

1. Which of the following functions are polynomials?

(a) $5\sqrt{x} + 3x^{-2}$

(d) $5x^{\frac{2}{3}} + 3x - 5$

(b) $x^3 + 5x^2 - 7x$

(e) $\frac{1}{x^3} + x$

(c) 9

(f) $3^x - x$

2. Consider the following polynomial $P(x) = (2x^5 + x + 1)(5 - 2x)$:

(a) What is the leading term?

(b) What is the coefficient of x^2 ?

(c) What is the constant term?

(d) What is the degree?

(e) Find the value of $P(-2)$.

(f) Is the polynomial monic, and why?

3. If $P(x) = 3x + 7$ and $Q(x) = x^3 - 4x + 2$: Find

(a) $P(x) + Q(x)$

(c) $Q(x) - P(x)$

(b) $P(x) - Q(x)$

(d) $Q(x)P(x)$

4. Find the cubic polynomial that has zeroes 0, 1 and 2, and in which the coefficient x^3 is 2.

5. Divide $4x^3 - 4x^2 + 7x + 14$ by $2x + 1$ and write out in terms of the dividend equalling the divisor times the quotient plus the remainder.

$$P(x) = D(x)Q(x) + R(x)$$

6. Find the remainder when $P(x) = 4x^3 - 7x + 2$ is divided by $x + 2$, using

(a) long division

(b) the remainder theorem

7. Find the value of k if $x - 1$ is a factor of $x^3 - 3x^2 + kx - 2$.

8. Solve the equations

(a) $x^3 + 2x^2 - 9x - 18 = 0$

(b) $2x^4 + 11x^3 + 19x^2 + 8x - 4 = 0$

9. Sketch the following polynomials by showing any x -intercepts and y -intercept:

(a) $y = x^2(x - 3)(x + 3)$

(b) $y = (x + 2)^2(x - 1)^3$

(c) $y = x^3(x + 3)(2 - x)$

(d) $y = -x(x - 3)^2(x - 6)^3$

10. Find the value of a if $3x^4 + ax^2 - 2$ is divisible by $x + 1$.

11. The polynomial $P(x) = x^4 - 2x^3 + ax + b$ has remainder 3 after division with $x - 1$ and has remainder -5 after division by $x + 1$. Find a and b .

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POLYNOMIALS

b) c) ✓

$$\begin{aligned}
 1. \quad P(x) &= (2x^5 + x + 1)(5 - 2x) \\
 &= 10x^5 - 4x^6 + \boxed{5x} - 2x^2 \\
 &\quad + 5 \boxed{(-2x)} \\
 &= -4x^6 + 10x^5 - 2x^2 + 3x + 5 \quad \checkmark
 \end{aligned}$$

a) leading term = $-4x^6$ ✓

b) coefficient of $x^2 = -2$ ✓

c) constant term = 5 ✓

d) degree = 6 ✓

$$\begin{aligned}
 e) \quad P(-2) &= -4(-2)^6 + 10(-2)^5 - 2(-2)^2 \\
 &\quad + 3(-2) + 5 \\
 &= -256 + (-320) - 8 - 6 + 5 \\
 &= -585 \quad \checkmark
 \end{aligned}$$

f) non-monic as coefficient of leading term $\neq 1$ ✓

3. $P(x) = 3x + 7$ $Q(x) = x^3 - 4x + 2$

a) $P(x) + Q(x)$
 $= 3x + 7 + x^3 - 4x + 2$
 $= x^3 - x + 9$ ✓

b) $P(x) - Q(x)$
 $= 3x + 7 - (x^3 - 4x + 2)$
 $= 3x + 7 - x^3 + 4x - 2$
 $= -x^3 + 7x + 5$ ✓

c) $Q(x) - P(x)$
 $= x^3 - 4x + 2 - (3x + 7)$
 $= x^3 - 4x + 2 - 3x - 7$
 $= x^3 - 7x - 5$ ✓

d) $Q(x)P(x)$
 $= (x^3 - 4x + 2)(3x + 7)$
 $= 3x^4 + 7x^3 - 12x^2 - 28x$
 $\quad + 6x + 14$
 $= 3x^4 + 7x^3 - 12x^2 - 22x + 14$ ✓

4. zeroes $x = 0, 1, 2$

coefficient $x^3 = 2$

$$y = 2x(x-1)(x-2) \quad \checkmark$$

$$\begin{array}{r}
 2x^2 - 3x + 5 \\
 2x + 1 \overline{) 4x^3 - 4x^2 + 7x + 14} \\
 \underline{4x^3 + 2x^2} \\
 -6x^2 - 7x \\
 \underline{-6x^2 - 3x} \\
 10x + 14 \quad \checkmark \\
 \underline{10x + 25} \\
 -29 \quad \checkmark
 \end{array}$$

$$\begin{aligned}
 4x^3 - 4x^2 + 7x + 14 \\
 = (2x + 1)(2x^2 - 3x + 5) + 9 \quad \checkmark
 \end{aligned}$$

