TEST 15 Mensuration

Marks: /80

Time: 1 hour 30 minutes

Name: .... Date: ....

INSTRUCTIONS TO CANDIDATES

Section A (40 marks) **Time: 45 minutes**

1. Answer **all** the questions in this section.
2. Calculators may **not** be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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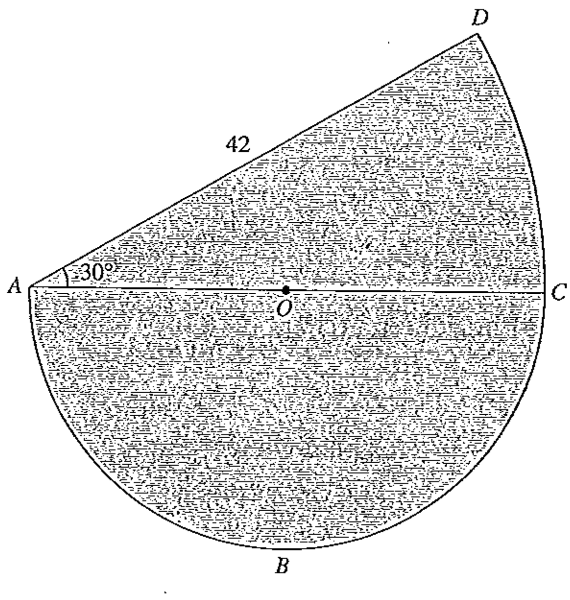
1 The diagram shows the cross-section of a solid. *ABC* is a semicircle with centre *O*. *ACD* is a sector of a circle, centre *A* and radius 42 cm with $C\hat{A}D = 30°$. Taking π to be $\frac{22}{7}$, calculate the perimeter of the cross-section of the solid.

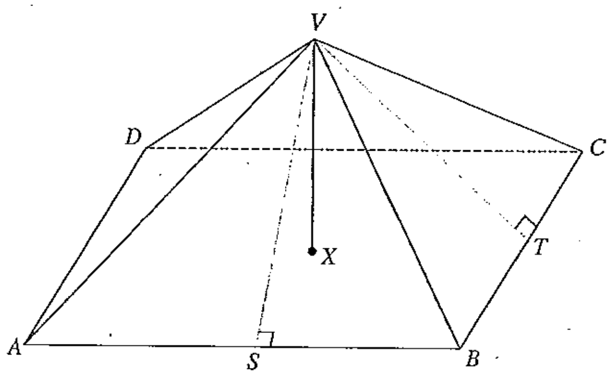


Answer .... cm [3]

2 The diagram shows a right pyramid with a horizontal rectangular base $ABCD$ and vertex, V. $AB = 30$ cm, $BC = 12$ cm and the slant heights, $VS = 10$ cm and $VT = 17$ cm. Given that the volume of the pyramid is 960 cm^3, find

(a) the height VX,

(b) the total surface area of the pyramid.



Answer (a) VX = ... cm [1]

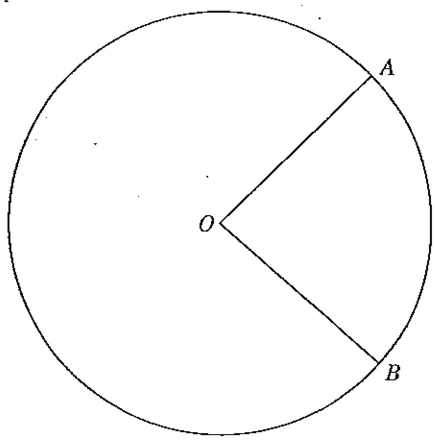
(b) .... cm^2 [2]

3 The diagram shows a circle, centre O and diameter 30 cm. Given that the minor arc AB is 22 cm, calculate

(a) $A\hat{O}B$,

(b) the area of the minor sector $A\hat{O}B$.

$\left[\text{Take } \pi \text{ to be } \frac{22}{7}.\right]$



Answer (*a*) $A\hat{O}B$ = ... ° [2]

(*b*) .... cm^2 [2]

4 A solid cylindrical metal rod of radius 4 cm and length 18 cm is melted down and recast to form a solid metal sphere. Find

(a) the volume of the cylindrical metal rod in terms of π,

(b) the radius of the sphere.

Answer (*a*) .... cm^3 [1]

(*b*) .... cm [2]

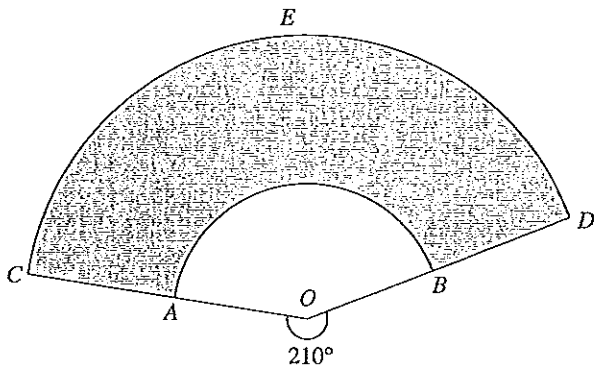
5 A piece of cardboard cut to the shape of a sector is shown below. Part of the sector is then painted orange as indicated by the shaded region. Reflex $C\hat{O}D = 210°$, $OC = 12$ cm and $OC = 2OA$.

(a) Calculate in terms of π,

(i) the area of the cardboard painted orange,

(ii) the length of the arc CED.

(b) The cardboard is then used to make a hollow cone by joining the edges OC and OD together. Calculate the base radius of the cone.



Answer (a) (i) .... cm^2 [2]

(ii) .... cm [1]

(b) .... cm [2]

6 A hemispherical pan of radius 9 cm is completely filled with water. The water is then poured into an empty cylindrical pot of radius 6 cm.

