Teacher..... Name .....

Sydney Technical High School

2015

Preliminary Examination

Mathematics

General Instructions

- Reading Time - 5 minutes.
- Working Time – 2 hours.
- Write using a black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in Question 11-18
- Begin each question on a fresh sheet of paper.

Total marks (82)

Section A Pages 2-5

- 10 Marks
- Attempt Questions 1 - 10
- All questions are of equal value.

Section B Pages 6-11

- 72 Marks
- Attempt Questions 11 - 18
- All questions are of equal value.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

1. Which of the following correctly expresses p as the subject of $q = \frac{4}{9p^2}$

   A. $p = \pm\frac{2p}{3}$

   B. $p = \pm\frac{2}{3q}$

   C. $p = \pm\frac{2\sqrt{q}}{3}$

   D. $p = \pm\frac{2}{3\sqrt{q}}$

2. What are the values of a and b if $\frac{5-2\sqrt{2}}{1+\sqrt{2}} = a + b\sqrt{2}$

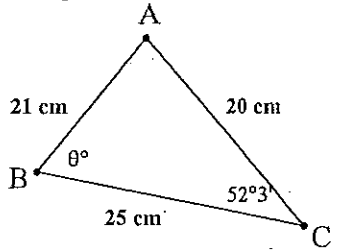
   A. $a = -9$ $b = 7$

   B. $a = 9$ $b = -7$

   C. $a = -7$ $b = 9$

   D. $a = 7$ $b = -9$

3. Which of the following is a correct expression involving θ in the triangle ABC?



   A. $20^2 = 21^2 + 25^2 + 2 \times 21 \times 25 \cos\theta$

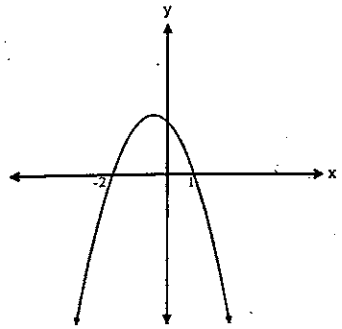
   B. $\cos\theta = \frac{21^2 + 25^2 - 20^2}{2 \times 21 \times 20}$

   C. $\frac{20}{\sin\theta} = \frac{21}{\sin 52°3'}$

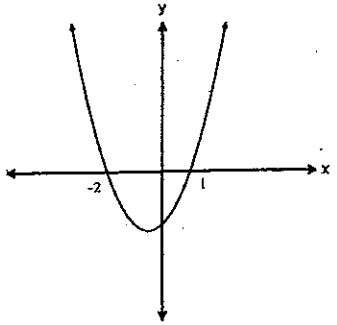
   D. $\frac{\sin\theta}{21} = \frac{\sin 52°3'}{20}$

4. Which of the graphs best represents $y = x^2 + x - 2$

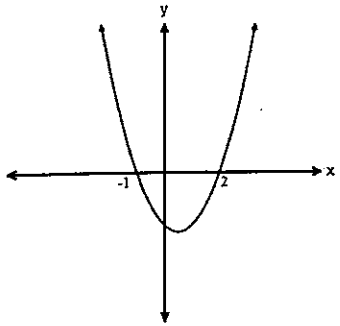
A.



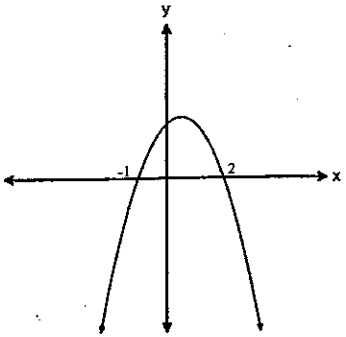
B.



C.



D.