$$\begin{array}{r}
 4x^2 - 8x + 9 \\
 x + 2 \overline{) 4x^3 - 7x + 2} \\
 \underline{4x^3 + 8x^2} \\
 -8x^2 - 7x \\
 \underline{-8x^2 - 16x} \\
 9x + 2 \quad \checkmark \\
 \underline{9x + 18} \\
 -16 \quad \checkmark
 \end{array}$$

$$\begin{aligned}
 P(-2) &= 4(-2)^3 - 7(-2) + 2 \\
 &= 4x - 8 + 14 + 2 \\
 &= -32 + 14 + 2 \\
 &= -16 \quad \checkmark
 \end{aligned}$$

$$\text{let } P(x) = 2x^4 + 11x^3 + 19x^2 + 8x - 4$$

7. $(x-1)$ factor of
 $P(x) = x^3 - 3x^2 + kx - 2$

$$P(1) = 0$$

$$\begin{aligned} P(1) &= 1^3 - 3(1) + k - 2 \\ &= 1 - 3 + k - 2 \\ &= -2 + k - 2 \\ &= -4 + k \end{aligned}$$

$$-4 + k = 0$$

$$k = 4 \quad \checkmark$$

8a) Solve

$$x^3 + 2x^2 - 9x - 18 = 0$$

Test factors of -18

$$\begin{aligned} x=2 \quad 2^3 + 2 \cdot 2^2 - 9(2) - 18 &= \\ 8 + 8 - 18 - 18 &= -40 \end{aligned}$$

$$\begin{aligned} x=-2 \quad (-2)^3 + 2 \cdot (-2)^2 - 9(-2) - 18 &= \\ -8 + 8 + 18 - 18 &= 0 \end{aligned}$$

factor $x+2$ \checkmark

$$x^2 - 9$$

$$\begin{array}{r} +2 \overline{) x^3 + 2x^2 - 9x - 18} \\ \underline{x^3 + 2x^2} \\ -9x - 18 \\ \underline{-9x - 18} \\ 0 \end{array}$$

$$x^3 + 2x^2 - 9x - 18 = 0 \quad \checkmark$$

$$(x+2)(x^2 - 9) = 0$$

$$(x+2)(x-3)(x+3) = 0$$

$$x = -2, x = 3, x = -3 \quad \checkmark$$

8b) $2x^4 + 11x^3 + 19x^2 + 8x - 4 = 0$

test factors of -4

$$x=1$$

$$2 \cdot 1 + 11 + 19 + 8 - 4 \neq 0$$

$$x=-1$$

$$\begin{aligned} 2 \cdot 1^4 + 11(-1)^3 + 19(1)^2 + 8(-1) - 4 &= \\ = 2 - 11 - 19 - 8 - 4 &= \\ = -9 - 19 - 8 - 4 &= \\ \neq 0 & \end{aligned}$$

$$x=2$$

$$\begin{aligned} 2 \cdot (-2)^4 + 11(-2)^3 + 19(-2)^2 + 8(-2) - 4 &= \\ = 2 \cdot 16 + 11(-8) + 76 - 16 - 4 &= \\ = 32 - 88 + 76 - 16 - 4 &= \\ = -56 + 76 - 16 - 4 &= \\ = 0 & \end{aligned}$$

factor $x+2$ \checkmark

$$\begin{array}{r} 2x^3 + 7x^2 + 5x - 2 \\ x+2 \overline{) 2x^4 + 11x^3 + 19x^2 + 8x - 4} \\ \underline{2x^4 + 4x^3} \\ 7x^3 + 19x^2 \\ \underline{7x^3 + 14x^2} \\ 5x^2 + 18x \\ \underline{5x^2 + 10x} \\ -2x - 4 \\ \underline{-2x - 4} \\ 0 \end{array}$$

let

$$Q(x) = 2x^3 + 7x^2 + 5x - 2$$

test factors of -2

$$\begin{aligned} Q(2) &= 2 \cdot 2^3 + 7 \cdot 2^2 + 5 \cdot 2 - 2 \\ &= 16 + 28 + 10 - 2 \\ &\neq 0 \end{aligned}$$

$$Q(-2) = 2(-2)^3 + 7(-2)^2 + 5(-2) - 2 = 0 \quad \checkmark$$

$$2x^2 + 3x - 1$$

$$\begin{array}{r} x+2 \overline{) 2x^3 + 7x^2 + 5x - 2} \\ \underline{2x^3 + 4x^2} \\ 3x^2 + 5x \\ \underline{3x^2 + 6x} \\ -x - 2 \\ \underline{-x - 2} \\ 0 \end{array}$$

$$2x^2 + 3x - 1 \quad \checkmark$$

$$\begin{aligned} \Delta &= b^2 - 4ac & a &= 2 \\ &= 3^2 - 4 \cdot 2 \cdot (-1) & b &= 3 \\ &= 9 + 8 & c &= -1 \\ &= 17 \end{aligned}$$

