

Logarithms

Logarithms, shortened to **logs**, are indices with special **base** numbers.


These special numbers are usually either:

base **10** which produces **common** logarithms.

On calculators such as the Texas Instruments TI-30X IIB, common logs are given by the button .

OR

base **e** a special number, which produces **natural** logarithms.

On the TI-30X IIB calculator, natural logs are given by the button .

The exponential function $y = e^x$ is studied in more detail in **Topic 24**.

In the days before calculators and computers, tables of logarithms were used to carry out tedious numerical calculations such as long multiplication and division.

The basic definition of a logarithm is:

If $b^a = c$ then $\log_b c = a$

What it means is that for every **index** statement there is an equivalent **logarithmic** statement
and
conversely
for every **logarithmic** statement there is an equivalent **index** statement.

e.g.

Index statement	Log statement
$100 = 10^2$	$\log_{10} 100 = 2$
$81 = 3^4$	$\log_3 81 = 4$
$32 = 2^5$	$\log_2 32 = 5$
$9 = 3^2$	$\log_3 9 = 2$

The calculator buttons **log** and **ln** only find logs to base 10 and e respectively.

Properties of Logarithms

Logarithms, to any base, have three main properties.

Property	Description	Example
$\log(ab) = \log a + \log b$	The log of two terms being multiplied together equals the sum of the logs of the two terms.	Simplify $\log 4 + \log 5$ $= \log 4 \times 5$ $= \log 20$
$\log \frac{a}{b} = \log a - \log b$	The log of two terms being divided equals the difference of the logs of the two terms	Simplify $\log 64 - \log 16$ $= \log 64/16$ $= \log 4$
$\log a^n = n \log a$	The log of a number to a power equals the log of the number multiplied by the power,	Write $3 \log 5$ as the log of a single number $3 \log 5 = \log 5^3 = \log 125$
$\log_a a = 1$	The log of a number with the same base equals 1.	$\log_{10} 10 = 1$ this is because $10^1 = 10$
$\log_a 1 = 0$	The log of 1 to any base is 0	$\log_{10} 1 = 0$

Using logs to solve index or exponential equations

Simple index equations such as $4^x = 64$ can be solved using the properties of exponents.

$$4^x = 64$$

$$4^x = 4^3$$

$$\text{Therefore } x = 3$$

[Click here](#) for a further example ($3^x = 81$)

Logarithms can be used to solve more complex equations involving indices, which are not able to be solved using other methods.

e.g. $3^x = 10$. (There is no conventional way to solve this equation.)

The method used is to "take logs" of both sides of the equation.

$$3^x = 10$$

$$\log(3^x) = \log(10) \quad \text{taking logs of both sides}$$

$$x \log 3 = \log 10 \quad \text{using property of logs}$$

$$x = \frac{\log 10}{\log 3} \quad \text{dividing both sides by } \log 3$$

$$x = \frac{1}{0.4771} \quad \text{from calculator}$$

$$x = 2.096 \quad (\text{to 4 significant figures})$$

[Click here](#) for a further example ($3^x = 80$).

Two more difficult examples:

Example 1	Example 2
$3^x = 10.2^x$ (remember the "." means multiply)	$2.3^{3x+2} = 50$ (remember the "." means multiply)
Click here	Click here