(a) Find the volume of water in the pan, leaving your answer in terms of π.

(b) Calculate the height of water in the pot.

Answer (a) .... cm^3 [1]

(b) .... cm [3]

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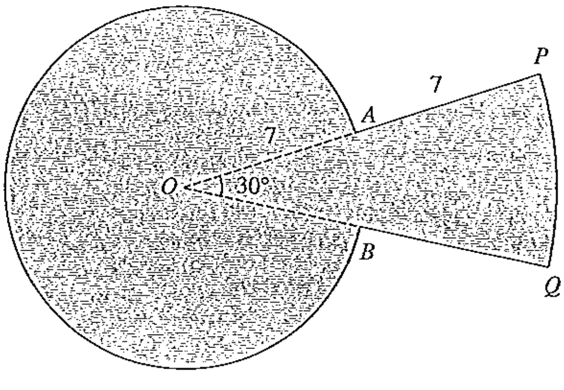
7 In the diagram, OA is the radius of the sector OAB and OP is the radius of the sector OPQ. Given that $OA = AP = 7$ m, calculate the perimeter of the figure.

$\left[\text{Take } \pi \text{ to be } \frac{22}{7}.\right]$



Answer .... m [3]

8 A solid wax cube of side 6 cm is melted down and the wax is poured into two separate moulds, filling them completely. The first mould is a cylinder of radius 3 cm and height 7 cm. The second mould is a pyramid with a square base of side x cm and height $2x$ cm.

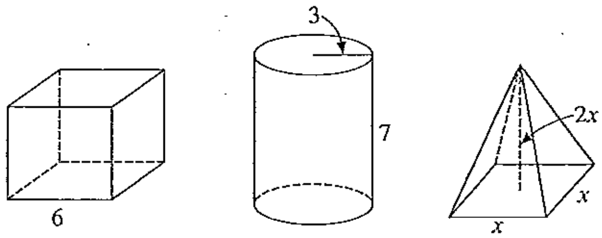
Calculate

(a) the volume of the cube,

(b) the volume of the cylinder,

(c) the value of x.

$\left[\text{Take } \pi \text{ to be } \frac{22}{7}.\right]$



Answer (a) .... cm³ [1]

(b) .... cm³ [1]

(c) $x =$ .... [2]

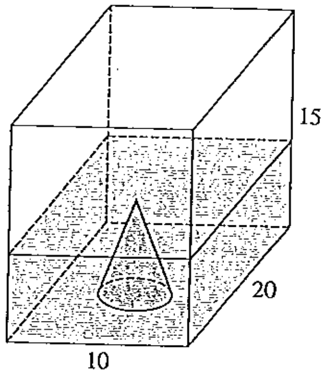
9 A solid cone of volume 66 cm^3 is placed into an empty rectangular tank measuring 10 cm by 20 cm by 15 cm. 1334 cm^3 of water is then poured into the tank. Given that the water just covers the vertex of the cone, calculate

(a) the height of the cone,

(b) the radius of the cone,

(c) the new water level in the tank when the cone is removed.

$\left[\text{Take } \pi \text{ to be } \frac{22}{7}.\right]$



Answer (a) .... cm [2]

(b) .... cm [2]

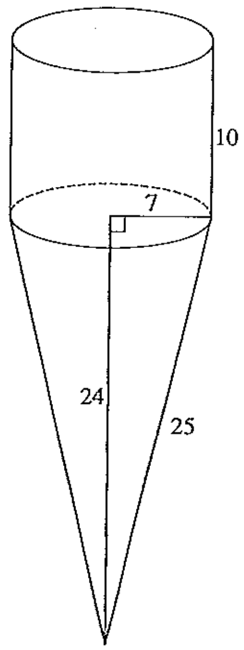
(c) .... cm [2]

10 An open container is made by joining together a cylinder of radius 7 cm and a cone with top radius 7 cm as shown in the diagram. The height of the cone is 24 cm and its slant height is 25 cm. The height of the cylinder is 10 cm.

(a) Calculate the outer surface area of the container.

(b) $490\pi\ \text{cm}^3$ of water is then poured into the container. Calculate the height of the water level in the container.

$\left[\text{Take } \pi \text{ to be } \dfrac{22}{7}.\right]$



Answer (a) .... cm^2 [2]

(b) .... cm [3]

INSTRUCTIONS TO CANDIDATES

Section B (40 marks) **Time: 45 minutes**

1. Answer **all** the questions in this section.
2. Calculators may be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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11 The solid shown below consists of a solid cone attached to a solid cylinder. The height of the cylinder is 8 cm and the area of its base is 140 cm^2. A 2 cm wide border is formed around the cone.

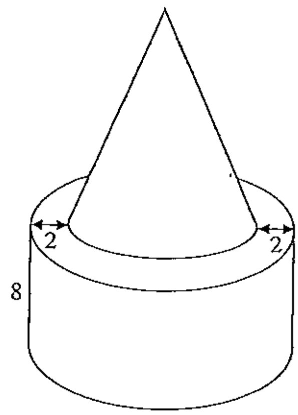
(a) Calculate the volume of the cylinder.

(b) Find the radius of the cone.

(c) Given that the volume of the cone is 280 cm^3, calculate the height of the cone.

(d) Given that the solid is made up of material of density 0.72 g/cm^3, calculate the mass of the solid, giving your answer correct to the nearest kilogram.

[Take π to be 3.142.]



Answer (a) .... cm^3 [1]

(b) .... cm [3]

(c) .... cm [2]

(d) .... kg [2]

12 The diagram shows 3 identical metal balls, each of radius 3 cm packed into a rectangular box. Each ball touches the top, bottom and sides of the box.

