TOTAL EDUCATION
Examination Papers

2010
PRELIMINARY
EXAMINATION PAPER

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- All necessary working should be shown in every question

Total Marks – 84

- Attempt Questions 1–7
- All questions are of equal value

Total Marks – 84
Attempt Questions 1 – 7
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Teams of 5 players are chosen from 9 basketball players which include A and B.

(i) How many different teams can be formed with no restrictions? **1**

(ii) If A and B do not play in the same team, how many teams can be formed? **2**

(b) The polynomial $P(x) = x^3 - 4x^2 + kx + 2$ is divisible by $x + 2$. Find the value of the constant k. **2**

(c) Fully factorise $x^6 - y^6$. **2**

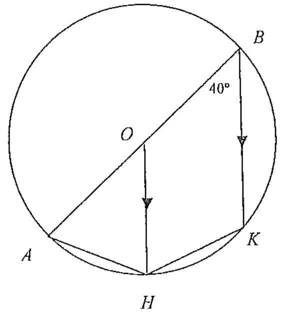
(d) (i) Factorise $(x+h)^3 - x^3$. **1**

(ii) Hence find $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ where $f(x) = x^3$. **1**

(e) Solve the inequality $\dfrac{x}{x-1} \geq 2$. **3**

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a)



In the diagram, AB is a daimeter of the circle with centre O. H and K are points on the circle with $OH//BK$ and $\angle OBK = 40°$.

Copy or trace this diagram into your writing booklet.

(i) Find $\angle OHA$, giving reasons. **1**

(ii) Find $\angle OHK$, giving reasons. **2**

(b) Solve $\sin 2\theta = \cos\theta$ for $0° \leq \theta \leq 360°$. **3**

(c) A and B are the points $(-1,2)$ and $(5,8)$ respectively.

(i) Find the coordinates of the point P which divides AB **externally** in the ratio $2:3$. **2**

(ii) $Q(1,4)$ is a point on AB. Find the ratio in which Q divides AB. **2**

(d) Find the acute angle between the lines **2**

$$y = 3x - 6 \text{ and } y = -2x + 1.$$

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) Find the exact value of $\cos 75°$. **2**

(b) If α is acute and β is obtuse with $\cos\alpha = \frac{4}{5}$ and $\sin\beta = \frac{12}{13}$, find the exact value of $\sin(\alpha + \beta)$. **3**

(c) Eight people, including a couple, sit at a round table. Find the number of ways this can be done,

(i) if there are no restrictions; **1**

(ii) if the couple must not sit together. **2**

(d) Consider the function $f(x) = 4\cos x - 3\sin x$.

(i) Express $f(x)$ in the form $R\cos(x + \alpha)$ where α is an acute angle. **3**

(ii) Hence find the maximum value of $4\cos x - 3\sin x + 2$. **1**

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) When $P(x) = 2x^3 + ax^2 + bx - 3$ is divided by $(x + 1)$ and $(x - 2)$, the remainders are 2 and 17 respectively. **3**

Find the values of the constants a and b.

(b) (i) Write down the domain of the function $y = \sqrt{4 - x^2}$. **1**

(ii) Draw the graph of $y = \sqrt{4 - x^2}$. **3**

Hence shade, in the same diagram, the region satisfying

$$y \leq \sqrt{4 - x^2}.$$

(c) (i) Let $t = \tan\frac{\theta}{2}$. **2**

Using double-angle formulae, prove that

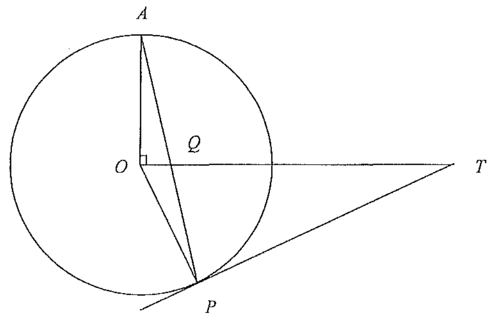
$$\sin\theta = \frac{2t}{1 + t^2} \text{ and } \cos\theta = \frac{1 - t^2}{1 + t^2}.$$

(ii) Use the substitution $t = \tan\frac{\theta}{2}$ to solve the equation **3**

$$\sin\theta + 2\cos\theta + 2 = 0 \quad \text{for } 0° \leq \theta \leq 360°.$$

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a)



In the diagram, O is the centre of the circle. TP is a tangent touching the circle at P. A is a point on the circle such that $\angle AOT$ is a right angle. AP cuts OT at Q.
Copy or trace this diagram into your writing booklet.