5. Which of the following is true for the equation $6x^2 + x - 2 = 0$

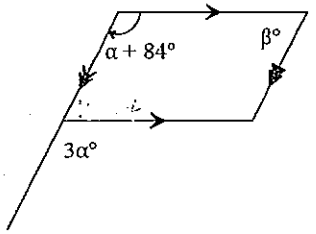
A. no real roots

B. one real root

C. two rational roots

D. two irrational distinct roots

6. What are the values of α and β



A. $\alpha = 42°$ $\beta = 54°$

B. $\alpha = 24°$ $\beta = 54°$

C. $\alpha = 24°$ $\beta = 108°$

D. $\alpha = 42°$ $\beta = 108°$

7. What is the perpendicular distance of the point (-3,1) from the line $3x - 2y = 4$?

A. $\frac{7}{\sqrt{13}}$

B. $\frac{7}{\sqrt{5}}$

C. $\frac{15}{\sqrt{13}}$

D. $\frac{15}{\sqrt{5}}$

8. Solve $|5x + 4| \leq 6$

A. $-\frac{2}{5} \leq x \leq 2$

B. $x \geq \frac{2}{5}$ or $x \leq -2$

C. $-2 \leq x \leq \frac{2}{5}$

D. $x \geq 2$ or $x \leq -\frac{2}{5}$

9. What is the derivative of $\frac{x}{2x+3}$?

A. $\frac{3}{(2x+3)^2}$

B. $\frac{1}{2}$

C. $\frac{4x+3}{(2x+3)^2}$

D. $\frac{1}{4}$

10. What is the solution to the equation $2\cos 2x - 1 = 0$ in the domain $0 \leq x \leq 2\pi$.

A. $60°$, $120°$

B. $120°$, $240°$

C. $30°$, $150°$, $210°$, $330°$

D. $60°$, $120°$, $240°$, $300°$

Question 11 (9 Marks) Use a Separate Sheet of paper **Marks**

a) Find the value of $\frac{1}{7.38}+\frac{1}{9.85}$, correct to 3 significant figures. **1**

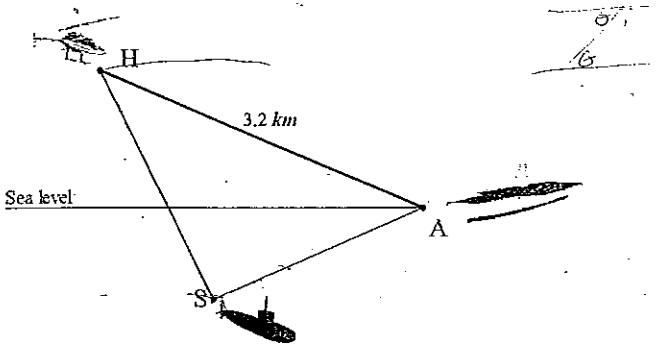
b) Express the decimal $0.4\dot{8}$ as a fraction in simplest form. **1**

c) Factorise $2x^2 + 9x - 5$ **2**

d) Simplify $\frac{3x^2}{x^2-9} \times \frac{x-3}{4x}$ **2**

e) From the helicopter (H), an aircraft carrier (A) is at a distance of 3.2 km and at an angle of depression of 15° and a submarine (S) is at an angle of depression of 58°. The angle of depression from the aircraft carrier to the submarine is 36°.

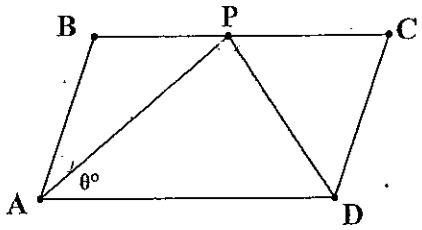
Copy the diagram into your answer booklet and show all information. **3**



Find, correct to the nearest 100m, the distance of the submarine (S) from the aircraft carrier (A)

End of Question 11.

