Polynomials and curve sketching (OPTION 6)

UNIT 1: Polynomials in general

QUESTION 1 Which of the following algebraic expressions is not a polynomial?

a x^2+3x-1 _____ b $2x^3+4x^2-8x-3$ _____ c 3^x+9 _____

d $5x^3-8x^2+7x$ _____ e $2x^2-3x^{\frac{1}{3}}+6$ _____ f $x^3-\frac{1}{x^2}+3x-1$ _____

QUESTION 2 For the following polynomials, write the degree, the leading term, the leading coefficient and the constant term.

a $6x^3+3x^2-6x+7$ _____ b $4x^3-3x^2-6x-2$ _____ c $8x^5+3x^4+5x^3-6x^2$ _____

d $5x^8-6x^3+7x^2-8x-3$ _____ e $5-x+3x^2+9x^3$ _____ f x^4-2x^3+9x-5 _____

g $9x^2+11x-8$ _____ h x^3+2x^2-7x+2 _____ i $6x^5-4x^4-x+9$ _____

QUESTION 3 Which of the following are monic polynomials?

a $3x^2+5x+7$ _____ b $x^5-4x^3+3x^2-x+1$ _____ c x^6+19 _____

d $8x^2-9x$ _____ e $2x^3+3x^2-9x+3$ _____ f x^4+3x^2-5x+7 _____

g x^3-3x^2+7x-8 _____ h x^3-x^2+7 _____ i $8x^2+6x-9$ _____

QUESTION 4 For the following polynomials, find the values indicated.

a $P(x)=x^2+x-1$ i $P(1)=$ _____ ii $P(3)=$ _____ iii $P(2)=$ _____

b $P(x)=x^3+3$ i $P(0)=$ _____ ii $P(1)=$ _____ iii $P(-1)=$ _____

c $P(x)=8-2x$ i $P(1)=$ _____ ii $P(2)=$ _____ iii $P(-5)=$ _____

d $P(x)=x^3+x^2$ i $P(0)=$ _____ ii $P(1)=$ _____ iii $P(2)=$ _____

QUESTION 5 What is the degree of each polynomial?

a $5x^3(8x^2+7x-6)$ _____ b $(6x^5+9x^3-6x^2)+(7x-3)$ _____

Polynomials and curve sketching (OPTION 6)

UNIT 2: Addition of polynomials

QUESTION 1 Add the following polynomials.

a $(x^3+2x+1)+(4x^3+3x^2-9)=$ _____

b $(3x^3-2x^2+5x-2)+(x^3+4x^2+9x-7)=$ _____

c $(6x^5+3x^4-8x^2)+(9x^4+9x^2+9x)=$ _____

d $(3x^6+8x^2+5x)+(x^2+6x-3)=$ _____

e $(8x^2+7x-6)+(x^5-3)=$ _____

f $(9x^3-8x^2+9)+(x^4-2x+7)=$ _____

QUESTION 2 If $P(x)=3x^2+2x-5$, $Q(x)=x^3+8x^2-9x+7$, $A(x)=2x^3+6x^2+x$ and $B(x)=3x^5+x^4-x^3+11$, find the following.

a $P(x)+Q(x)=$ _____

b $A(x)+B(x)=$ _____

c $P(x)+A(x)=$ _____

d $Q(x)+B(x)=$ _____

e $P(x)+B(x)=$ _____

f $Q(x)+A(x)=$ _____

QUESTION 3 Add the following polynomials and write the degree of $P(x)+Q(x)$.

a $P(x)=x^3+2x^2-9$, $Q(x)=2x^4+9x-6=$ _____

b $P(x)=x^4+9x^2-5$, $Q(x)=x^3+x^2+x+1=$ _____

c $P(x)=5x^3-1$, $Q(x)=2x^2-9=$ _____

d $P(x)=6x^3+4x^2+5x$, $Q(x)=3x^2-6x-9=$ _____

e $P(x)=x^4-3x^2+8x$, $Q(x)=x^3-3x^2+8x=$ _____

f $P(x)=5x^3+6x^2-5x+7$, $Q(x)=8x^2+7x-9=$ _____

g $P(x)=7x^4+3x-6$, $Q(x)=x^4+3x^3+2x^2-x+12=$ _____

Polynomials and curve sketching (OPTION 6)

UNIT 3: Subtraction of polynomials

QUESTION 1 Subtract the following polynomials.

a $(x^3+2x^2-x+1)-(5x^2+7x-9)=$ _____

b $(4x^3-3x^2+7x-3)-(x^3-x^2+x+1)=$ _____

c $(6x^5+3x^4-9x^2)-(8x^4+6x^2-3x)=$ _____

d $(2x^6+7x^3-8x)-(5x^4+9x^3-7x)=$ _____

e $(9x^2+5x-7)-(x^2-3x-11)=$ _____

f $(5x^3-7x^2+7)-(x^4-3x-6)=$ _____

QUESTION 2 If $P(x)=5x^2+4x-7$, $Q(x)=2x^3+6x^2-7x+9$, $A(x)=3x^3+7x^2-x$ and $B(x)=7x^5+x^4-x^3+2$, find the following.