Show that $TP = TQ$. **3**

(b) How many triangles can be formed with 12 straight lines in a plane,

(i) if no 2 lines are parallel and no 3 lines are concurrent? **1**

(ii) if 2 lines are parallel and 4 other lines are concurrent? **3**

(c) α, β and γ are the roots of the equation $2x^3 + 4x^2 + 3x - 5 = 0$.

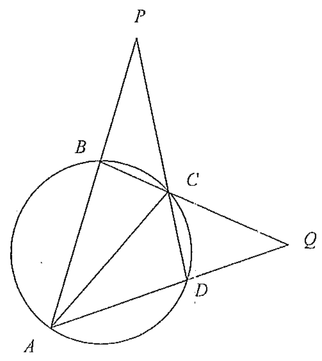
(i) Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. **2**

(ii) Find the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$. **1**

(iii) Hence find the cubic equation with roots $\frac{1}{\alpha}, \frac{1}{\beta}$ and $\frac{1}{\gamma}$. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a)



In the diagram, the chords AB and DC are produced to meet at P, and the chords AD and BC are produced to meet at Q. $\angle APD = \angle AQB$.

Copy or trace this diagram into your writing booklet.

(i) Explain why $BPQD$ is a cyclic quadrilateral. **1**

(ii) Show that AC is a diameter. **3**

(b) (i) Draw the graph of $y = |x - 1|$. **1**

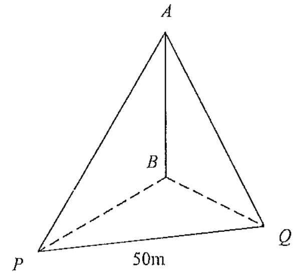
(ii) Hence, or otherwise, solve the inequality **2**

$$|x - 1| > x.$$

Question 6 continues on page 8

Question 6 (continued)

(c)



In the diagram, P and Q are 50m apart in a horizontal plane. AB is a vertical tower standing on the plane. The angles of elevation of the top of the tower A from P and Q are 35° and 28° respectively. The bearings of the tower from P and Q are 058° and 337° respectively.

Let the height of the tower be h metres.

(i) Show that $BP = h \tan 55°$. **1**

(ii) Use a diagram to show that $\angle PBQ = 81°$. **1**

(iii) Find the value of h. **3**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Show that $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$. **1**

(ii) Hence solve $x^2 + \frac{1}{x^2} - x - \frac{1}{x} - 4 = 0$. **3**

(b) P and Q are two variable points on the parabola $x^2 = 4ay$ with parameters p and q respectively.

(i) Show that the eqaution of the tangent to the parabola at the point P is **2**

$$y = px - ap^2.$$

(ii) Write down the eqaution of the tangent to the parabola at the point Q. **1**

(iii) The tangents to the parabola at P and Q meet at T. Show that the coordinates of T are **2**

$$(a(p+q), apq).$$

(iv) If PQ always passes through the point $(-a, 0)$, find the equation of the locus of T. **3**

END OF PAPER

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Examination Papers

2010 Preliminary Yearly Examination – Mathematics Extension 1
Suggested Solutions

Question 1 (12 marks)
(a) (i) (1 mark)
Outcomes Assessed: PE3

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$C_5^9 = 126$ 1

(a) (ii) (2 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Correctly works out the number of teams which include A and B. • Gives the correct answer.	1 1

Sample answer

A and B play together: $C_3^7 = 35$ 1

Answer=126−35=91 1

(b) (2 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Correctly applies the Factor Theorem. • Gives the correct answer.	1 1

Sample Answer

$P(-2) = 0$
$-8 - 16 - 2k + 2 = 0$ 1
$k = -11$ 1

(c) (2 marks)
Outcomes Assessed: P3

Criteria	Mark
• Applies the formula for the difference of 2 squares or 2 cubes. • Gives the correct answer.	1 1

Sample Answer

$x^6 - y^6$
$= (x^3 + y^3)(x^3 - y^3)$ 1
$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$ 1

(d) (i) (1 mark)
Outcomes Assessed: P8

Criteria	Mark
• Gives the correct answer.	1

Sample answer

$(x + h)^3 - x^3$
$= (x + h - x)[(x + h)^2 + (x + h)x + x^2]$
$= h(3x^2 + 3hx + h^2)$ 1

