi}{3} - \sqrt{3}\sin\frac{4\pi}{3}\right) - \left(-\cos\frac{\pi}{3} - \sqrt{3}\sin\frac{\pi}{3}\right)$		
	$= \left(\frac{1}{2} - \sqrt{3}\cdot\left(\frac{-\sqrt{3}}{2}\right)\right) - \left(-\frac{1}{2} - \sqrt{3}\cdot\frac{\sqrt{3}}{2}\right)$		
	$= \frac{1}{2} + \frac{3}{2} + \frac{1}{2} + \frac{3}{2}$		
	$= 4\ u^2$		

Qn	Solutions	Marks	Comments; Criteria
9	a) (i) $399 \times 60 = \$23940$. $\therefore$ Interest $= \$23940 - \$20000$ $= \$3940$.	1	
	(ii). $A_1 = 20000(1+0.006)^1 - 399$ 7.2% pa $= \frac{7.2}{12}$ $= 0.6\%$ per month $= 0.006$ $= \$19721$, as required.	1	
	(iii) $A_2 = A_1(1.006) - 399$ $= (20000(1.006) - 399)1.006 - 399$ $= 20000(1.006)^2 - 399(1 + 1.006)$, as required.	2	
	(iv) $A_n = 20000(1.006)^n - 399(1 + 1.006 + \ldots + 1.006^{n-1})$ $= 20000(1.006)^n - 399\left[\frac{1(1.006^n - 1)}{1.006 - 1}\right]$ $= 20000(1.006)^n - 399\frac{(1.006^n - 1)}{0.006}$	2	
	$= 20000(1.006)^n - 66500(1.006^n - 1)$ $= 20000(1.006)^n - 66500(1.006)^n + 66500$ $= 66500 - 46500(1.006)^n$, as required		

Qn	Q9 continued. Solutions	Marks	Comments: Criteria
	(v) 2 years = 24 months $\therefore A_{24} = 66500 - 46500 \times 1.006^{24}$ $= \$12820.99$ $\therefore$ Mark will pay \$12820.99 to repay the loan.	1	
(b)	(i) $\varphi = \varphi_0 e^{-kt}$ $\frac{d\varphi}{dt} = -k\varphi_0 e^{-kt}$ $= -k\varphi$, as required.	1	
	(ii) $1 = 2e^{-k.5500}$. $\frac{1}{2} = e^{-5500k}$ $\therefore \ln\frac{1}{2} = -5500k$ $\therefore k = \frac{\ln\frac{1}{2}}{-5500}$ $= \frac{\ln 2^{-1}}{-5500}$ $= \frac{-\ln 2}{-5500}$ $\therefore k = \frac{\ln 2}{5500}$, as required.	2	
	(iii). $0.15 = 1.e^{-\frac{\ln 2}{5500}t}$ $\therefore \ln 0.15 = -\frac{\ln 2}{5500}t$ $\therefore t = \ln 0.15 \times \frac{5500}{-\ln 2}$ $\therefore t = 15\,000$ years (nearest 1000yrs)	2	

Qn	Solutions	Marks	Comments: Criteria
10	(a) $\log_2 x^{-1} + \log_2 x^{-2} + \log_2 x^{-3} + \dots + \log_2 x^{-10}$ $= -\log_2 x - 2\log_2 x - 3\log_2 x - \dots - 10\log_2 x$ Series is an AP, with $a = -\log_2 x$, $d = -\log_2 x$, $n = 10$. $S_n = \frac{n}{2}(a+l)$ $440 = \frac{10}{2}[-\log_2 x - 10\log_2 x]$ $440 = 5(-11\log_2 x)$ $88 = -11\log_2 x$ $-8 = \log_2 x$ $\therefore x = 2^{-8}$ $\therefore x = \frac{1}{256}$	3	
	(b)(i) $OR = OS = x$ $\therefore SR = 2x$ units. Area of rectangle $= lb$ $= 2x \times e^{-x^2}$ $A = 2xe^{-x^2}$, as required.	1	
	(ii) $\frac{dA}{dx} = 2x(-2x).e^{-x^2} + 2(e^{-x^2})$ $= 2e^{-x^2}(-2x^2+1)$ Let $\frac{dA}{dx} = 0$. i.e. $2e^{-x^2}(-2x^2+1) = 0$. $2e^{-x^2} = 0$ or $1 - 2x^2 = 0$ No solution. $2x^2 = 1$ $x^2 = \frac{1}{2}$ $\therefore x = \pm\frac{1}{\sqrt{2}}$. But $x > 0$. $\therefore x = \frac{1}{\sqrt{2}}$		

Qn	Solutions	Marks	Comments; Criteria
	Test $x = \frac{1}{\sqrt{2}}$		
	$\begin{array}{c\|c\|c\|c} x & 0 & \frac{1}{\sqrt{2}} & 1 \\ \hline \frac{dA}{dx} & >0 & - & <0 \\ & / & - & \backslash \end{array}$ $\therefore$ maximum	3	
	$\therefore$ when $x = \frac{1}{\sqrt{2}}$, PQRS has maximum area.		
	c) 		
	(i) $\angle ADC$ is exterior angle of ΔABD = sum of opposite 2 interior angles α and β. $\therefore \angle ADC = \angle ABD + \angle BAD$ $= \alpha + \beta$.	1	
	(ii) $\angle DAC = 180 - (\alpha + \beta + \alpha)$ (angle sum of ΔADC) $= 180 - (2\alpha + \beta)$ $= 180 - 2\alpha - \beta$	2	
	(iii) $\angle ADE = \angle AED$ (angles opposite equal sides). and $\angle ADE + \angle AED + \angle DAC = 180°$ (angle sum ΔADE) $\therefore 2 \times \angle ADE = 180 - \angle DAC$ (since $\angle ADE = \angle AED$) $\angle ADE = \frac{180 - (180 - 2\alpha - \beta)}{2}$ $= \frac{2\alpha + \beta}{2}$ $= \alpha + \frac{\beta}{2}$	2	
	Now $\angle ADC = \angle ADE + \angle EDC$ $\therefore \alpha + \beta = \alpha + \frac{\beta}{2} + \angle EDC$ $\therefore \angle EDC = \alpha + \beta - \alpha - \frac{\beta}{2}$ $= \frac{\beta}{2}$		

— End of Paper —