36 Polynomials

Question 1 For each of the following polynomials, state the degree and the leading coefficient of the polynomial:

(a) $2x^3 - x^2 + 5$

(b) $5x^4 - x^3 + x + 4$

(c) $7 - x^2 - 4x^3 - 6x^5$

(d) $\frac{x^2}{3} + 2x + 7$

(e) $3 - x + x^2 + x^3$

(f) $3 + x^2 - 2x^4 - \frac{x^6}{10}$

Question 2 Carry out the following additions and subtractions:

(a) $5x^3 - x^2 + 7x + 3$ + $2x^3 - 3x^2 - 4x - 5$

(b) $7x^3 - 8x^2 + x - 3$ − $2x^3 - 3x^2 - 4x + 5$

(c) $(9x^4 + 8x^2 + x + 4) + (5x^4 + 3x^3 + 5)$

(d) $(7x^4 - 3x^3 + x^2 - 5) - (x^4 + x^3 - 3x)$

(e) $(3x^3 + 2x^2 + x) + (2x^3 - x^2 - 5) - (4x^3 + x + 7)$

(f) $(5x^4 + x^3 - 7x^2 + x + 4) - (2x^3 + x^2 + 4) + (x^4 - x^2 - 2x + 1)$

Question 3 Carry out the following multiplications:

(a) $(x^2 - 5x + 2)(2x^2 + x)$

(b) $(2x + 1)(x^3 + 8x^2 - 7x + 2)$

(c) $(3 - 7x - x^2)(4 + x + x^3)$

(d) $(x^2 + 8x + 3)(2x^4 + 3x^3 - x + 4)$

Question 4 Carry out the following divisions:

(a) $x-1\overline{)x^3+2x^2+5x-1}$

(b) $x+2\overline{)3x^4-7x^3+8x^2-2x-5}$

(c) $x-3\overline{)5x^3-7x^2+x+7}$

(d) $2x+1\overline{)4x^3+6x^2-8x+2}$

(e) $x^2+5\overline{)7x^4-3x^3+6x^2+7x-2}$

(f) $x^2-7x+1\overline{)x^3+8x^2-3x+2}$

(g) $\left(5x^3+7x^2+2x-3\right)\div\left(x-2\right)$

(h) $\left(7x^5+x^4-2x^3+x+3\right)\div\left(x+1\right)$

Question 5 Use the remainder theorem to find the remainder when $P(x)$ is divided by $A(x)$.

(a) $P(x) = x^3 - 3x^2 + 8x + 1$ $A(x) = x - 1$

(b) $P(x) = 4x^2 - 8x + 2$ $A(x) = x + 2$

(c) $P(x) = 6x^4 - x^3 + x^2 - 5$ $A(x) = x - 3$

(d) $P(x) = 7 - x + 2x^2 + x^3 - x^4$ $A(x) = x + 1$

(e) $P(x) = 4x^3 - 8x^2 + 7x - 3$ $A(x) = x - 2$

(f) $P(x) = x^5 + 2x^4 - 3x^3 - x^2 + x - 7$ $A(x) = x - 2$

Question 6 Use the factor theorem to show that the linear polynomial is a factor of $P(x)$.

(a) $P(x) = x^3 + 2x^2 - x - 2$, $x + 2$

(b) $P(x) = 6x^3 - 17x^2 + 6x + 8$, $x - 2$

Question 7 Use the factor theorem to factorise the following polynomials completely:

(a) $P(x) = x^3 - 2x^2 - 5x + 6$

(b) $P(x) = 2x^4 - 4x^3 - 3x^2 - 6x - 9$

(c) $P(x) = 18 + 27x + x^2 - 4x^3$

(d) $P(x) = 10x^4 - x^3 - 37x^2 + 16x + 12$

Question 8 SYNTHETIC DIVISION

For example: $\left(3x^3-2x^2+7x-20\right)\div\left(x-2\right)$

In synthetic division, only the coefficients of the dividend and the constant of the divisor are used.