(d) (ii) (1 mark)
Outcomes Assessed: P8

Criteria	Mark
Gives the correct answer.	1

Sample Answer

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{h(3x^2 + 3hx + h^2)}{h}$$

$$= \lim_{h \to 0} (3x^2 + 3hx + h^2)$$

$$= 3x^2$$

1

(e) (3 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Gives a correct method to get rid of the denominator. • Gives the correct solution for the quadratic inequality. • Gives the correct answer by excluding $x = 1$.	1 1 1

Sample Answer

Multiplying both sides by $(x - 1)^2$,
$x(x - 1) \geq 2(x - 1)^2$ 1
$(x - 1)[x - 2(x - 1)] \geq 0$
$(x - 1)(-x + 2) \geq 0$
$1 \leq x \leq 2$ 1
$x \neq 1, \therefore 1 < x \leq 2$ 1

Question 2 (12 marks)
(a) (i) (1 mark)
Outcomes Assessed: PE3

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$\angle AOH = 40°$ (corresponding angles, $OH//BK$)
$OA = OH$ (radii)
$\angle OAH = \angle OHA$ (base angles of isosceles triangle)
$\angle OAH + \angle OHA + 40° = 180°$ (angle sum of triangle)
$\angle OHA = 70°$ 1

(a) (ii) (2 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Correctly works out $\angle AHK$ with the property stated.	1
• Gives the correct answer.	1

Sample answer

$\angle AHK = 180° - 40°$ (opposite angles of cyclic quadrilateral) 1
$= 140°$
$\angle OHA = 70°$
$\angle OHK + 70° = 140°$
$\angle OHK = 70°$ 1

(b) (3 marks)
Outcomes Assessed: P4

Criteria	Mark
• Correctly applies the double-angle formula.	1
• Gives the correct answers for $\cos\theta = 0$.	1
• Gives the correct answers for $\sin\theta = 0.5$.	1

Sample Answer

$2\sin\theta\cos\theta = \cos\theta$ 1
$\cos\theta(2\sin\theta - 1) = 0$
$\cos\theta = 0 \text{ or } \sin\theta = \frac{1}{2}$
$\theta = 90°, 270° \text{ or } 30°, 150°$ 1+1

(c) (i) (2 marks)
Outcomes Assessed: P4

Criteria	Mark
• Applies the division formula.	1
• Gives the correct answer.	1

Sample Answer

$P = \left(\frac{-2\times 5 + 3\times -1}{-2+3}, \frac{-2\times 8 + 3\times 2}{-2+3}\right)$ 1

$P = (-13, -10)$ 1

(c) (ii) (2 marks)
Outcomes Assessed: P4

Criteria	Mark
• Applies the correct method.	1
• Gives the correct answer.	1

Sample answer

Let the ratio be $1:r$.
$Q = \left(\frac{1\times 5 + r\times -1}{1+r}, \frac{1\times 8 + r\times 2}{1+r}\right) = (1,4)$

$\frac{5-r}{1+r} = 1$ 1
$5 - r = 1 + r$
$r = 2$ 1
The ratio is 1:2.

(d) (2 marks)
Outcomes Assessed: P4

Criteria	Mark
• Applies the $\tan(A - B)$ formula.	1
• Gives the correct answer.	1

Sample Answer

Let the acute angle be θ.
$\tan\theta = \left|\frac{3 - -2}{1 + 3(-2)}\right|$ 1
$\tan\theta = 1$
$\theta = 45°$ 1

Question 3 (12 marks)
(a) (2 marks)
Outcomes Assessed: P4

Criteria	Mark
• Correctly expands $\cos(A+B)$.	1
• Gives the correct answer.	1

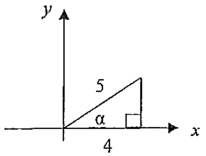
Sample Answer

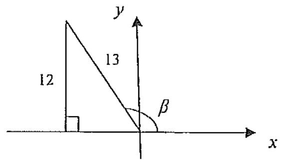
$\cos 75°$
$= \cos(30° + 45°)$
$= \cos 30° \cos 45° - \sin 30° \sin 45°$ 1
$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$
$= \frac{\sqrt{3}-1}{2\sqrt{2}}$
$= \frac{\sqrt{2}(\sqrt{3}-1)}{4}$ 1

(b) (3 marks)
Outcomes Assessed: P4

Criteria	Mark
• Correctly works out either $\sin\alpha$ or $\cos\beta$.	1
• Correctly expands $\sin(\alpha+\beta)$ and substitutes values of trigonometric ratios.	1
• Gives the correct answer.	1