a $P(x)-Q(x)=$ _____

b $A(x)-B(x)=$ _____

c $P(x)-A(x)=$ _____

d $Q(x)-B(x)=$ _____

e $P(x)-B(x)=$ _____

f $Q(x)-A(x)=$ _____

QUESTION 3 For the following polynomials find $P(x)-Q(x)$ and state its degree.

a $P(x)=x^3+3x-7$, $Q(x)=3x^4+7x-6=$ _____

b $P(x)=x^5+7x^2-9$, $Q(x)=3x^2-2x+1=$ _____

c $P(x)=8x^3-2x^2+5$, $Q(x)=9x^2-7x+3=$ _____

d $P(x)=x^4-3x^3+7x^2$, $Q(x)=x^3-4x^2+7x=$ _____

e $P(x)=x^5-3x^3+7x$, $Q(x)=9x^2-6x+3=$ _____

f $P(x)=6x^3+7x^2-9x+2$, $Q(x)=9x^2-6x+3=$ _____

g $P(x)=8x^3+5x-4$, $Q(x)=x^2-3x+7=$ _____

Polynomials and curve sketching (OPTION 6)

UNIT 4: Multiplication of polynomials

QUESTION **1** Expand and simplify the following.

a $(x+1)(x^2+5x+2)$

b $(x+2)(x^2-3x+4)$

c $(x-2)(x^2-3x-5)$

d $(x+3)(x^2+7x-8)$

e $(2x+1)(x^2-5x-3)$

f $(3x-2)(x^3+x^2-x-2)$

QUESTION **2** Find the following products.

a $(x^2-3)(x^2+7x-9) =$

b $(x^2+2x)(x^2+2x+1) =$

c $(6x^2-7)(x^3+2x^2-3x+4) =$

d $(4x^3-2x)(8x^2-3x+5) =$

e $(2x^3-3)(3x^2-8x+1) =$

f $(3x^2+2x+1)^2 =$

QUESTION **3** Find the product of the following polynomials and state the degree of the product.

a $5x^2+3x-9 \times 4x+1$

b $2x^3+3x^2-x \times x^2-2$

c $3x^4-5x^2+3 \times 2x+1$

Polynomials and curve sketching (OPTION 6)

UNIT 5: Division of polynomials

QUESTION **1** Complete the following divisions and express each result in the form: dividend = divisor × quotient + remainder.

a $x+1\overline{)x^2+3x+5}$

b $x-1\overline{)x^3+3x^2+2x-1}$

c $x+5\overline{)x^3+2x^2-8}$

d $x^2+x\overline{)x^5+3x^3+2x^2}$

QUESTION **2** Find the quotient and remainder for the following divisions.

a $(5x^2+6x+3)\div(x+2)$

b $(x^2-5x+16)\div(x-1)$

c $(6x^3+4x^2-7x+5)\div(x+1)$

d $(x^3+3x^2-4x+9)\div(x+1)$

QUESTION **3** Show that the first polynomial is exactly divisible by the second polynomial and hence write the first polynomial as a product of two factors.

a $(x^2-x-6)\div(x+2)$

b $(x^3+6x^2+12x+7)\div(x+1)$

c $(2x^3+3x^2+10x+15)\div(2x+3)$

d $(x^3+5x^2+7x+3)\div(x+3)$

Polynomials and curve sketching (OPTION 6)

UNIT 6: The remainder theorem

QUESTION 1 Use the remainder theorem to find the remainder for the following divisions.

a $(2x^2 - 3x + 6) \div (x + 1)$

b $(x^2 + 5x + 6) \div (x + 1)$

c $(3x^2 + 4x - 5) \div (x + 2)$

d $(2x^2 + 3x - 14) \div (x - 1)$

e $(2x^3 + 5x^2 - 7x + 6) \div (x - 3)$

f $(3x^4 - 2x^3 + 9x - 5) \div (x - 1)$

QUESTION 2 Find the remainder when $P(x)$ is divided by $A(x)$.

a $P(x) = 3x^2 + 9x - 6,\ A(x) = x + 2$

b $P(x) = 2x^2 + 7x - 8,\ A(x) = x + 1$

c $P(x) = x^3 + 5x^2 - 10x - 6,\ A(x) = x + 4$

d $P(x) = x^4 - 7x^3 + 6x^2,\ A(x) = x + 1$

e $P(x) = 2x^3 + 2x^2 - 8x + 3,\ A(x) = x + 1$

f $P(x) = 5x^3 + 7x^2 - 8x + 1,\ A(x) = x - 2$

QUESTION 3

a If $px^3 + x^2 - 3x - 2$ is divided by $(x + 2)$, the remainder is zero. Find the value of p.

b If $x^3 + x^2 - mx + 4$ is divided by $(x - 2)$, the remainder is 8. Find the value of m.