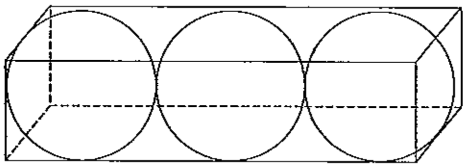
(a) Calculate the total surface area of the box.

(b) Calculate the volume of the 3 balls, giving your answer in terms of π.

(c) Calculate the space in the box not occupied by the balls.

(d) The balls are then melted down and recast to form a hemisphere. Find the radius of the hemisphere.

[Take π to be 3.142.]



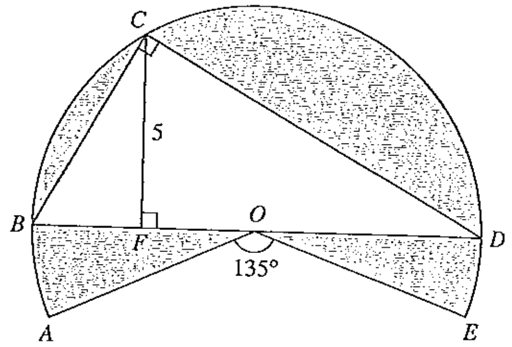
Answer (a) .... cm^2 [2]

(b) .... cm^3 [2]

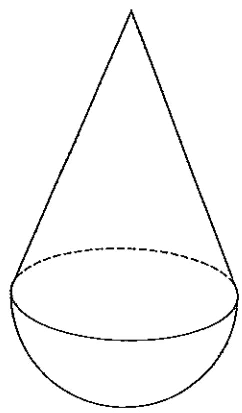
(c) .... cm^3 [1]

(d) .... cm [2]

13 (a) In the diagram, AOE is a sector of a circle, centre O. $B\hat{C}D = C\hat{F}D = 90°$, $A\hat{O}E = 135°$, $CF = 5$ cm and the area of triangle BCD is 30 cm^2.
Calculate the area of the shaded region, giving your answer correct to 3 significant figures.
[Take π to be 3.142.]



(b) The diagram below shows a solid consisting of a cone and a hemisphere. The volume of the solid is 4950π cm^3 and the diameter of the cone is 30 cm.
Find
(i) the height of the cone,
(ii) the total surface area of the solid, giving your answer correct to the nearest square centimetres.
[Take π to be 3.142.]



Answer *(a)* .... cm^2 [3]

(b) (i) .... cm [3]

(ii) .... cm^2 [3]

14 The diagrams below show three different positions of a closed container which is half-filled with water. The container is made by joining a hemisphere of radius 6 cm to a cylinder of radius 6 cm and length 14 cm.

(a) Calculate the volume of water in the container, giving your answer in terms of π.

(b) Calculate the surface area of the container in Diagram I which is in contact with water. Give your answer correct to the nearest square centimetres.

(c) Calculate the depth of water in the container shown in

(i) Diagram II,

(ii) Diagram III.

[Take π to be 3.142.]

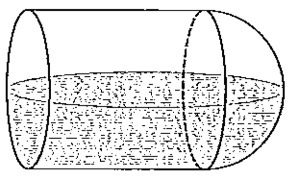


Diagram I

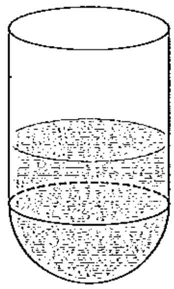


Diagram II

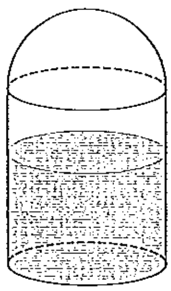


Diagram III

Answer (a) .... cm^3 [2]

(b) .... cm^2 [3]

(c) (i) .... cm [3]

(ii) .... cm [1]

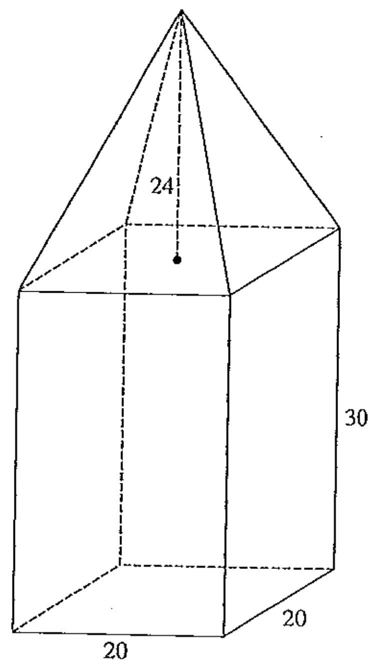
15 The diagram shows a solid made by joining together a pyramid and a cuboid. The cuboid has a square base of sides 20 cm and height 30 cm. The perpendicular height of the pyramid is 24 cm.

(a) Calculate

(i) the volume of the solid,

(ii) the total surface area of the sides and top of the solid.

(b) 124 of these solids are made. The sides and tops of these solids are painted. The paint used is sold in tins, each containing enough paint to cover 6 m^2. How many tins are needed?



Answer (a) (i) .... cm^3 [2]

(ii) .... cm^2 [3]

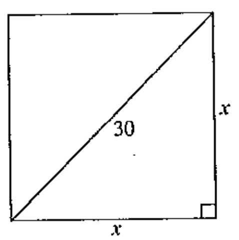
(b) .... tins [2]

Using Pythagoras' Theorem on $\triangle PQC$,
$PQ^2 = PC^2 + QC^2$
$x^2 = (y-2)^2 + (3+5)^2$
$x^2 = y^2 - 4y + 4 + 64$
$x^2 = y^2 - 4y + 68$ —— (2)

Substitute (1) into (2):
$y^2 + 25 = y^2 - 4y + 68$
$4y = 43$
$y = \frac{43}{4} = 10.75$

Substitute $y = 10.75$ into (1):
$x^2 = 10.75^2 + 25$
$= 140.5625$
$x = \sqrt{140.5625}$
$\approx$ **11.9** (correct to 3 sig. fig.)