The coefficients are written down in a row, with a vertical line drawn left of the leading coefficient and a horizontal line drawn leaving a vacant row.

That is:

	3	–2	7	–20

Next, the opposite of the divisor's constant is written down left of the vertical line.

2	3	–2	7	–20

Next, the leading coefficient is transferred to below the horizontal line directly beneath itself.

2	3	–2	7	–20
	3			

We now multiply the 2 and 3, write the product beneath the second coefficient (–2), and add it to the second coefficient.

2	3	–2	7	–20
		6		
	3	4		

This sum is multiplied by the 2, written beneath the next coefficient (7), and is added to it.

2	3	–2	7	–20
		6	8	
	3	4	15	

This sum is multiplied by the 2, written beneath the next coefficient (–20), and is added to it.

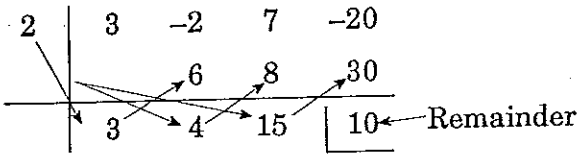
2	3	–2	7	–20
		6	8	30
	3	4	15	10

Since we have run out of coefficients, we stop multiplying. All that remains is to interpret the numbers below the horizontal line.

The last digit is the remainder and the other digits are the coefficients of powers of x, starting with one degree less than the dividend.

Thus, the quotient is $3x^2 + 4x + 15$ and the remainder is 10.

That is,



Use synthetic division to divide the polynomial $P(x)$ by the linear polynomial in each case:

(a) $P(x) = 2x^3 + 3x^2 - 8x - 2$, $x - 1$

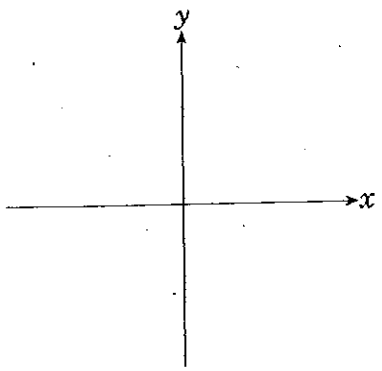
(b) $P(x) = 5x^4 - 3x^3 + 8x^2 + 7x - 6$, $x + 2$

(c) $P(x) = x^3 + 8x^2 - 15x - 3$, $x - 3$

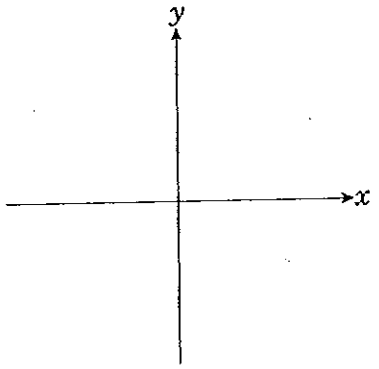
(d) $P(x) = x^5 - 3x^3 + 8x^2 + x + 1$, $x + 1$