Sample answer





$\sin\alpha = \frac{3}{5}$ $\cos\beta = \frac{-5}{13}$ 1

$\sin(\alpha+\beta)$
$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$= \frac{3}{5} \times \frac{-5}{13} + \frac{4}{5} \times \frac{12}{13}$ 1
$= \frac{33}{65}$ 1

(c) (i) (1 mark)
Outcomes Assessed: PE3

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

7! or 5040 1

(c) (ii) (2 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Gives the number of ways of sitting together.	1
• Gives the correct answer.	1

Sample Answer

Number of ways of seating if the couple sit together= $6! \times 2$ or 1440 1
Answer= $7! - 6! \times 2 = 3600$ 1

(d) (i) (3 marks)
Outcomes Assessed: P4

Criteria	Mark
• Applies the correct method.	1
• Gives the correct answer for R and α.	1+1

Sample answer

$4\cos x - 3\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$
$R\cos\alpha = 4$ and $R\sin\alpha = 3$ 1
Solving, $R = 5, \alpha = 36°52'$ 1+1

$f(x) = 5\cos(x + 36°52')$

(d) (ii) (1 mark)
Outcomes Assessed: P4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$4\cos x - 3\sin x + 2 = 5\cos(x + 36°52') + 2$

The maximum value of $5\cos(x + 36°52')$ is 5.

The maximum value of $4\cos x - 3\sin x + 2$ is 7. 1

Question 4 (12 marks)
(a) (3 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Correctly applies the Remainder Theorem once.	1
• Solves the simultaneous equation.	1
• Gives the correct answer.	1

Sample Answer

$P(-1) = -2 + a - b - 3 = 2$ 1
$P(2) = 16 + 4a + 2b - 3 = 17$

$a - b = 7$ 1
$4a + 2b = 4$
Solving, $a = 3, b = -4$ 1

(b) (i) (1 mark)
Outcomes Assessed: P5

Criteria	Mark
• Gives the correct answer.	1

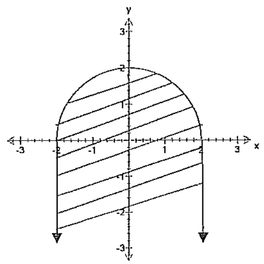
Sample answer

$4 - x^2 \geq 0$
$-2 \leq x \leq 2$ 1

(b) (ii) (3 marks)
Outcomes Assessed: P5

Criteria	Mark
• Gives the correct graph.	1
• Shades the interior of the semi-circle.	1
• Shades the correct region below the x-axis.	1

Sample Answer



1
1
1

(c) (i) (2 marks)
Outcomes Assessed: P4

Criteria	Mark
• Gives the proof for $\sin\theta$.	1
• Gives the proof for $\cos\theta$.	1

Sample Answer

$$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2 \times \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} \times \cos^2\theta = 2\tan\frac{\theta}{2} \times \frac{1}{\sec^2\frac{\theta}{2}} = \frac{2t}{1+t^2}$$ 1

$$\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = \frac{2}{\sec^2\frac{\theta}{2}} - 1 = \frac{2}{1+t^2} - 1 = \frac{1-t^2}{1+t^2}$$ 1

(c) (ii) (3 marks)
Outcomes Assessed: P4

Criteria	Mark
• Substitutes the results in (i) and simplifies the equation.	1
• Gives the first correct answer.	1
• Gives the other answer.	1

Sample answer

$$\frac{2t}{1+t^2} + 2\left(\frac{1-t^2}{1+t^2}\right) + 2 = 0$$

$2t + 2 - 2t^2 + 2 + 2t^2 = 0$ 1
$2t + 4 = 0$
$t = -2$ 1
$\theta = 233°8'$

Substitutes $\theta = 180°$,
LHS= $\sin 180° + 2\cos 180° + 2 = 0$=RHS 1
$\theta = 180°$ is a solution.