Question 12 (9 Marks) Start a NEW Sheet of paper **Marks**



a) ABCD is a parallelogram. P is a point chosen on side BC such that AP bisects $\angle DAB$ and $\angle APD = 90^\circ$

Let $\angle PAD = \theta$

i) Prove that $\angle CPD = (90 - \theta)$ **1**

ii) Prove that PC=DC. **2**

b) Find $\lim_{x \to 3} \dfrac{x^2 - 2x - 3}{x - 3}$ **1**

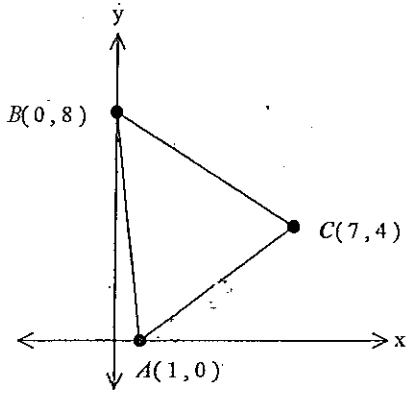
c) Find the values of a,b and c if $2x^2 + 3x + 1 = ax(x + 2) + b(x + 2) + c$ **3**

d) If $f(x) = 5x - x^2$, find $\dfrac{f(x + h) - f(x)}{h}$ **2**

End of Question 12.

Question 13 (9 Marks) Start a NEW Sheet of paper **Marks**

a) The points A, B and C have co-ordinates (1,0), (0,8) and (7,4) as shown on the diagram. The angle between CA and the positive axis is θ.



i) Find the gradient of CA **1**

ii) Calculate the size of θ to the nearest degree **1**

iii) Find the equation of CA **1**

iv) Find the co-ordinates of D, the midpoint of CA **1**

v) Show that CA ⊥ BD **2**

vi) Calculate the area of ΔABC **2**

b) $f(x) = \begin{cases} -x - 3 & x \leq -3 \\ x + 3 & x > -3 \end{cases}$

Evaluate $f(-4) + f(1) - f(5)$ **1**

End of Question 13.

Question 14 (9 Marks) Start a NEW Sheet of paper **Marks**

a) Differentiate with respect to x:

i) $\frac{1}{\sqrt{x}}$ **2**

ii) $x^2(1-x)^9$ **2**

iii) $\frac{x}{(x-1)^3}$ **2**

b) Find the equation of the tangent to the curve $y=x^3$ at the point where $x=-2$ **3**

End of Question 14.

Question 15 (9 Marks) Start a NEW Sheet of paper **Marks**

a) The quadratic equation $2x^2-5x-3=0$ has roots α and β. Find:

i) $\alpha+\beta$ **1**

ii) $\alpha\beta$ **1**

iii) $\frac{1}{\alpha}+\frac{1}{\beta}$ **2**

iv) $\alpha^2+\beta^2$ **2**

b) i) Sketch the graph of $y=x^2-7x+12$ **1**
(Hint: Graph should be one-third of your page, Use a ruler)

ii) State the domain and range **2**

End of Question 15.

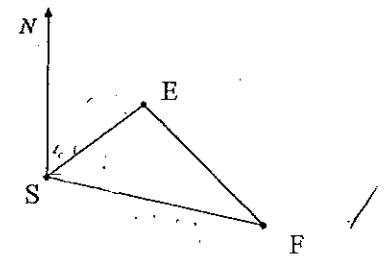
Question 16 (9 Marks) Start a NEW Sheet of paper **Marks**

a) Prove that $\frac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta} \equiv \tan^2\theta+\cos^2\theta$ **2**

b) Show the region of the number plane where the following hold simultaneously: **3**

$$\begin{cases} (x+1)^2+y^2 \le 4 \\ y \le (x+3) \\ y \ge 0 \end{cases}$$

c) Two cruise ships set sail from Sydney Harbour (S). The Elvis Presley Tribute cruise (E) sails at 18km/h on a bearing of 049° while the Frank Sinatra Tribute Cruise (F) sails at 21km/h along a bearing of 151°.



i) Show that $\angle ESF = 102°$ **1**

ii) Calculate the distance between the cruise ships to the nearest kilometre after 3 hours. **3**

End of Question 16.