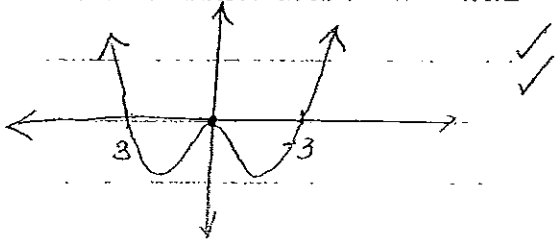
$$\begin{aligned} x &= \frac{-b \pm \sqrt{\Delta}}{2a} \\ &= \frac{-3 \pm \sqrt{17}}{4} \quad \checkmark \end{aligned}$$

$$x = -2, \quad \frac{-3 \pm \sqrt{17}}{4}$$

$$a) \quad y = x^2(x-3)(x+3)$$

$$y=0 \quad x=0, 3, -3$$

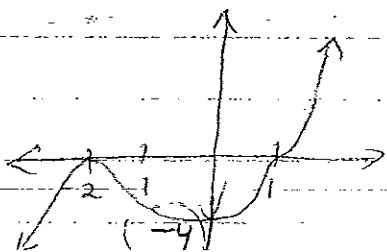
double root $x=0$



$$b) \quad y = (x+2)^2(x-1)^3$$

double root $x = -2$ triple root $x = 1$

$$x=0$$



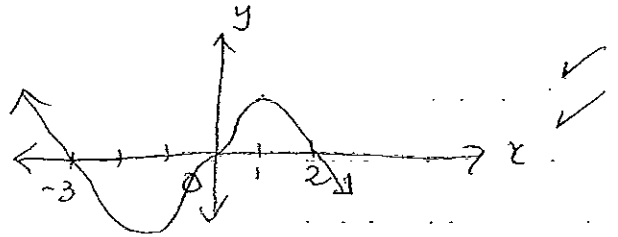
$$\begin{aligned} x &= 0 \\ y &= 2^2 \cdot (-1)^3 \\ &= 4 \cdot (-1) \\ &= -4 \end{aligned}$$

$$c) \quad y = x^3(x+3)(2-x)$$

$$y=0$$

$$x=0, -3, 2$$

triple root at $x=0$



$$x=1$$

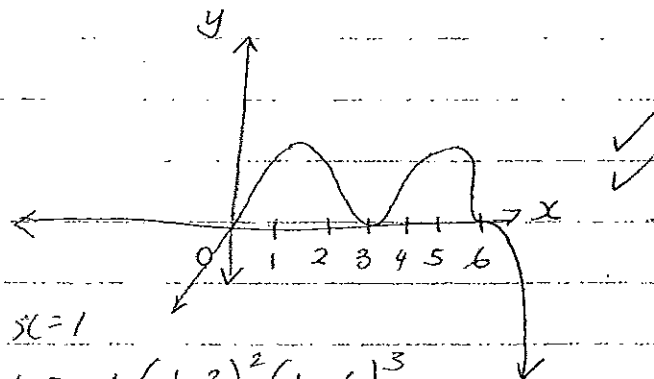
$$\begin{aligned} y &= 1^3(1+3)(2-1) \\ &= 1 \cdot 4 \cdot 1 \end{aligned}$$

$$d) \quad y = -x(x-3)^2(x-6)^3$$

$$y=0 \quad x=0, 3, 6$$

double root at $x=3$

triple root at $x=6$



$$x=1$$

$$\begin{aligned} y &= -1(1-3)^2(1-6)^3 \\ &= -1 \cdot 2^2 \cdot (-5)^3 \\ &= -1 \cdot 4 \cdot -125 \\ &= 500 \\ &> 0 \end{aligned}$$

4

$$P(x) =$$

$$10. 3x^4 + ax^2 - 2$$

factor $x+1$

$$\therefore P(-1) = 0$$

$$3(-1)^4 + a(-1)^2 - 2$$

$$3 + a - 2 = 0$$

$$a + 1 = 0$$

$$a = -1 \quad \checkmark$$

$$11. P(1) = 3$$

$$P(-1) = -5$$

$$P(x) = x^4 - 2x^3 + ax + b$$

$$P(1) = 1^4 - 2(1)^3 + a(1) + b$$

$$= 1 - 2 + a + b$$

$$= -1 + a + b$$

$$\underline{-1 + a + b = 3}$$

$$\underline{a + b = 4} \quad (1)$$

$$P(-1) = (-1)^4 - 2(-1)^3 + a(-1) + b$$

$$= 1 + 2 - a + b$$

$$1 + 2 - a + b = -5$$

$$3 - a + b = -5$$

$$\underline{-a + b = -8} \quad (2)$$

$$\text{①} \quad \underline{a + b = 4} \quad (1) + (2)$$

$$\text{sub } \underline{1a + b = 4} \quad (2)$$

$$2b = -4$$

$$b = -2 \quad \checkmark$$

$$a = 6 \quad \checkmark$$