(b) (i)

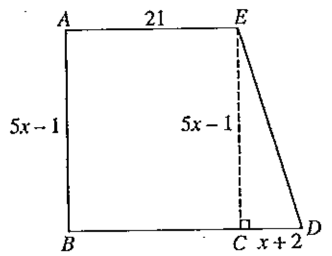


Let the length of each side of the square be x cm.
Using Pythagoras' Theorem,
$x^2 + x^2 = 30^2$
$2x^2 = 900$
$x^2 = \frac{900}{2} = 450$
$x = \sqrt{450}$

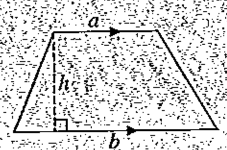
Perimeter of square $= 4x$
$= 4 \times \sqrt{450}$
$\approx$ **84.9 cm** (correct to 3 sig. fig.)

(ii) Area of square $= x^2$
$=$ **450 cm^2**

15. (a)



Teacher's Tip
Area of trapezium
$A = \frac{1}{2}(a+b)h$
a, b = length of parallel sides
h = height



Area of $\triangle ECD = \frac{1}{6} \times$ Area of rectangle $ABCE$

$\frac{1}{2}(x+2)(5x-1) = \frac{1}{6_3}(5x-1)(21^7)$ Multiply both sides by 2.
$(x+2)(5x-1) = 7(5x-1)$
$5x^2 - x + 10x - 2 = 35x - 7$
$5x^2 - 26x + 5 = 0$ Shown

(b) $5x^2 - 26x + 5 = 0$
$(5x-1)(x-5) = 0$
$x = \frac{1}{5}$ (rejected) or $x = 5$

When $x = \frac{1}{5}$,
$AB = 5\left(\frac{1}{5}\right) - 1$
$= 0$
$\therefore x = \frac{1}{5}$ is rejected.

When $x = 5$,
$AB = 5(5) - 1 = 24$ cm
$CD = 5 + 2 = 7$ cm

Using Pythagoras' Theorem on $\triangle ECD$,
$ED^2 = EC^2 + CD^2$
$= 24^2 + 7^2$ $EC = AB = 24$ cm
$= 625$
$ED = \sqrt{625} = 25$ cm

Perimeter of trapezium $ABDE$
$= 21 + 24 + (21 + 7) + 25$
= 98 cm

Test 15: Mensuration

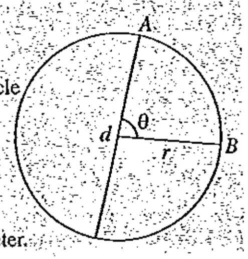
Section A

1. Length of arc CD
$= \frac{30°}{360°} \times 2\pi(42)$ Radius, $r = AD = 42$ cm
$= \frac{30°}{360°} \times 2 \times \frac{22}{7} \times 42$
$= 22$ cm

Length of semicircle ABD
$= \frac{1}{2} \times \pi(42)$
$= \frac{1}{2} \times \frac{22}{7} \times 42$
$= 66$ cm

Total perimeter $= AD +$ arc $CD +$ semicircle ABD
$= 42 + 22 + 66$
= 130 cm

Teacher's Tip
Length of arc AB
$= \frac{\theta}{360°} \times$ Circumference of circle
$= \frac{\theta}{360°} \times 2\pi r$
or $= \frac{\theta}{360°} \times \pi d$
where r = radius and d = diameter.



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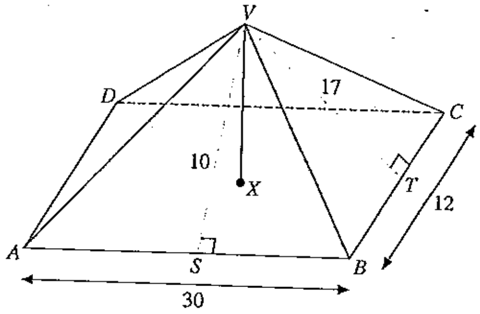


(a) Volume of pyramid $= \frac{1}{3} \times$ Base area $\times$ Height

$$960 = \frac{1}{3} \times (30 \times 12) \times VX$$

$$VX = \frac{960 \times 3}{30 \times 12}$$

$$= \mathbf{8\ cm}$$

(b) Area of base $ABCD = 30 \times 12 = 360\ \text{cm}^2$

Area of faces VAB and $VDC = 2 \times \left(\frac{1}{2} \times 30 \times 10\right)$

$= 300\ \text{cm}^2$

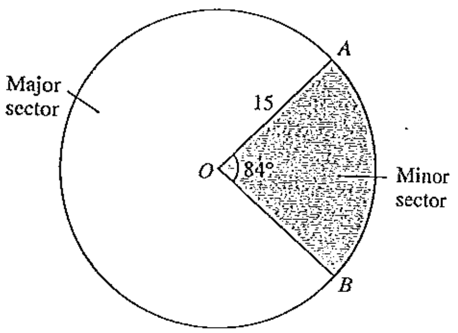
Area of faces VBC and $VAD = 2 \times \left(\frac{1}{2} \times 12 \times 17\right)$

$= 204\ \text{cm}^2$

Total surface area $= 360 + 300 + 204$

$= \mathbf{864\ cm^2}$

3.



(a) Length of minor arc $AB = 22$ cm Given

$\frac{A\hat{O}B}{360°} \times \pi(30) = 22$ — Circumference of circle $= \pi d$ or $2\pi r$, d = diameter, r = radius.