Question 17 (9 Marks) Start a NEW Sheet of paper **Marks**

a) Sketch the graphs of the following, stating the domain and range of each.
(Graph should be one-third of your page)
(Hint: Use a ruler)

i) $y = 2^x$ **2**

ii) $2x - 3y + 12 = 0$ **3**

b) i) Write down the discriminant of $2x^2 + 4x + k$ **1**

ii) For what values of k does $2x^2 + 4x + k = 0$ have real roots **1**

c) Solve $2\cos x + 1 = 0$ for $0° \leq x \leq 360°$ **2**

End of Question 17.

Question 18 (9 Marks) Start a NEW Sheet of paper **Marks**

a) Find the exact value of $\sec 60°$. **1**

b) If $\sin\theta = -\frac{5}{13}$ and $\tan\theta < 0$, find $\cos\theta$ **2**

c) Determine if the function $f(x) = \frac{3x}{x^2 - 1}$ is an odd or even function **2**

d) For the function $f(x) = \sqrt{4 - x^2} + 3$ find;
i) the domain **1**

ii) the range **1**

e) Solve the equation $(x^2 + x) + \frac{12}{x^2 + x} - 8 = 0$ **2**

End of Examination.

Preliminary Examination
2015 Mathematics

Multiple Choice

1	D	6	A
2	A	7	C
3	C	8	C
4	B	9	A
5	C	10	C

Question 11

a) 0·237

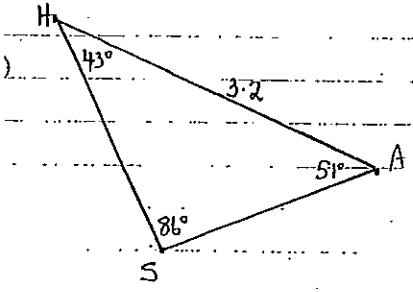
b) $\frac{16}{33}$

c) $2x^2+9x-5 = (2x-1)(x+5)$

d) $\frac{3x^2}{x^2-9} \times \frac{x-3}{4x} = \frac{3x^2}{(x-3)(x+3)} \times \frac{x-3}{4x}$

$= \frac{3x}{4(x+3)}$

e)



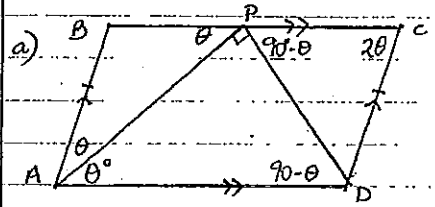
$\frac{x}{\sin 43°} = \frac{3.2}{\sin 86}$

$x = 2.19$ km

$x = 2200$ m

Question 12

a)



i) $\angle PAD = 90-\theta$ (angle sum of ΔAPD)

$\angle CPD = 90° - \theta$ (alternate angles, BC//A

ii) $\angle PCD = 2\theta$ (opposite angles are equal in a parallelogram)