Polynomials and curve sketching (OPTION 6)

UNIT 7: The factor theorem

QUESTION 1 In each of the following, show that the first polynomial is a factor of $P(x)$ and hence find all the factors of $P(x)$.

a $(x+1),\ P(x)=5x^2+8x+3$

b $(x-1),\ P(x)=x^3+x^2-5x+3$

c $(2x-1),\ P(x)=2x^3-11x^2+17x-6$

d $(x+3),\ P(x)=x^3+12x^2+47x+60$

QUESTION 2 Use the factor theorem to show that $A(x)$ is a factor of $P(x)$ and hence find all the factors of $P(x)$.

a $P(x)=x^3+6x^2+11x+6,\quad A(x)=x+2$

b $P(x)=2x^3+5x^2-23x+10,\quad A(x)=x-2$

c $P(x)=2x^3+5x^2+7x+6,\quad A(x)=2x+3$

d $P(x)=2x^3-3x^2-2x+3,\quad A(x)=2x-3$

QUESTION 3 Use the factor theorem to factorise the following.

a $4x^3-17x^2-16x+5$

b $3x^3+4x^2-5x-2$

c $2x^3+7x^2+2x-3$

d $4x^3-27x^2+33x+10$

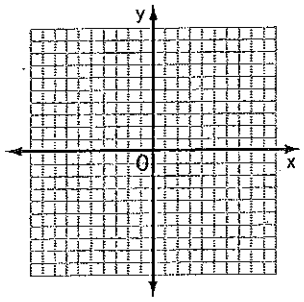
QUESTION 4 If x^3-x^2+px+2 is divisible by $x-1$, find the value of p.

Polynomials and curve sketching (OPTION 6)

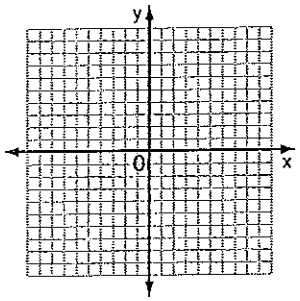
UNIT 8: Curve sketching

QUESTION 1 Sketch the following curves.

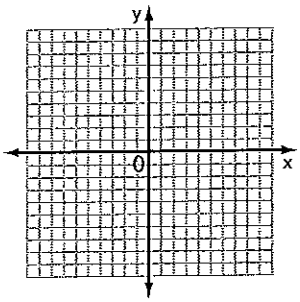
a $y = x$



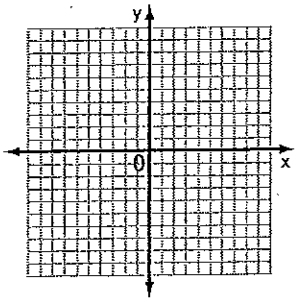
b $y = x^2$



c $y = x^3$

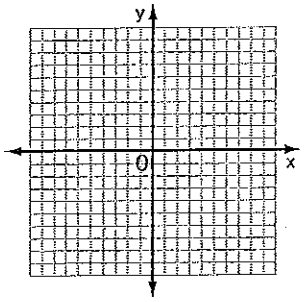


d $y = x^4$

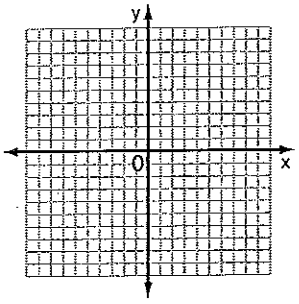


QUESTION 2 Make sketches of the following curves.

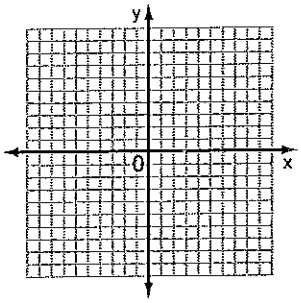
a $y = 3x + 5$



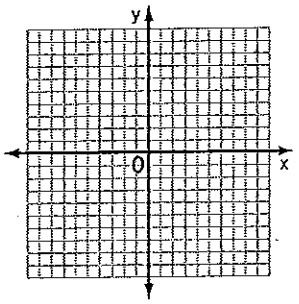
b $y = x^2 + 3$



c $y = 2x^3$

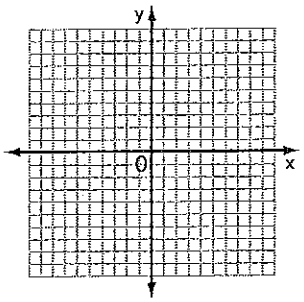


d $y = -x^3 + 1$

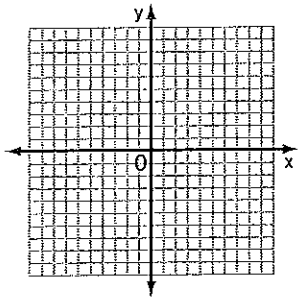


QUESTION 3 Sketch each of the following curves.