$\frac{A\hat{O}B}{360°} \times \frac{22}{7} \times 30 = 22$

$$A\hat{O}B = \frac{22 \times 7 \times 360°}{22 \times 30}$$

$$= \mathbf{84°}$$

(b) Area of minor sector AOB

$= \frac{84°}{360°} \times \pi(15)^2$

$= \frac{84°}{360°} \times \frac{22}{7} \times 15 \times 15$

$= \mathbf{165\ cm^2}$

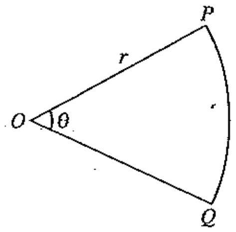
Teacher's Tip

Area of sector POQ

$= \frac{\theta}{360°} \times$ Area of circle

$= \frac{\theta}{360°} \times \pi r^2$

where r = radius.



4. (a) Volume of cylindrical metal rod

$= \pi(4)^2 18$

$= \mathbf{288\pi\ cm^3}$

(b) Let r be the radius of the sphere.

Vol. of sphere = Vol. of metal rod

$$\frac{4}{3}\pi r^3 = 288\pi$$

$$r^3 = \frac{288 \times 3}{4}$$

$$= 216$$

$$r = \sqrt[3]{216}$$

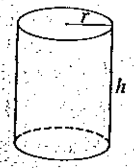
$$= 6\ \text{cm}$$

$\therefore$ the radius of the sphere is **6 cm**.

Teacher's Tip

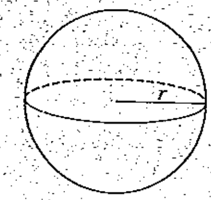
Volume of cylinder $= \pi r^2 h$

where r = radius and h = height.



Volume of sphere $= \frac{4}{3}\pi r^3$

where r = radius.



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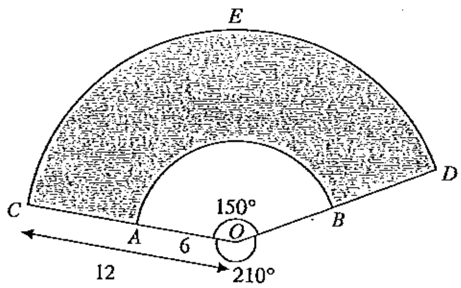


(a) (i) $OA = \frac{1}{2}OC = \frac{1}{2}(12) = 6$ cm

Obtuse $C\hat{O}D = 360° - 210°$ $\angle$s at a pt.

$= 150°$

$$\text{Painted area} = \begin{pmatrix}\text{Area of}\\ \text{sector } COD\end{pmatrix} - \begin{pmatrix}\text{Area of}\\ \text{sector } AOB\end{pmatrix}$$
$$= \frac{150°}{360°} \times \pi(12)^2 - \frac{150°}{360°} \times \pi(6)^2$$
$$= \frac{150°}{360°} \times \pi(12^2 - 6^2)$$
$$= \frac{150°}{360°} \times \pi \times 108$$
$$= \mathbf{45\pi \ cm^2}$$

(ii) Length of arc CED

$$= \frac{150°}{360°} \times 2\pi(12)$$
$$= \mathbf{10\pi \ cm}$$

(b) If R is the base radius of the cone, then the circumference of the base of the cone is $2\pi R$.

$$\therefore\ 2\pi R = \text{Length of arc } CED$$
$$= 10\pi$$
$$R = \frac{10\pi}{2\pi}$$
$$= 5 \text{ cm}$$

$\therefore$ the base radius of the cone is **5 cm**.

6. (a) Volume of water in the pan

$$= \frac{2}{3}\pi(9)^3$$
$$= \mathbf{486\pi \ cm^3}$$

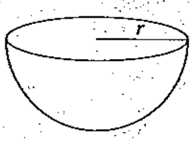
(b) Let h be the height of water in the pot.

Vol. of water in pot = Vol. of water in pan

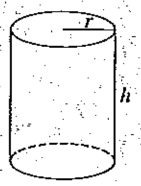
$$\pi(6)^2(h) = 486\pi$$
$$h = \frac{486\pi}{36\pi}$$
$$= 13.5 \text{ cm}$$

$\therefore$ the height of water in the pot is **13.5 cm**.

Teacher's Tip

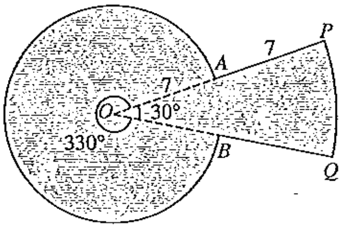


Volume of hemisphere $= \frac{2}{3}\pi r^3$ where r = radius.



Volume of cylinder $= \pi r^2 h$ where r = radius and h = height.

7.



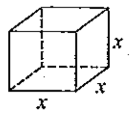
Perimeter of figure

$$= 2AP + \begin{pmatrix}\text{Length of}\\ \text{arc } AB\end{pmatrix} + \begin{pmatrix}\text{Length of}\\ \text{arc } PQ\end{pmatrix}$$
$$= 2(7) + \left(\frac{330°}{360°} \times 2 \times \frac{22}{7} \times 7\right)$$
$$+ \left(\frac{30°}{360°} \times 2 \times \frac{22}{7} \times 14\right)$$
$$= 14 + 40\frac{1}{3} + 7\frac{1}{3}$$
$$= \mathbf{61\frac{2}{3} \ m}$$