$\therefore \angle CDP = 180° - [2\theta + (90-\theta)]$

$= 90 - \theta$

$\therefore PC = DC$ (sides opposite equal angl

ΔCDP is isosceles

b) $\lim_{x\to 3} \frac{x^2-2x-3}{x-3}$

$= \lim_{x\to 3} \frac{(x-3)(x+1)}{x-3}$

$= 4$

c) $2x^2+3x+1 = ax(x+2) + b(x+2) + c$

$= ax^2 + 2ax + bx + 2b + c$

$= ax^2 + x(2a+b) + (2b+c)$

$a = 2$

$2a+b = 3$ $\quad 2b+c = 1$

$4+b = 3$ $\quad -2+c = 1$

$b = -1$ $\quad c = 3$

$a = 2 \quad b = -1 \quad c = 3$

d) $f(x) = 5x - x^2$

$f(x+h) = 5(x+h) - (x+h)^2$

$= 5x + 5h - x^2 + 2xh + h^2$

$= 5x + 5h - x^2 - 2xh - h^2$

$\frac{f(x+h) - f(x)}{h}$

$= \frac{5x + 5h - x^2 - 2xh - h^2 - 5x + x^2}{h}$

$= \frac{5h - 2x - h^2}{h}$

$= \frac{h(5 - 2x - h)}{h}$

$= 5 - 2x - h$

Question 13

a i) $M_{CA} = \frac{4-0}{7-1} = \frac{4}{6} = \frac{2}{3}$

ii) $\tan\theta = \frac{4}{6}$

$\theta = 34°$

iii) Equation of CA

$y - 0 = \frac{2}{3}(x-1)$

$3y = 2x - 2$ or

$2x - 3y - 2 = 0$

iv) Midpoint of $CA = \left(\frac{1+7}{2}, \frac{0+4}{2}\right)$

$D = (4, 2)$

v) $M_{BD} = \frac{6}{-4} = -\frac{3}{2}$

$M_{BD} \times M_{CA} = -\frac{3}{2} \times \frac{2}{3} = -1$

$\therefore BD \perp CA$

vi) area ΔABC

$AC = \sqrt{(7-1)^2 + (4-0)^2}$

$= \sqrt{52}$

$\frac{1}{2} \times \sqrt{52} \times \sqrt{52} = 26$ units

b) $f(-4) + f(1) + f(5)$

$1 + 4 - 8$

$= -3$

Question 14

a) i) $y = x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$ or $\frac{-1}{2\sqrt{x^3}}$

ii) $y = x^2(1-x)^9$

$u = x^2 \qquad v = (1-x)^9$

$du = 2x \qquad dv = -9(1-x)^8$

$\frac{dy}{dx} = x^2 \times -9(1-x)^8 + 2x(1-x)^9$

$= -9x^2(1-x)^8 + 2x(1-x)^9$

$= x(1-x)^8[-9x + 2x(1-x)]$

iii) $y = \frac{x}{(x-1)^3}$

$u = x \qquad v = (x-1)^3$

$du = 1 \qquad dv = 3(x-1)^2$

$\frac{dy}{dx} = \frac{(x-1)^3 - x[3(x-1)^2]}{(x-1)^6}$

$= \frac{(x-1)^2[x-1-3x]}{(x-1)^6}$

$= \frac{(x-1)^2(-2x-1)}{(x-1)^6}$

c) $y = x^3$

$\frac{dy}{dx} = 3x^2$ $(-2, -8)$

When $x = -2$ $m = 12$

$y + 8 = 12(x + 2)$

$y + 8 = 12x + 24$

$y = 12x + 16$

Question 15

a i) $\alpha + \beta = \frac{-b}{a} = \frac{5}{2}$

ii) $\alpha\beta = \frac{c}{a} = \frac{-3}{2}$

iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha\beta}{\alpha+\beta} = \frac{5}{2} \div \frac{-3}{2}$

$= \frac{-5}{3}$

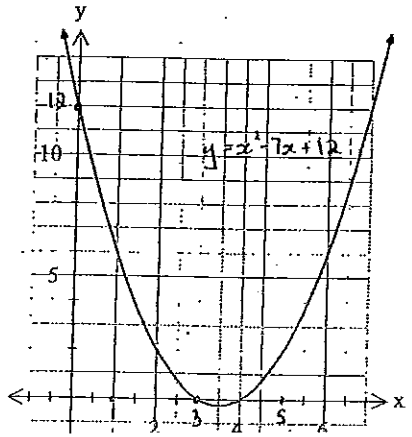
iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{5}{2}\right)^2 - 2 \times \frac{-3}{2}$

$= \frac{25}{4} + 3$

$= \frac{37}{4}$ or $9\frac{1}{4}$

d)