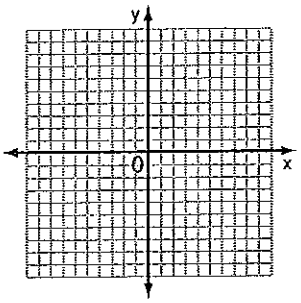
a $y = (x-1)^2$



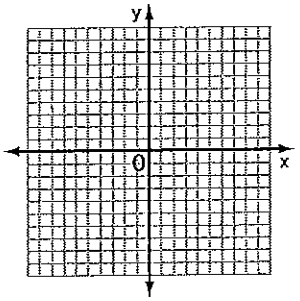
b $y = 3(x+2)^2$



c $y = (x-3)^3$

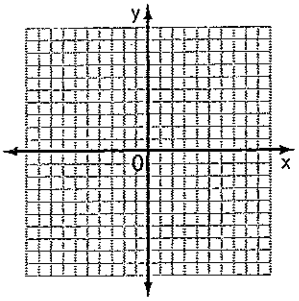


d $y = 3(x-1)^2$

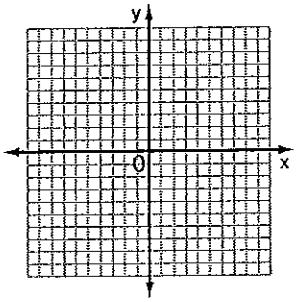


QUESTION 4 Sketch each of the following curves.

a $y = (x-1)(x+2)(x-3)$



b $y = 3(x-1)(x-4)(x+2)$



UNIT 9: TOPIC TEST

SECTION 1

Polynomials and curve sketching (OPTION 6)

Instructions for SECTION 1

- You have 15 minutes to answer Section 1
- Each question is worth 2 marks
- Attempt ALL questions
- Calculators are NOT to be used
- Fill in only ONE CIRCLE for each question

Marks

1 A polynomial is an algebraic expression consisting of

Ⓐ one term Ⓑ two terms Ⓒ three terms Ⓓ many terms — 2

2 The degree of the polynomial $7-2x-3x^2+x^4$ is

Ⓐ 0 Ⓑ 1 Ⓒ 2 Ⓓ 4 — 2

3 In the polynomial $2x^3+8x^2-9x+7$, the leading coefficient is

Ⓐ 2 Ⓑ 8 Ⓒ 9 Ⓓ 7 — 2

4 In the polynomial x^5+3x^2+7x+5, the constant term is

Ⓐ x^5 Ⓑ $3x^2$ Ⓒ $7x$ Ⓓ 5 — 2

5 The polynomial is monic if the leading coefficient is

Ⓐ 1 Ⓑ 2 Ⓒ 3 Ⓓ 4 — 2

6 The degree of the polynomial $6x^2\left(x^5+3x^2-1\right)$ is

Ⓐ 2 Ⓑ 4 Ⓒ 5 Ⓓ 7 — 2

7 For a graph to be a parabola, the highest power of x is

Ⓐ 1 Ⓑ 2 Ⓒ 3 Ⓓ 4 — 2

8 For any polynomial $f(x)$, if $f(a)=0$ then $(x-a)$ is a

Ⓐ factor of $f(x)$ Ⓑ remainder of $f(x)$ Ⓒ quotient of $f(x)$ Ⓓ none of these — 2

9 The zeros of any polynomial are also called the

Ⓐ roots Ⓑ terms Ⓒ coefficients Ⓓ none of these — 2

10 The graph of every linear equation is a

Ⓐ straight line Ⓑ parabola Ⓒ hyperbola Ⓓ circle — 2

Total marks achieved for SECTION 1 ____/20

UNIT 9: TOPIC TEST — SECTION 2

Polynomials and curve sketching (OPTION 6)

Instructions for SECTION 2

- You have 20 minutes to answer ALL of Section 2
- Each question is worth 2 marks
- Attempt ALL questions
- Calculators may be used

Questions	Answers	Marks
1 State whether the algebraic expression $2x^3 + x^2 - x + 7$ is a polynomial or not.	_____	2
For the polynomial $x^3 + 4x^2 - 9$, write:		
2 the degree.	_____	2
3 the leading coefficient.	_____	2
4 the coefficient of x^2.	_____	2
5 the constant term.	_____	2
6 whether it is monic, or not.	_____	2
For the polynomial $P(x) = x^2 + 3x + 7$, find:		
7 $P(2)$	_____	2
8 $P(-3)$	_____	2
9 $P\left(\frac{1}{a}\right)$	_____	2
Work out the following.		
10 $\left(x^3 - x^2 + 2x - 8\right) + \left(5x^3 + 3x^2 - 9x + 11\right)$	_____	2
11 $\left(4x^2 - 8x - 12\right) - \left(2x^2 + 7x + 9\right)$	_____	2
12 $(x + 2)\left(x^2 - 5x + 6\right)$	_____	2
13 $\left(x^2 + 8x + 15\right) \div (x + 3)$	_____	2
Sketch the following curves.		
14 $y = 2x + 1$	_____	2
15 $y = x^2$	_____	2

Total marks achieved for SECTION 2 ____/30

Answers

Page 1 1 c, e, f 2 a $3, 6x^3, 6, 7$ b $3, 4x^3, 4, -2$ c $5, 8x^5, 8, 0$ d $8, 5x^8, 5, -3$ e $3, 9x^3, 9, 5$ f $4, x^4, 1, -5$ g $2, 9x^2, 9, -8$ h $3, x^3, 1, 2$ i $5, 6x^5, 6, 9$ 3 b, c, f, g, h 4 a $P(1) = 1, P(3) = 11, P(2) = 5$ b $P(0) = 3, P(1) = 4, P(-1) = 2$ c $P(1) = 6, P(2) = 4, P(-5) = 18$ d $f(0) = 0, f(1) = 2, f(2) = 12$ 5 a 5 b 6