8. (a) Vol. of cube $= 6^3$

$$= \mathbf{216 \ cm^3}$$

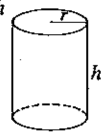
Vol. of cube $= x^3$



(b) Vol. of cylinder $= \pi(3)^2(7)$

$$= \frac{22}{7} \times 9 \times 7$$
$$= \mathbf{198 \ cm^3}$$

Vol. of cylinder $= \pi r^2 h$

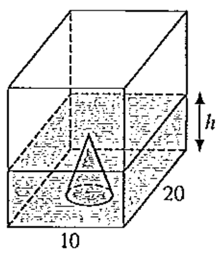


(c) Vol. of pyramid $= 216 - 198$

$$= 18 \text{ cm}^3$$

$$\therefore\ \frac{1}{3} \times (\text{Area of square base}) \times \text{Height} = 18 \text{ cm}^3$$
$$\frac{1}{3}(x^2)(2x) = 18$$
$$x^3 = \frac{18 \times 3}{2}$$
$$= 27$$
$$x = \sqrt[3]{27}$$
$$= \mathbf{3}$$

9.



(a) Let h be the height of the water in the tank.

Vol. of cone ↘ Vol. of water ↙

$$\therefore\ 10 \times 20 \times h = 66 + 1334$$
$$200h = 1400$$
$$h = \frac{1400}{200}$$
$$= 7 \text{ cm}$$

$\therefore$ height of cone is **7 cm**.

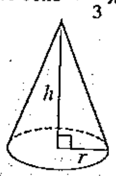
Teacher's Tip

Since the water just covers the vertex of the cone, the height of the cone is the same as the height of the water level in the tank.

(b) Let the radius of the cone be r cm.

Vol. of cone = 66 cm^3

$\frac{1}{3}\pi r^2(7) = 66$ Vol. of cone $= \frac{1}{3}\pi r^2 h$



$\frac{1}{3} \times \frac{22}{\not 7} \times r^2 \times \not 7 = 66$

$r^2 = \frac{66 \times 3}{22}$

$= 9$

$r = \sqrt{9} = 3$

$\therefore$ the radius of the cone is **3 cm**.

(c) Let H be the new water level of the tank.
When the cone is removed, 1334 cm^3 of water remained in the tank.

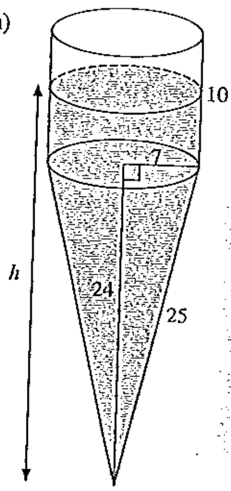
$\therefore 10 \times 20 \times H = 1334$

$H = \frac{1334}{200}$

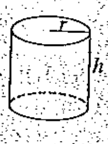
$= 6.67$ cm

$\therefore$ the new water level in the tank is **6.67 cm**.

10. (a)



Curved surface area of cylinder $= 2\pi rh$



Curved surface area of cone $= \pi rl$

Outer surface area of container

$= \begin{pmatrix}\text{Curved surface} \\ \text{area of} \\ \text{cylinder}\end{pmatrix} + \begin{pmatrix}\text{Curved surface} \\ \text{area of} \\ \text{cone}\end{pmatrix}$

$= 2\pi(7)(10) + \pi(7)(25)$

$= 2 \times \frac{22}{7} \times 7 \times 10 + \frac{22}{7} \times 7 \times 25$

$= 440 + 550$

= 990 cm²

(b) Let the height of water in the container be h.
Volume of water in container $= 490\pi$ cm^3

$\begin{pmatrix}\text{Volume of} \\ \text{water in} \\ \text{conical part}\end{pmatrix} + \begin{pmatrix}\text{Volume of} \\ \text{water in} \\ \text{cylindrical} \\ \text{part}\end{pmatrix} = 490\pi$

$\frac{1}{3}\not\pi(7)^2(24) + \not\pi(7)^2(h - 24) = 490\not\pi$

$392 + 49(h - 24) = 490$

$49(h - 24) = 98$

$h - 24 = \frac{98}{49}$

$= 2$

$h = 2 + 24$

$= 26$ cm

$\therefore$ the height of the water in the container is **26 cm**.

Section B

11. (a) Vol. of cylinder = Base area × Height

$= 140 \times 8$

= 1120 cm³

(b) Let the radius of the cylinder be r cm.

Base area of cylinder = 140 cm^2 Given

$\pi r^2 = 140$

$3.142r^2 = 140$

$r^2 = \frac{140}{3.142}$

$r = \sqrt{\frac{140}{3.142}}$

$= 6.675$

$\therefore$ radius of cone $= 6.675 - 2$

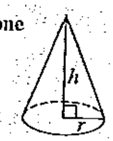
$= 4.675$

$\approx$ **4.68 cm** (correct to 3 sig. fig.)

(c) Let h be the height of the cone.

Vol. of cone = 280 cm^3 Given

$\frac{1}{3}(3.142)(4.675)^2 h = 280$ **Vol. of cone** $= \frac{1}{3}\pi r^2 h$



$h = \frac{280 \times 3}{3.142 \times 4.675^2}$

$= 12.23$

≈ 12.2 cm (correct to 3 sig. fig.)

$\therefore$ the height of the cone is **12.2 cm**.

(d) Volume of solid

Vol. of cylinder + Vol. of cone

$= 1120 + 280$

$= 1400$ cm^3

Mass of solid

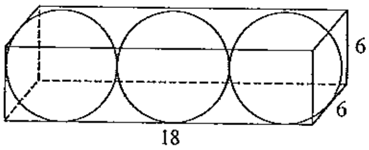
= Density × Volume   Density $= \frac{\text{Mass}}{\text{Volume}}$

$= 0.72$ g/cm$^3 \times 1400$ cm^3 Mass = Density × Volume

$= 1008$ g

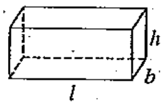
$\approx$ **1 kg** (correct to the nearest kg)   1 kg = 1000 g

12.