Domain : all real for x

Range : $y \geq -\frac{1}{4}$

Question 16

$\frac{1 - \sin^2\theta\cos^2\theta}{\cos^2\theta} = \tan^2\theta + \cos^2\theta$

L.H.S $= \frac{1 - \sin^2\theta\cos^2\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta\cos^2\theta}{\cos^2\theta}$

$= \sec^2\theta - \sin^2\theta$

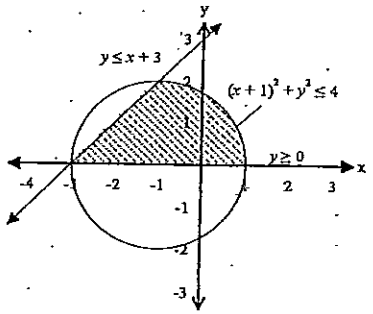
$= \sec^2\theta - (1 - \cos^2\theta)$

$= \sec^2\theta - 1 + \cos^2\theta$

$= \tan^2\theta + \cos^2\theta$

$= R.H.S$

b)



c i) $\angle ESF = 151° - 49° = 102°$

ii) $(EF)^2 = 54^2 + 63^2 - 2 \times 54 \times 63 \times \cos 102°$

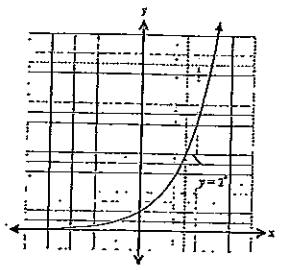
$(EF)^2 = 8299.63$

$EF = \sqrt{8299.63}$

$EF = 91\text{ km}$

Question 17

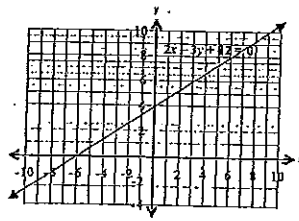
a i)



Domain: all real x

Range: $y > 0$

ii)



Domain all real x

Range all real y

b i) $2x^2 + 4x + k$

$\Delta = 4^2 - 4 \times 2 \times k$

$\Delta = 16 - 8k$

ii) Real Roots $\Delta \geq 0$

$16 - 8k \geq 0$

$16 \geq 8k$

$k \leq 2$

c) $2\cos x + 1 = 0$

$2\cos x = -1$

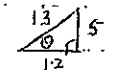
$\cos x = -\frac{1}{2}$

$x = 120°, 240°$

Question 18

a) $\sec 60° = \frac{1}{\cos 60°} = 2$

b) $\sin\theta = -\frac{5}{13}$



$\cos\theta = \frac{12}{13}$

c) $f(x) = \frac{3x}{x^2 - 1}$

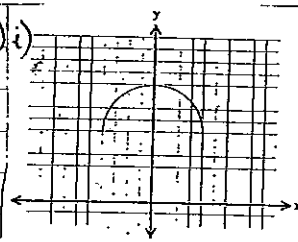
$f(-x) = \frac{-3x}{x^2 - 1}$

$= -\left[\frac{3x}{x^2 - 1}\right]$

$= -f(x)$

$\therefore$ function is odd

d) i)



$f(x) = \sqrt{4 - x^2} + 3$

$4 - x^2 \geq 0$

$x^2 \leq 4$

$-2 \leq x \leq 2$

Domain

ii) Range $0 \leq \sqrt{4 - x^2} + 3 \leq 2$

$-3 \leq f(x) \leq 5$

e) Let $m = (x^2 + x)$ $m + \frac{12}{m} - 8 = 0$

$m^2 + 12 - 8m = 0$

$(m-6)(m-2) = 0$

$m = 6$ $m = 2$

$x^2 + x - 6 = 0$ $x^2 + x - 2 = 0$

$(x+3)(x-2) = 0$ $(x+2)(x-1) = 0$

$x = -3$ $x = -2$

$x = 2$ $x = 1$