Page 2 1 a $5x^3 + 3x^2 + 2x - 8$ b $4x^3 + 2x^2 + 14x - 9$ c $6x^5 + 12x^4 + x^2 + 9x$ d $3x^6 + 9x^2 + 11x - 3$ e $x^5 + 8x^2 + 7x - 9$ f $x^4 + 9x^3 - 8x^2 - 2x + 16$ 2 a $x^3 + 11x^2 - 7x + 2$ b $3x^5 + x^4 + x^3 + 6x^2 + x + 1$ c $2x^3 + 9x^2 + 3x - 5$ d $3x^5 + x^4 + 8x^2 - 9x + 18$ e $3x^5 + x^4 - x^3 + 3x^2 + 2x + 6$ f $3x^3 + 14x^2 - 8x + 7$ 3 a $2x^4 + x^3 + 2x^2 + 9x - 15$, degree = 4 b $x^4 + x^3 + 10x^2 + x - 4$, degree = 4 c $5x^3 + 2x^2 - 10$, degree = 3 d $6x^3 + 7x^2 - x - 9$, degree = 3 e $x^4 + x^3 - 6x^2 + 16x$, degree = 4 f $5x^3 + 14x^2 + 2x - 2$, degree = 3 g $8x^4 + 3x^3 + 2x^2 + 2x + 6$, degree = 4

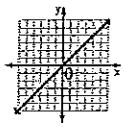
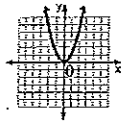
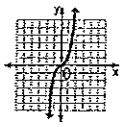
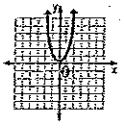
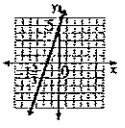
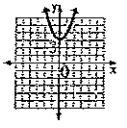
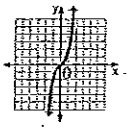
Page 3 1 a $x^3 - 3x^2 - 8x + 10$ b $3x^3 - 2x^2 + 6x - 4$ c $6x^5 - 5x^4 - 15x^2 + 3x$ d $2x^6 - 5x^4 - 2x^3 - x$ e $8x^2 + 8x + 4$ f $-x^4 + 5x^3 - 7x^2 + 3x + 13$ 2 a $-2x^3 - x^2 + 11x - 16$ b $-7x^5 - x^4 + 4x^3 + 7x^2 - x - 2$ c $-3x^3 - 2x^2 + 5x - 7$ d $-7x^5 - x^4 + 3x^3 + 6x^2 - 7x + 7$ e $-7x^5 - x^4 + x^3 + 5x^2 + 4x - 9$ f $-x^3 - x^2 - 6x + 9$ 3 a $-3x^4 + x^3 - 4x - 1$, degree = 4 b $x^5 + 4x^2 + 2x - 10$, degree = 5 c $8x^3 - 11x^2 + 7x + 2$, degree = 3 d $x^4 - 4x^3 + 11x^2 - 7x$, degree = 4 e $x^5 - 3x^3 - 9x^2 + 13x - 3$, degree = 5 f $6x^3 - 2x^2 - 3x - 1$ degree = 3 g $8x^3 - x^2 + 8x - 11$, degree = 3

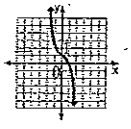
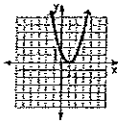
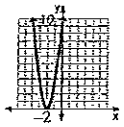
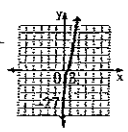
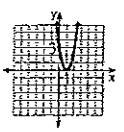
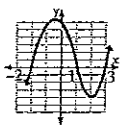
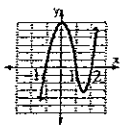
Page 4 1 a $x^3 + 6x^2 + 7x + 2$ b $x^3 - x^2 - 2x + 8$ c $x^3 - 5x^2 + x + 10$ d $x^3 + 10x^2 + 13x - 24$ e $2x^3 - 9x^2 - 11x - 3$ f $3x^4 + x^3 - 5x^2 - 4x + 4$ 2 a $x^4 + 7x^3 - 12x^2 - 21x + 27$ b $x^4 + 4x^3 + 5x^2 + 2x$ c $6x^5 + 12x^4 - 25x^3 + 10x^2 + 21x - 28$ d $32x^5 - 12x^4 + 4x^3 + 6x^2 - 10x$ e $6x^5 - 16x^4 + 2x^3 - 9x^2 + 24x - 3$ f $9x^4 + 12x^3 + 10x^2 + 4x + 1$ 3 a $20x^3 + 17x^2 - 33x - 9$, degree = 3 b $2x^5 + 3x^4 - 5x^3 - 6x^2 + 2x$, degree = 5 c $6x^5 + 3x^4 - 10x^3 - 5x^2 + 6x + 3$, degree = 5