Question 9 **Sketch the following polynomial functions:**

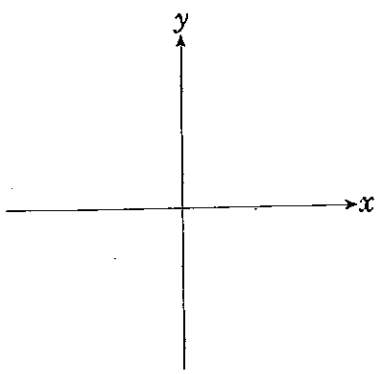
(a) $y = (x+1)(x-2)(x-3)$



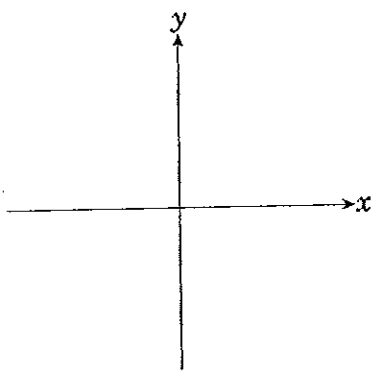
(b) $y = (x+3)(x+1)(x-2)(x-5)$



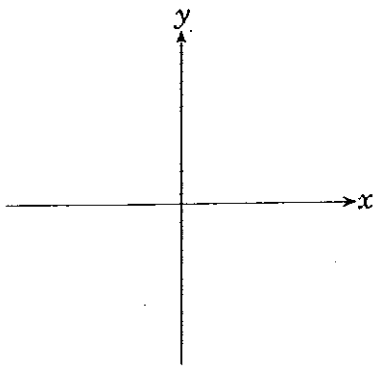
(c) $y = (x+2)(x-1)(4-x)$



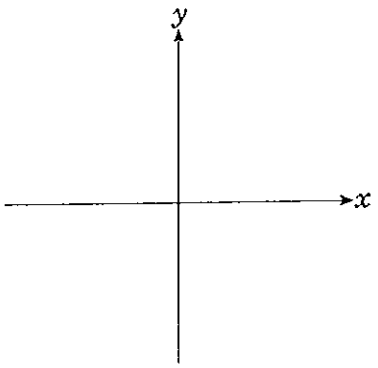
(d) $y = x(x-1)(x-2)(x-3)$



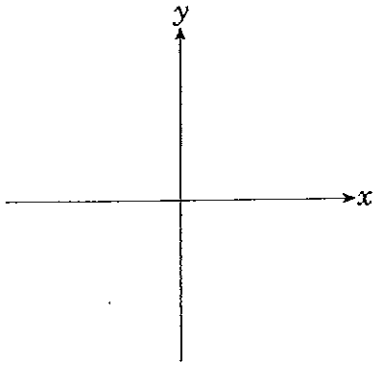
(e) $y = (x+2)^2(x-1)(x-4)^3$



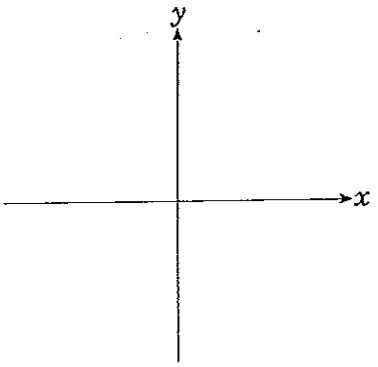
(f) $y = x^2(x+3)^3(x-1)(x-3)^2$



(g) $y = x^2(x+2)^3(4-x)$

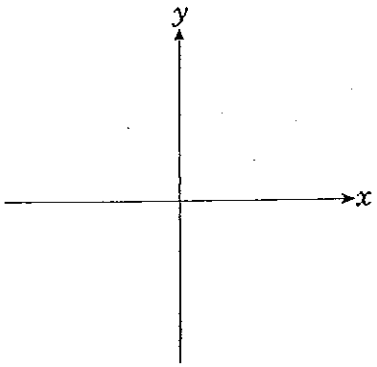


(h) $y = (x+3)(x-1)(x-2)^2$

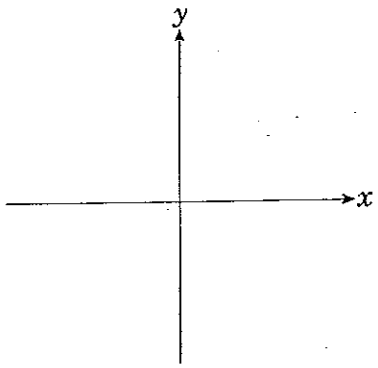


Question 10 Sketch the following polynomial functions by first factorising them using the factor theorem:

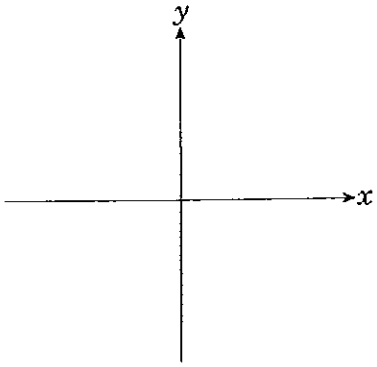
(a) $y = x^3 + 8x^2 + 17x + 10$



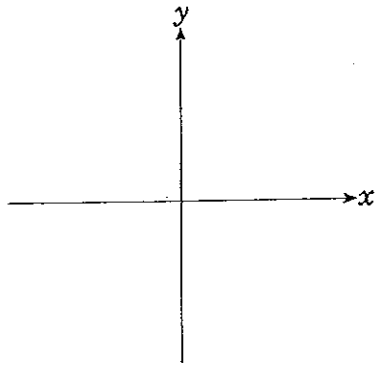
(b) $y = x^4 - 2x^3 - 15x^2 - 4x + 20$



(c) $y = x^4 - x^3 - 10x^2 - 8x$



(d) $y = x\left(x^4 - 17x^2 + 16\right)$



Question 11

Find the values of a and b if $(x-1)$ and $(x-2)$ are factors of

$$P(x) = 2x^3 - 5ax^2 + bx + 14.$$

Question 12

Find the values of m and n if $(x-2)$ is a factor of $P(x) = 6x^3 - 5mx^2 + 7nx + 10$ and there is a remainder of 10 when $P(x)$ is divided by $(x-3)$.

(d) $x = 95°$, $y = 93°$
(e) $a = 100°$, $b = 160°$
(f) $m = 50°$, $n = 65°$

2 (a) 10 cm (b) 8 cm (c) 24 cm

3 and 4 Proofs

5 (a) 42 cm (b) $x = 15$ cm, $y = 68°$

6 Proofs

7 (a) 36° (b) $a = 58°$, $b = 58°$ $c = 58°$, $d = 39°$

8 Proofs

9 (a) 3 cm (b) 6 cm

10 Proofs

34 Logarithms

1 (a) $x = \frac{3}{2}$ (b) $x = \frac{2}{3}$ (c) $x = -2$
(d) $x = \frac{5}{2}$ (e) $x = \frac{5}{4}$ (f) $x = \frac{7}{9}$
(g) $x = \frac{2}{9}$ (h) $x = 0$ (i) $x = \frac{7}{5}$

2 (a) $\log_2 64 = 6$ (b) $\log_{10} 100 = 2$
(c) $\log_5 125 = 3$ (d) $\log_2 3 = x$
(e) $\log_{10} 5 = x$ (f) $\log_4 15 = x$

3 (a) 4 (b) 2 (c) 4 (d) $\frac{3}{2}$
(e) $\frac{3}{2}$ (f) $\frac{5}{3}$ **4** (a) 5 (b) 3
(c) $\frac{3}{2}$ (d) 5 (e) $\frac{5}{2}$ (f) 2
(g) 4096 (h) 128 (i) $12\frac{1}{4}$

5 (a) 2 (b) 3 (c) 2 (d) 3
(e) 1 (f) 2

6 (a) $x = 14$ (b) $x = \frac{9}{8}$ (c) $x = \frac{1}{4}$
(d) $x = 7$ (e) $x = 4$ (f) $x = 3$

7 (a) 1·7712 (b) 1·6826 (c) 2·1699
(d) 2·5850 (e) 38·5682 (f) 5·3013

35 Functions and mappings

1 (a) function (b) function (c) non-function
(d) non-function (e) function (f) function

2 (a) 4 (b) 5 (c) 8 (d) 13

3 (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{1+a}$ (d) $\frac{x}{x+1}$

4 Proof

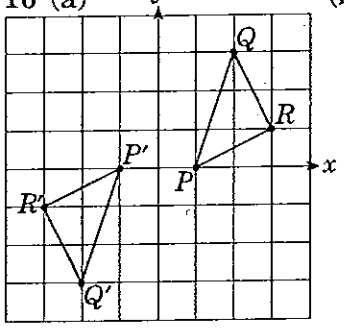
5 (a) 2 (b) 4 (c) 2^{a+1} (d) $\frac{1}{4}$