(a) Length of box = 3 × (2 × 3) = 18 cm
Breadth of box = 2 × 3 = 6 cm
Height of box = 2 × 3 = 6 cm
Total surface area of box
= 2[(18 × 6) + (18 × 6) + (6 × 6)]
= **504 cm^2**

Teacher's Tip



Total surface area of a cuboid
= 2(*lb* + *lh* + *bh*)

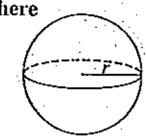
(b) Vol. of 3 balls

$= 3 \times \left(\frac{4}{3} \times \pi \times 3^3\right)$

= **108π cm^3**

Vol. of sphere

$= \frac{4}{3}\pi r^3$



(c) Vol. of box = 18 × 6 × 6
= 648 cm^3
Space in box not occupied by the balls
= 648 − 108π
= 648 − 108(3.142)
= 308.664
≈ **309 cm^3** (correct to 3 sig. fig.)

(d) Let the radius of the hemisphere be *r* cm.
Vol. of hemisphere = Vol. of 3 balls

$\frac{2}{3}\pi r^3 = 108\pi$

$r^3 = \frac{108\pi \times 3}{2\pi}$

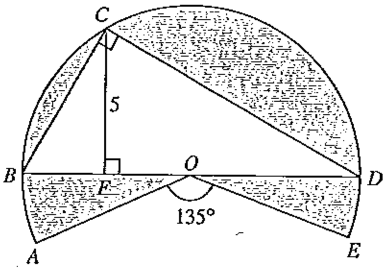
$= 162$

$r = \sqrt[3]{162}$

≈ 5.45 (correct to 3 sig. fig.)

∴ the radius of the hemisphere is **5.45 cm.**

13. (a)



Area of $\triangle BCD$ = 30 cm^2 Given

$\frac{1}{2} \times BD \times 5 = 30$

$BD = \frac{30 \times 2}{5}$

= 12 cm

Radius, $OB = \frac{12}{2} = 6$ cm

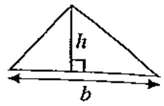
Area of shaded region

$= \begin{pmatrix}\text{Area of} \\ \text{sector} \\ OAE\end{pmatrix} - \begin{pmatrix}\text{Area of} \\ \triangle BCD\end{pmatrix}$

$= \left(\frac{360° - 135°}{360°} \times 3.142 \times 6^2\right) - 30$

= 40.695

≈ **40.7 cm^2** (correct to 3 sig. fig.)



Area of △

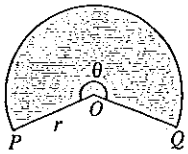
$= \frac{1}{2} \times b \times h$

b = base, *h* = height

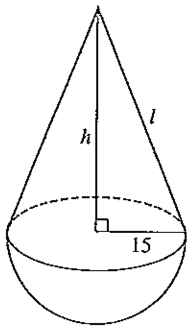
Area of sector *OPQ*

$= \frac{\theta}{360°} \times \pi r^2$

where *r* = radius.



(b) (i)



Radius of hemisphere = Radius of cone

$= \frac{30}{2} = 15$ cm

Let *h* be the height of the cone.
Vol. of solid = Vol. of cone + Vol. of hemisphere

$4950\pi = \frac{1}{3}\pi(15)^2h + \frac{2}{3}\pi(15)^3$

$= 75\pi h + 2250\pi$

$75\pi h = 2700\pi$

$h = \frac{2700\pi}{75\pi}$

= 36 cm

∴ the height of the cone is **36 cm.**

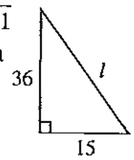
(ii) Let *l* be the slant height of the cone.
Using Pythagoras' Theorem,

$l^2 = h^2 + 15^2$

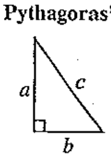
$l^2 = 36^2 + 15^2$

$l = \sqrt{1521}$

= 39 cm



Pythagoras' Theorem



$c^2 = a^2 + b^2$

Total surface area of solid

$= \begin{pmatrix}\text{Curved surface}\\ \text{area of cone}\end{pmatrix} + \begin{pmatrix}\text{Curved surface}\\ \text{area of hemisphere}\end{pmatrix}$

$= \pi(15)(39) + 2\pi(15)^2$

$= 585\pi + 450\pi$

$= 1035\pi$

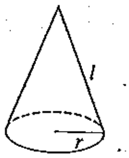
$= 1035(3.142)$

$= 3251.97$

$\approx$ **3252 cm²** (correct to the nearest cm²)

Teacher's Tip

Curved surface area of cone $= \pi rl$



Curved surface area of hemisphere $= 2\pi r^2$

14.

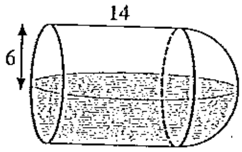


Diagram I

(a) Vol. of water in container

$= \frac{1}{2}\begin{pmatrix}\text{Vol. of water}\\ \text{in cylinder}\end{pmatrix} + \begin{pmatrix}\text{Vol. of water}\\ \text{in hemisphere}\end{pmatrix}$

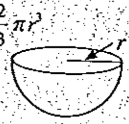
$= \frac{1}{2}\left[\pi(6)^2(14) + \frac{2}{3}\pi(6)^3\right]$