Page 5 1 a $x^2 + 3x + 5 = (x + 1)(x + 2) + 3$ b $x^3 + 3x^2 + 2x - 1 = (x - 1)(x^2 + 4x + 6) + 5$ c $x^3 + 2x^2 - 8 = (x + 5)(x^2 - 3x + 15) - 83$ d $x^5 + 3x^3 + 2x^2 = (x^2 + x)(x^3 - x^2 + 4x - 2) + 2x$ 2 a quotient = $5x - 4$, remainder = 11 b quotient = $x - 4$, remainder = 12 c quotient = $6x^2 - 2x - 5$, remainder = 10 d quotient = $x^2 + 2x - 6$, remainder = 15 3 a $x^2 - x - 6 = (x + 2)(x - 3)$ b $x^3 + 6x^2 + 12x + 7 = (x + 1)(x^2 + 5x + 7)$ c $2x^3 + 3x^2 + 10x + 15 = (2x + 3)(x^2 + 5)$ d $x^3 + 5x^2 + 7x + 3 = (x + 3)(x + 1)^2$

Page 6 1 a 11 b 2 c –1 d –9 e 84 f 5 2 a –12 b –13 c 50 d 14 e 11 f 53 3 a $p = 1$ b $m = 4$

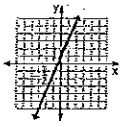
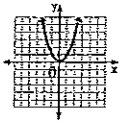
Page 7 1 a $(x + 1)(5x + 3)$ b $(x - 1)^2(x + 3)$ c $(x - 2)(x - 3)(2x - 1)$ d $(x + 3)(x + 4)(5x + 5)$ 2 a $(x + 1)(x + 2)(x + 3)$ b $(x - 2)(x + 5)(2x - 1)$ c $(2x + 3)(x^2 + x + 2)$ d $(x - 1)(x + 1)(2x - 3)$ 3 a $(x - 5)(x + 1)(4x - 1)$ b $(x - 1)(x + 2)(3x + 1)$ c $(x + 1)(x + 3)(2x - 1)$ d $(x - 2)(x - 5)(4x + 1)$ 4 $p = -2$

Page 8 1 a  b  c  d  2 a  b  c 

d  3 a  b  c  d  4 a  b 

Page 9 1 D 2 D 3 A 4 D 5 A 6 D 7 B 8 A 9 A 10 A

Page 10 1 Polynomial 2 3 3 1 4 4 5 –9 6 Monic 7 17 8 7 9 $\frac{1}{a^2} + \frac{3}{a} + 7$ 10 $6x^3 + 2x^2 - 7x + 3$ 11 $2x^2 - 15x - 21$

12 $x^3 - 3x^2 - 4x + 12$ 13 $x + 5$ 14  15 

Page 2

Q14 (a)(i) $P(1) = 1^2+1-1$
$= 1+1-1$
$= 1$ ✓

(ii) $P(3) = 3^2+3-1$
$= 9+3-1$
$= 11$ ✓

(iii) $P(2) = 2^2+2-1$
$= 4+2-1$
$= 5$ ✓

(b) (i) $P(0) = 0^3+3$
$= 3$ ✓

(ii) $P(1) = 1^3+3$
$= 4$ ✓

(iii) $P(-1) = (-1)^3+3$
$= -1+3$
$= 2$ ✓

(c) (i) $P(1) = 8-(2\times1)$
$= 8-2$
$= 6$ ✓

(ii) $P(2) = 8-(2\times2)$
$= 8-4$
$= 4$ ✓

(iii) $P(-5) = 8-(2\times-5)$
$= 8--10$
$= 18$ ✓

(d) (i) $P(0) = 0$ ✓

(ii) $P(1) = 1^3+1^2$
$= 2$ ✓

(iii) $P(2) = 2^3+2^2$
$= 8+4$
$= 12$ ✓

Q11 (a) $(x^3+2x+1)+(4x^3+3x^2-9)$
$= x^3+2x+1+4x^3+3x^2-9$ ✓
$= 5x^3+3x^2+2x-8$ ✓

(b) $(3x^3-2x^2+5x-2)+(x^3+4x^2+9x-7)$
$= 3x^3-2x^2+5x-2+x^3+4x^2+9x-7$
$= 4x^3+2x^2+14x-9$ ✓

(c) $(6x^5+3x^4-8x^2)+(9x^4+9x^2+9x)$
$= 6x^5+12x^4+x^2+9x$ ✓

(d) $(3x^6+8x^2+5x)+(x^3+6x-3)$
$= 3x^6+9x^2+11x-3$ ✓

(e) $(8x^2+7x-6)+(x^5-3)$
$= 8x^2+7x-6+x^5-3$
$= x^5+8x^2+7x-9$ ✓

(f) $(9x^3-8x^2+9)+(x^4-2x+7)$
$= 9x^3+x^4-8x^2+(7)$ ✓

Q12 (a) $P(x)+Q(x) = (3x^2+2x-5)+(x^3+8x^2-9x+7)$
$= x^3+11x^2-7x+2$ ✓

(b) $A(x)+B(x) = (2x^3+6x^2+x)+(3x^5+x^4-7x^3+11)$
$= 3x^5+x^4+x^3+x+11+6x^2$

(c) $P(x)+A(x) = (3x^2+2x-5)+(2x^3+6x^2+x)$
$= 2x^3+9x^2+3x-5$ ✓

(d) $Q(x)+B(x) = (x^3+8x^2-9x+7)+(3x^5+x^4-x^3+11)$
$= 3x^5+x^4+8x^2-9x+18$ ✓

(e) $P(x)+B(x) = (3x^2+2x-5)+(3x^5+x^4-x^3+11)$
$= 3x^5+x^4-x^3+3x^2+2x+6$

(f) $Q(x)+A(x) = (x^3+8x^2-9x+7)+(2x^3+6x^2+x)$
$= 3x^3+14x^2-8x+7$

Page 1

Q13 (a) $P(x)+Q(x) = (x^3+2x^2-9)+(2x^4+9x-6)$
$= 2x^4+x^3+2x^2+9x-15$
∴ Degree is 4 ✓