6 (a) 7 (b) $2x^2 + 4xh + 2h^2 - x - h + 1$
(c) $2h^2 + 4xh - h$ (b) $2h + 4x - 1$

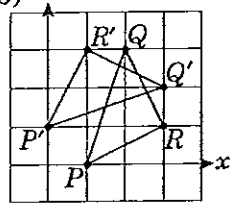
7 (a) $9x^2 - 12x + 3$ (b) $3x^2 - 5$

8 (a) $\frac{1}{3}x - \frac{4}{3}$ **9** (a) $\frac{3x-2}{7}$
(b) $-\frac{2}{7}$ (c) $\frac{4}{7}$ (d) $-\frac{8}{7}$

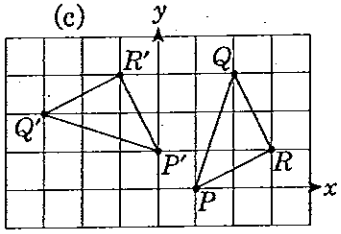
10 (a)



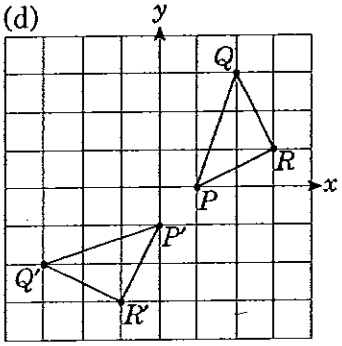
(b)



(c)



(d)



11 (a) 3 (b) 9 (c) 9 (d) 9 (e) −1 (f) 1

36 Polynomials

1 (a) deg. = 3, l.c. = 2 (b) deg. = 4, l.c. = 5
(c) deg. = 5, l.c. = −6 (d) deg. = 2, l.c. = $\frac{1}{3}$
(e) deg. = 3, l.c. = 1 (f) deg. = 6, l.c. = $-\frac{1}{10}$

2 (a) $7x^3 - 4x^2 + 3x - 2$
(b) $5x^3 - 5x^2 + 5x - 8$
(c) $14x^4 + 3x^3 + 8x^2 + x + 9$
(d) $6x^4 - 4x^3 + x^2 + 3x - 5$
(e) $x^3 + x^2 - 12$
(f) $6x^4 - x^3 - 9x^2 - x + 1$

3 (a) $2x^4 - 9x^3 - x^2 + 2x$
(b) $2x^4 + 17x^3 - 6x^2 - 3x + 2$
(c) $12 - 25x - 11x^2 + 2x^3 - 7x^4 - x^5$
(d) $2x^6 + 19x^5 + 30x^4 + 8x^3 - 4x^2 + 29x + 12$

4 (a) $x^2 + 3x + 8 + \frac{7}{x-1}$
(b) $3x^3 - 13x^2 + 34x - 70 + \frac{135}{x+2}$
(c) $5x^2 + 8x + 25 + \frac{82}{x-3}$
(d) $2x^2 + 2x - 5 + \frac{7}{2x+1}$
(e) $7x^2 - 3x - 29 + \frac{22x+143}{x^2+5}$
(f) $x + 15 + \frac{101x-13}{x^2-7x+1}$
(g) $5x^2 + 17x + 36 + \frac{69}{x-2}$
(h) $7x^4 - 6x^3 + 4x^2 - 4x + 5 - \frac{2}{x+1}$

5 (a) $P(1) = 7$ (b) $P(-2) = 34$
(c) $P(3) = 463$ (d) $P(-1) = 8$
(e) $P(2) = 11$ (f) $P(2) = 31$