Question 5 (12 marks)
(a) (3 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Correctly finds $\angle OPT$ with reason given.	1
• Correctly expresses $\angle TQP$ in terms of $\angle OAQ$.	1
• Completes the proof.	1

Sample Answer

$\angle OPT = 90°$ (Radius perpendicular to tangent.) 1
$\angle TPQ = 90° - \angle OPQ$

$\angle TQP = \angle AQO$ (vertically opposite angles)
$= 180° - 90° - \angle OAQ$ (angle sum of triangle)
$= 90° - \angle OAQ$ 1

$\angle OAQ = \angle OPQ$ (base angles of isosceles triangle)

$\therefore \angle TPQ = \angle TQP$
$TP = TQ$ (sides opposite equal angles in a triangle are equal.) 1

(b) (i) (1 mark)
Outcomes Assessed: PE3

Criteria	Mark
• Gives the correct answer.	1

Sample answer

$C_3^{12} = 220$ 1

(b) (ii) (3 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Gives the number of triangles formed with 2 chosen lines and a third line from the remaining 10 lines..	1
• Gives the number of triangles formed with 4 lines.	1
• Gives the correct answer.	1

Sample Answer
2 lines are parallel:
If 2 of the lines are NOT parallel, then $(12 - 2) = 10$ triangles can be formed with these 2 lines and a line from the remaining 10 lines. 1

4 lines are concurrent:
If 4 of the lines are not concurrent, then they can form $C_3^4 (= 4)$ triangles. 1

Answer= $220 - 10 - 4 = 206$ 1

(c) (i) (2 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Gives the value of either $\beta\gamma + \gamma\alpha + \alpha\beta$ or $\alpha\beta\gamma$.	1
• Gives the correct answer.	1

Sample Answer

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$$
$$= \frac{\beta\gamma + \gamma\alpha + \alpha\beta}{\alpha\beta\gamma}$$

$$= \frac{3}{2} \div \frac{5}{2}$$ 1

$$= \frac{3}{5}$$ 1

(c) (ii) (1 mark)
Outcomes Assessed: PE3

Criteria	Mark
• Gives the correct answer.	1

Sample answer

$$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$$
$$= \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma}$$
$$= -2 \div \frac{5}{2} = -\frac{4}{5}$$ 1

(c) (iii) (2 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Gives the correct value of $\frac{1}{\alpha} \times \frac{1}{\beta} \times \frac{1}{\gamma}$.	1
• Gives the correct answer.	1

Sample answer

$$\frac{1}{\alpha} \times \frac{1}{\beta} \times \frac{1}{\gamma} = 1 \div \frac{5}{2} = \frac{2}{5}$$ 1

The required equation is
$$x^3 - \frac{3}{5}x^2 - \frac{4}{5}x - \frac{2}{5} = 0$$
$$5x^3 - 3x^2 - 4x - 2 = 0$$ 1

Question 6 (12 marks)
(a) (i) (1 mark)
Outcomes Assessed: PE3

Criteria	Mark
• Gives the correct reason.	1

Sample Answer

BD subtends equal angles at P and Q. 1

(a) (ii) (3 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Show that $\angle CDQ = \angle PBC$ with reason given.	1
• Show that $\angle CDQ = \angle ABC$ with reason given.	1
• Completes the proof.	1

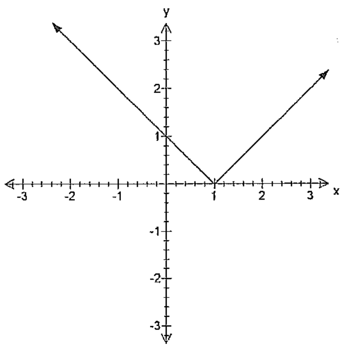
Sample answer

$BPQD$ is a cyclic quadrilateral.		
$\angle CDQ = \angle PBC$	(angles in the same segment.)	1
$\angle CDQ = \angle ABC$	(exterior angle of a cyclic quadrilateral)	1
$\angle PBC = \angle ABC$		
$\angle PBC + \angle ABC = 180°$	(adjacent angles on a straight line)	
$\therefore \angle ABC = 90°$		
AC is a diameter		1

(b) (i) (1 mark)
Outcomes Assessed: P5

Criteria	Mark
• Draws the correct graph.	1

Sample Answer

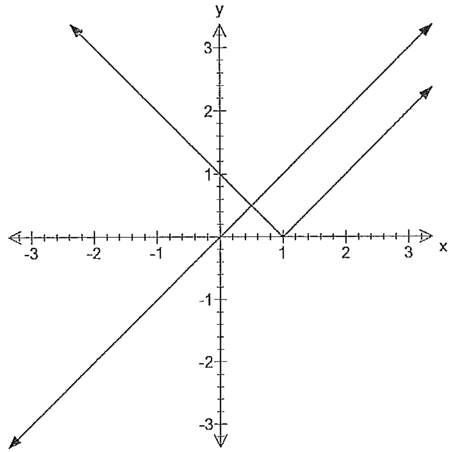


1

(b) (ii) (2 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Draws the graph of $y = x$ in the same diagram.	1
• Gives the correct answer.	1