Page 3

Q11 (a) $(x^3+2x^2-x+1)-(5x^2+7x-9)$
$= x^3+2x^2-x+1-5x^2-7x+9$
$= x^3-3x^2-8x+10$ ✓

(b) $(4x^3-3x^2+7x-3)-(x^3-x^2+x+1)$
$= 4x^3-3x^2+7x-3-x^3+x^2-x-1$
$= 3x^3-2x^2+6x-4$ ✓

(c) $(6x^6+3x^4-9x^2)-(8x^4+6x^2+3x)$
$= 6x^6+3x^4-9x^2-8x^4-6x^2-3x$
$= 6x^6-5x^4-15x^2-3x$ ✓

(d) $(2x^6+7x^3-8x)-(5x^4+9x^3-7x)$
$= 2x^6+7x^3-8x-5x^4-9x^3+7x$
$= 2x^6-5x^4-2x^3-x$ ✓

(e) $(9x^2+5x-7)-(x^2-3x-11)$
$= 9x^2+5x-7-x^2+3x+11$
$= 8x^2+8x+4$ ✓

(f) $(5x^3-7x^2+7)-(x^4+3x-6)$
$= 5x^3-7x^2+7-x^4+3x+6$
$= -x^4+5x^3-7x^2+3x+13$ ✓

Q12 (a) $P(x)-Q(x) = (5x^2+6x-7)-(2x^3+6x^2-7x+9)$
$= 5x^2+6x-7-2x^3-6x^2+7x-9$
$= -2x^3-x^2+11x-16$ ✓

(b) $A(x)-B(x) = (3x^3+7x^2-x)-(7x^5+x^4-x^3+2)$
$= 3x^3+7x^2-x-7x^5-x^4+x^3-2$
$= -7x^5-x^4+4x^3+7x^2-x-2$

(c) $P(x)-A(x) = (5x^2+4x-7)-(3x^3+7x^2-x)$
$= 5x^2+4x-7-3x^3-7x^2+x$
$= -3x^3-2x^2+5x-7$ ✓

(d) $Q(x)-B(x) = (2x^3+6x^2-7x+9)-(7x^5+x^4-x^3+2)$
$= 2x^3+6x^2-7x+9-7x^5-x^4+x^3-2$
$= -7x^5-x^4+3x^3+6x^2-7x+7$

(e) $P(x)-B(x) = (5x^2+4x-7)-(7x^5+x^4-x^3+2)$
$= 5x^2+4x-7-7x^5-x^4+x^3-2$
$= -7x^5-x^4+x^3+5x^2+4x-9$

(f) $Q(x)-A(x) = (2x^3+6x^2-7x+9)-(3x^3+7x^2-x)$
$= 2x^3+6x^2-7x+9-3x^3-7x^2+x$
$= -x^3-x^2-6x+9$ ✓

Page 4

Q12 (f) let $a = 3x^2$ and $b = (2x+1)$
$= (3x^2)^2+2(3x^2)(2x+1)+(2x+1)^2$
$= 9x^4+6x^2(2x+1)+4x^2+4x+1$
$= 9x^4+12x^3+6x^2+4x^2+4x+1$
$= 9x^4+12x^3+10x^2+4x+1$ ✓

Q13 (a) $(4x+1)(5x^2+3x-9)$
$= 20x^3+12x^2-36x+5x^2+3x-9$
$= 20x^3+17x^2-33x-9$ ✓

(b) $(x^2-2)(2x^3+3x^2-x)$
$= 2x^5+3x^4-x^3-4x^2-6x^2+2x$
$= 2x^5+3x^4-5x^3-6x^2+2x$ ✓

(c) $(2x+1)(3x^4-5x^2+3)$
$= 6x^5-10x^3+6x+3x^4-5x^2+3$
$= 6x^5+3x^4-10x^3-5x^2+6x+3$ ✓

Page 5

Q11 (a)
$$x+1 \overline{)\, x^2+3x+5}$$
$x+2$
x^2+x
$2x+5$
$2x+2$
3 ✓

(b)
$$x-1 \overline{)\, x^3+3x^2+2x-1}$$
x^2+4x+6
x^3-x^2
$4x^2+2x$
$4x^2-4x$
$6x-1$
$6x-6$ ✓
5