$= \frac{1}{2}(648\pi)$

$=$ **324π cm³**

Vol. of cylinder $= \pi r^2 h$

Vol. of hemisphere $= \frac{2}{3}\pi r^3$



(b) Required surface area

$= \frac{1}{2}\pi(6)^2 + \frac{1}{2}[2\pi(6)(14)] + \frac{1}{2}[2\pi(6)^2]$

$= 18\pi + 84\pi + 36\pi$

$= 138\pi$

$= 138(3.142)$

$= 433.596$

$\approx$ **434 cm²** (correct to the nearest cm²)

Required surface area

$\underbrace{\frac{1}{2}\pi r^2 + \frac{1}{2}(2\pi rh)}_{\text{Cylindrical part}} + \underbrace{\frac{1}{2}(2\pi r^2)}_{\text{Hemispherical part}}$

(c) (i)

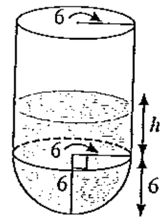


Diagram II

Vol. of water in the hemispherical part

$= \frac{2}{3}\pi(6)^3$

$= 144\pi$ cm³

Vol. of water in cylindrical part

$= \begin{pmatrix}\text{Vol. of water}\\ \text{in container}\end{pmatrix} - \begin{pmatrix}\text{Vol. of water}\\ \text{in hemispherical}\\ \text{part}\end{pmatrix}$

$= 324\pi - 144\pi$

$= 180\pi$ cm³

Let the depth of the water in the cylindrical part be h.

$\therefore\ \pi(6)^2 h = 180\pi$

$h = \frac{180\pi}{36\pi}$

$= 5$ cm

Depth of water in container

$= 5 + 6$

$=$ **11 cm**

(ii)

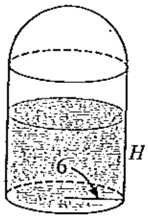


Diagram III

Let H be the depth of water in the container.

$\pi(6)^2 H = 324\pi$

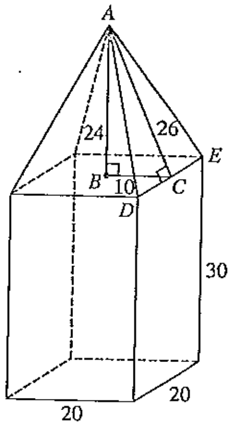
$H = \frac{324\pi}{36\pi}$

$= 9$ cm

Since the water only fills the cylindrical part of the container, use $\pi r^2 h$.

$\therefore$ the depth of water in the container is **9 cm**.

15.



(a) (i) Vol. of solid

= Vol. of pyramid + Vol. of cuboid

$= \left(\frac{1}{3} \times 20^2 \times 24\right) + (20 \times 20 \times 30)$

$= 3200 + 12\ 000$

$= \mathbf{15\ 200\ cm^3}$

Vol. of pyramid
$= \frac{1}{3} \times$ Base area $\times$ Height

Vol. of cuboid
= Length × Breadth × Height

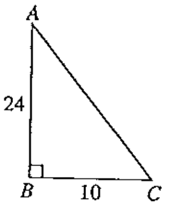
(ii) $BC = \frac{20}{2} = 10$ cm

Using Pythagoras' Theorem on $\triangle ABC$,

$AC^2 = 24^2 + 10^2$

$= 676$

$AC = \sqrt{676} = 26$ cm



Required total surface area

$= \left(\text{Surface area of sides}\right) + (\text{Surface area of tops})$

$= [4 \times (20 \times 30)] + \left[4 \times \left(\frac{1}{2} \times 20 \times 26\right)\right]$

$= 2400 + 1040$

$= \mathbf{3440\ cm^2}$

(b) Total surface area to be painted

$= 124 \times 3440$

$= 426\ 560\ \text{cm}^2$

No. of tins required

$= \frac{426\ 560\ \text{cm}^2}{60\ 000\ \text{cm}^2}$

$= 7.109$

∴ **8 tins** of paint are required.

1 m = 100 cm
$1\ \text{m}^2 = (100\ \text{cm})^2$
$= 10\ 000\ \text{cm}^2$
$\therefore 6\ \text{m}^2 = 60\ 000\ \text{cm}^2$

Test 16: Statistics – Measures of Central Tendency

Section A

1. (a) 4, 5, 5, 5, 7, 8, 12, 12, 16, 21

(two middle numbers: 7, 8)

Arrange the numbers in ascending order first.

(i) Mode = **5**

The **mode** is the number which occurs most often.
5 occurs the most often.

(ii) Median $= \frac{7+8}{2}$

$= \mathbf{7.5}$

- The **median** is the middle value of the numbers arranged in order.
- When there are two middle numbers, the median is the mean of the two numbers.

(iii) Mean $= \frac{4+5+5+5+7+8+12+12+16+21}{10}$

$= \frac{95}{10}$

$= \mathbf{9.5}$

The **mean** is obtained by dividing the sum of all the numbers by how many numbers there are.

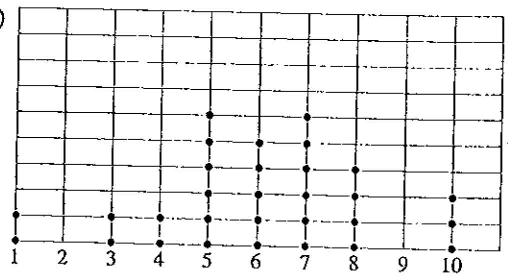
(b) Sum of the 10 numbers = 95

Sum of the 11 numbers = 11 × 10 = 110

∴ $x = 110 - 95$

$= \mathbf{15}$

2. (a)



Teacher's Tip

- In a dot diagram, the information is displayed using dots.
- Each dot represents a data.

(b) (i) Mode = **5 tomatoes and 7 tomatoes**

There are two modes. 5 and 7 are repeated 6 times.

(ii) Median = **6 tomatoes**

The middle term is the mean of the 15th and 16th term which are both 6.