6 (a) $P(-2) = 0$ (b) $P(2) = 0$

7 (a) $(x-1)(x-3)(x+2)$

(b) $(x+1)(x-3)(2x^2+3)$

(c) $-(x+2)(x-3)(4x+3)$

(d) $(x-1)(x+2)(2x-3)(5x+2)$

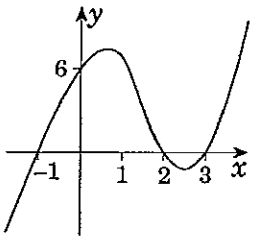
8 (a) $2x^2+5x-3-\frac{5}{x-1}$

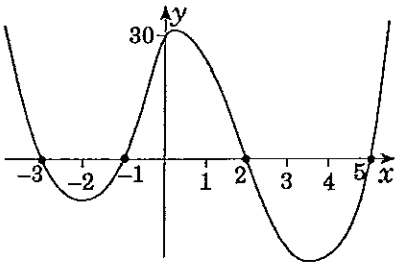
(b) $5x^3-13x^2+34x-61+\frac{116}{x+2}$

(c) $x^2+11x+18+\frac{51}{x-3}$

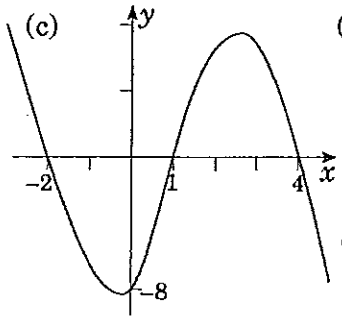
(d) $x^4-x^3-2x^2+10x-9+\frac{10}{x+1}$

9 (a) (b)

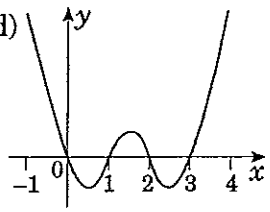




(c)



(d)



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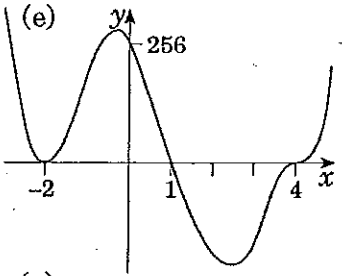
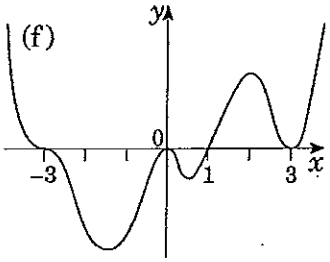
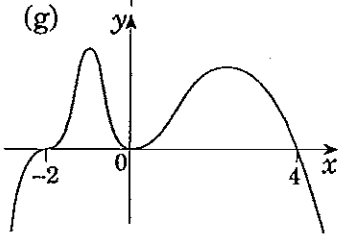


(f)



(g)



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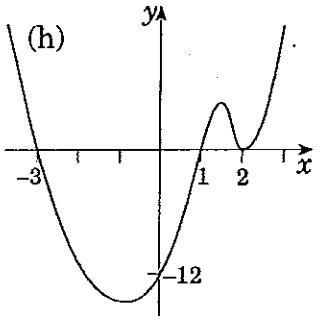
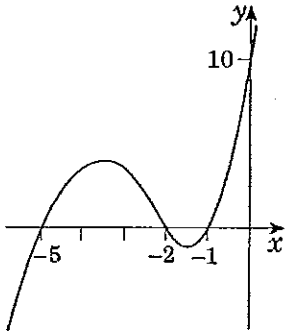
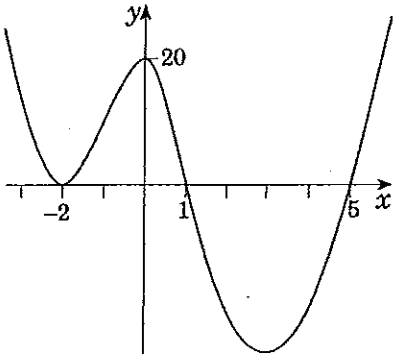
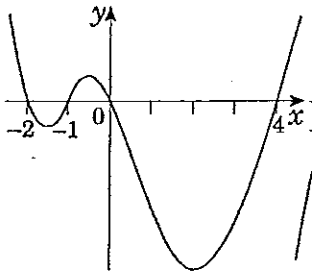