Logarithms

1. Write the equivalent logarithmic statement:

a. $5^3 = 125$

b. $3^4 = 81$

c. $2^3 = 8$

d. $6^3 = 216$

e. $5^{-1} = \frac{1}{5}$

f. $1000 = 10^3$

2. Write the equivalent index statement:

a. $\log_2 64 = 6$

b. $\log_3 243 = 5$

c. $\log_2 \frac{1}{8} = -3$

d. $\log_3 9 = 2$

e. $\log_{25} 125 = 1.5$

f. $\log_3 \frac{1}{3} = -1$

3. Calculate the following:

a. $\log_{10} 100$

b. $\log_9 3$

c. $\log_{625} 125$

d. $\log_{16} 32$

e. $\log_{0.1} 10$

4. Solve the following equations for x:

a. $\log_2 x = 6$

b. $\log_3 x = 4$

c. $\log_x 3 = 1$

d. $\log_3 27 = x + 2$

e. $\log_5 25 = 3x + 1$

5. Write as the log of a single number:

a. $\log 6 + \log 5$

b. $\log 21 - \log 7$

c. $\log 12 - \log 2 + \log 6$

d. $\log 54 - (\log 9 + \log 6)$

e. $2\log 6 + 5\log 2$

f. $3\log 5 + 3\log 2 - \log 10$

g. $2\log 10 + \frac{1}{2}\log 25 - 3\log 5$

h. $\frac{4}{3}\log 64 + \frac{1}{3}\log 27$

i. $\log a + 3\log b - 2\log c$

6. Evaluate:

a. $2\log_6 6$

b. $\log_2 16 + \log_2 4$

7. Solve the following index equations:

a. $2^x = 128$

b. $2^{x+1} = 32$

c. $9^x = 3$

d. $64^x = 32$

8. Solve the following index equations (give the answers to 3 decimal places):

a. $10^x = 9$

b. $3^x = 4$

c. $4^{2x} = 5^{x+2}$

d. $3.6^{4x+5} = 2.9^{2x+13}$



Logarithms

1.	a. $\log_5 125 = 3$	b. $\log_3 81 = 4$	c. $\log_2 8 = 3$
	d. $\log_6 216 = 3$	e. $\log_5 \frac{1}{5} = -1$	f. $\log_{10} 1000 = 3$

2.	a. $2^6 = 64$	b. $3^5 = 243$	c. $2^{-3} = \frac{1}{8}$
	d. $3^2 = 9$	e. $25^{1.5} = 125$	f. $3^{-1} = \frac{1}{3}$

3.	a. 2	b. 0.5	c. 3/4	d. 5/4	e. -1
4.	a. 64	b. 81	c. 3	d. 1	e. 1/3

5.	a. $\log 30$	b. $\log 3$	c. $\log 36$
	d. 0	e. $\log 1152$	f. $\log 100$
	g. $\log 4$	h. $\log 768$	i. $\log \frac{ab^3}{c^2}$

6.	a. 2	b. 6	
7.	a. $x = 7$	b. $x = 4$	c. $x = 0.5$ d. $x = 5/6$

8.	a. $x = 0.954$	b. $x = 1.262$	c. $x = 2.767$	d. $x = 2.44$
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Logarithms

Unit Test #5

Select your answers to the following 10 questions from the pop-up menus in the right hand column. When you are satisfied with your answers, fill in your name in the space provided below the test, and click the "Submit Test" button. Clicking the "Begin Test Again" button will clear all the answers.

Q1:	$\log_5 25 =$	A. 2 B. 5 C. 25 D. 125	Answer 1:	<input type="text"/>
Q2:	Solve the equation $\log_3 x = \frac{p-1}{2}$	A. 6 B. 1.5 C. 9 D. 2/3	Answer 2:	<input type="text"/>
Q3:	Simplify $\log 8 + \log 2$	A. $\log 16$ B. $\log 4$ C. $\log 64$ D. $\log 0.25$	Answer 3:	<input type="text"/>
Q4:	Simplify $\log 8 - \log 2$	A. $\log 16$ B. $\log 4$ C. $\log 64$ D. $\log 0.25$	Answer 4:	<input type="text"/>
Q5:	$3\log 2 =$	A. $\log 6$ B. $\log 9$ C. $\log 8$ D. $\log 1.5$	Answer 5:	<input type="text"/>
Q6:	$\log_a a =$	A. 0 B. a C. 1 D. a^2	Answer 6:	<input type="text"/>
Q7:	Simplify $\frac{\log 64}{\log 4}$	A. 3 B. $\log 16$ C. 16 D. $\log 3$	Answer 7:	<input type="text"/>
Q8:	Solve $2^x = 32$	A. 16 B. 5 C. 4 D. 6	Answer 8:	<input type="text"/>
Q9:	Solve $4^x = 8$	A. 2 B. 1.5 C. 0.5 D. 4	Answer 9:	<input type="text"/>

Q10:	$\log_x 1 =$	A. 1 B. x C. 0 D. 1/x	Answer 10:	<input type="checkbox"/>
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Enter your initial
and surname here:

Submit Test

Begin Test Again