Sample Answer



1

Solution: $x < \frac{1}{2}$ 1

(c) (i) (1 mark)
Outcomes Assessed: P4

Criteria	Mark
• Completes the proof.	1

Sample answer

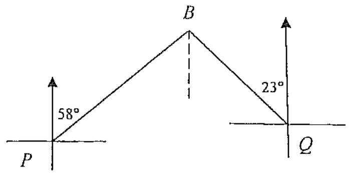
$\angle APB = 35°$
$\angle PAB = 55°$
$\tan 55° = \frac{BP}{h}$
$BP = h \tan 55°$ 1

(c) (ii) (1 mark)
Outcomes Assessed: P4

Criteria	Mark
• Draws a diagram to complete the proof.	1

Sample answer

$360° - 337° = 23°$



$\angle PBQ = 58° + 23° = 81°$ 1

(c) (iii) (3 marks)
Outcomes Assessed: P4

Criteria	Mark
• Applies the Cosine Rule to ΔPBQ.	1
• Makes h^2 the subject.	1
• Gives the correct answer.	1

Sample answer

Using the Cosine Rule,
$50^2 = (h\tan 55°)^2 + (h\tan 62°)^2 - 2(h\tan 55°)(h\tan 62°)\cos 81°$ 1

$$h^2 = \frac{50^2}{(\tan 55°)^2 + (\tan 62°)^2 - 2(\tan 55°)(\tan 62°)\cos 81°}$$ 1

$h \approx 22.97(\text{m})$ 1

Question 7 (12 marks)
(a) (i) (1 mark)
Outcomes Assessed: P4

Criteria	Mark
• Gives the correct working.	1

Sample Answer

$\left(x + \frac{1}{x}\right)^2 - 2 = x^2 + 2 + \frac{1}{x^2} - 2 = x^2 + \frac{1}{x^2}$ 1

(a) (ii) (3 marks)
Outcomes Assessed: P4

Criteria	Mark
• Transforms the given equation into a quadratic equation.	1
• Gives the first set of answers.	1
• Gives the second set of answers.	1

Sample answer

$x^2 + \frac{1}{x^2} - x - \frac{1}{x} - 4 = 0$
$\left(x + \frac{1}{x}\right)^2 - 2 - x - \frac{1}{x} - 4 = 0$
Let $m = x + \frac{1}{x}$.
$m^2 - m - 6 = 0$ 1
$m = -2$ or 3
$x + \frac{1}{x} = -2$ or $x + \frac{1}{x} = 3$

$x^2 + 2x + 1 = 0$ | $x^2 - 3x + 1 = 0$

$x = -1$ | $x = \frac{3 \pm \sqrt{5}}{2}$ 1+1

(b) (i) (2 marks)
Outcomes Assessed: PE4

Criteria	Mark
• Shows that the gradient is p.	1
• Completes the proof.	1

Sample Answer

$y = \frac{x^2}{4a}$
$\frac{dy}{dx} = \frac{2x}{4a}$
At $P, m = p$ 1

The equation of the tangent at P is
$y - ap^2 = p(x - 2ap)$
$y = px - ap^2$ 1

(b) (ii) (1 mark)
Outcomes Assessed: PE4

Criteria	Mark
• Gives the correct answer.	1

Sample Answer

$y = qx - aq^2$ 1

(b) (iii) (2 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Eliminates y to obtain an equation in x only.	1
• Completes the proof.	1

Sample answer

Solving simultaneously,
$px - ap^2 = qx - aq^2$ 1

$(p-q)x = a(p^2 - q^2)$
$x = a(p+q)$
$y = p[a(p+q)] - ap^2$ 1
$y = apq$

(b) (iv) (3 marks)
Outcomes Assessed: PE3

Criteria	Mark
• Equates the gradients of AP and AQ where $A = (-a, 0)$.	1
• Correctly finds the relation between p and q.	1
• Gives the correct answer.	1

Sample answer

PQ passes $(-a, 0)$. 1
$\frac{ap^2 - 0}{2ap + a} = \frac{aq^2 - 0}{2aq + a}$

$\frac{p^2}{2p+1} = \frac{q^2}{2q+1}$

$2p^2q + p^2 = 2pq^2 + q^2$
$p^2 - q^2 = 2pq(q - p)$
$p \neq q, \therefore -2pq = p + q$ 1

For $T, x = a(p+q)$, $y = apq$

The locus of T is $-2y = x$ or $y = -\frac{1}{2}x$. 1

END OF PAPER