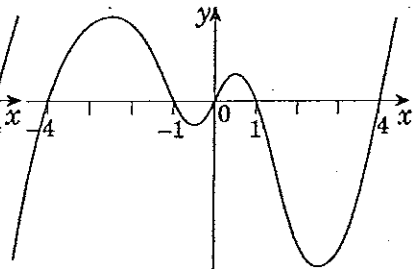
10 (a) $(x+1)(x+2)(x+5)$ (b) $(x-1)(x-5)(x+2)^2$





(c) $x(x+1)(x+2)(x-4)$





(d) $x(x-1)(x+1)(x-4)(x+4)$

11 $a=-\frac{1}{5}$, $b=-17$ **12** $m=5$, $n=3$

37 Matrices

1 (a) $\begin{bmatrix}11\\7\end{bmatrix}$ (b) $\begin{bmatrix}2&-3\\4&5\end{bmatrix}$ (c) $\begin{bmatrix}5&7\\10&9\end{bmatrix}$ (d) $\begin{bmatrix}1&1\end{bmatrix}$

(e) $\begin{bmatrix}8&3\\2&3\\0&4\end{bmatrix}$ (f) $\begin{bmatrix}2&7&10\\7&15&7\\10&7&7\end{bmatrix}$ (g) $\begin{bmatrix}5&18\\12&10\end{bmatrix}$

(h) $\begin{bmatrix}12&11\\-27&3\end{bmatrix}$ **2** (a) $\begin{bmatrix}6&1\\-2&3\end{bmatrix}$ (b) $\begin{bmatrix}4&1\\4&5\end{bmatrix}$

(c) $\begin{bmatrix}30&6\\6&24\end{bmatrix}$ (d) $\begin{bmatrix}-4&-1\\-4&-5\end{bmatrix}$ (e) $\begin{bmatrix}17&3\\-3&10\end{bmatrix}$

(f) $\begin{bmatrix}18&5\\26&27\end{bmatrix}$ **3** (a) $\begin{bmatrix}18\\8\end{bmatrix}$ (b) $\begin{bmatrix}10&13\\22&29\end{bmatrix}$ (c) $\begin{bmatrix}6\\-2\end{bmatrix}$

(d) $\begin{bmatrix}4\\3\end{bmatrix}$ (e) $\begin{bmatrix}-11&-11\\17&43\end{bmatrix}$ (f) $\begin{bmatrix}6&-2\\-2&-6\end{bmatrix}$

4 (a) $\begin{bmatrix}12&-14\\-7&-1\end{bmatrix}$ (b) $\begin{bmatrix}2&16\\8&9\end{bmatrix}$ (c) $\begin{bmatrix}16&-26\\14&-14\end{bmatrix}$

(d) $\begin{bmatrix}-1&-13\\11&3\end{bmatrix}$ (e) $\begin{bmatrix}76&-106\\-26&16\end{bmatrix}$ (f) $\begin{bmatrix}76&-106\\-26&16\end{bmatrix}$

(g) $\begin{bmatrix}\frac{3}{11}&\frac{2}{11}\\\frac{1}{11}&-\frac{3}{11}\end{bmatrix}$ (h) $\begin{bmatrix}-8&-4\\3&-11\end{bmatrix}$

5 (a) $x=5$, $y=10$ (b) $x=2$, $y=-1$

(c) $x=8$, $y=-1$ (d) $x=1$, $y=3$

6 (a) $Q\begin{bmatrix}2\\5\end{bmatrix}$, $R\begin{bmatrix}6\\4\end{bmatrix}$ (b) $\begin{bmatrix}-4\\1\end{bmatrix}$ (c) $P(1,2)$

7 (a) $A'(0,4)$, $B'(-2,11)$, $C'(4,10)$

(b)