Page 5

Qn1 (c) $x+5 \overline{)x^3+2x^2+0x-8}$

x^3+5x^2

$-3x^2+0x$

$-3x^2-15x$

$15x-8$

$15x+75$

-83

(d) $x^2+x \overline{)x^5+0x^4+3x^3+2x^2+0x+0}$

x^5+x^4

$-x^4+3x^3$

$-x^4-x^3$

$4x^3+2x^2$

$4x^3+4x^2$

$-2x^2+0x$

$-2x^2-2x$

$2x+0$

x^3-x^2+4x-2

Qn2 (a) $x+2 \overline{)5x^2+6x+3}$ — $5x-4$

$5x^2+10x$

$-4x+3$

$-4x-8$

11

(b) $x-1 \overline{)x^2-5x+16}$ — $x-4$

x^2-x

$-4x+16$

$-4x+4$

12

(c) $x+1 \overline{)6x^3+4x^2-7x+5}$ — $6x^2-2x-5$

$6x^3+6x^2$

$-2x^2-7x$

$-2x^2-2x$

$-5x+5$

$-5x-5$

10

(d) $x+1 \overline{)x^3+2x^2+0x-8}$ — x^2+x-1

x^3+x^2

$-3x^2+0x$

$-3x^2-15x$

$15x-8$

$15x+15$

-23

Qn3 (a) $x-2 \overline{)x^2-x-6}$ — $x-3$

x^2+2x

$-3x-6$

$-3x-6$

0

(b) $x+1 \overline{)x^3+6x^2+12x+7}$ — x^2+5x+7

x^3+x^2

$5x^2+12x$

$5x^2+5x$

$7x+7$

$7x+7$

0

(c) $2x+3 \overline{)2x^3+3x^2+10x+15}$ — x^2+0+5

$2x^3+3x^2$

$0+10x$

$0+0$

$10x+15$

$10x+15$

0

Qn3 (d) $x+3 \overline{)x^3+5x^2+7x+3}$ — x^2+2x+1

x^2+3x^2

$2x^2+7x$

$2x^2+6x$

$x+3$

$x+3$

0

$(x+1)^2$

Page 6

Qn1 (a) $x+1 \overline{)2x^2-3x+6}$ — $2x-5$

$2x^2+2x$

$-5x+6$

$-5x-5$

11

(b) $(-1)^2+(-5)+6$

$= (1-5+6)$

$= 2$

(c) $P(-2) = 3x^2+4x-5$

$= 3(-2)^2+4(-2)-5$

$= 3\times4+-8-5$

$= 12-8-5$

$= -1$

(d) $P(\#1) = 2x^2+3x-14$

$= 2(\#1)^2+3(1)-14$

$= 2+3-14$

$= 5-14$

$= -9$

(e) $P(3) = 2x^3+5x^2-7x+6$

$= 2(3)^3+5(3)^2-21+6$

$= 2\times27+5\times9-21+6$

$= 54+45-21+6$

$= 84$

f) $P(1) = 3x^4-2x^3+9x-5$

$= 3(1)^4-2(1)^3+9-5$

$= 3-2+9-5$

$= 1+9-5$

$= 5$

Qn2 (a) $x=-2$

$P(-2) = 3(-2)^2+9(-2)-6$

$= 3\times4-18-6$

$= 12-18-6$

$= -12$

(b) $x=-1$

$P(-1) = 2(-1)^2+7(-1)-8$

$= 2-7-8$

$= -13$

(c) $x=-4$

$P(-4) = (-4)^3+5(-4)^2-10(-4)-6$

$= -64+80+40-6$

$= 16+40-6$

$= 50$

(d) $x=-1$

$P(-1) = (-1)^4-7(-1)^3+6(-1)^2$

$= 1+7+6$

$= 14$

(e) $x=-1$

$P(-1) = 2(-1)^3+2(-1)^2-8(-1)+3$

$= -2+2+8+3$

$= 11$

f) $x=2$

$P(2) = 5(2)^3+7(2)^2-8(2)+1$

$= 40+28-16+1$

$= 68-16+1$

$= 53$

Page 6

Q13 (a) $x=-2$

$P(-2) = p(-2)^3 + (-2)^2 - 3(-2) - 2$

$= p(-8) + 4 + 6 - 2$

$= -8p + 8$

$-8p + 8 = 0$

$-8p = -8$

$\therefore p = 1$

(b) $x = 2$

$P(2) = 2^3 + 2^2 - m(2) + 4$

$= 8 + 4 - 2m + 4$

$= 12 - 2m + 4$

$= 16 - 2m + 4$

$16 - 2m = 0$

$-2m = -16$

$\therefore m = 8$

Page 7

Q11 (a) $P(-1) = 5(-1)^2 + 8(-1) + 3$

$= 5 - 8 + 3$

$= 0$

$(x+1)(5x+3)$

$\therefore$ factors are $(x+1)$, $(5x+3)$

(b) $P(1) = 1^3 + 1^2 - 5(1) + 3$

$= 1 + 1 - 5 + 3$

$= -3 + 3$

$= 0$

$x^2 + 2x - 3$

$x-1 \;\overline{)}\; x^3 + x^2 - 5x + 3$

$x^3 - x^2$

$2x^2 - 5x$

$2x^2 - 2x$

$-3x + 3$

$-3x + 3$

0

Continued...

(b) $(x=1)(x+3)$

$\therefore$ the factors are $(x-1)(x-1)(x+3)$

$= (x-1)^2(x+3)$

(c) $2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

$P(\frac{1}{2}) = 2(\frac{1}{2})^3 - 11(\frac{1}{2})^2 + 17(\frac{1}{2}) - 6$

$= (2 \times \frac{1}{8}) - (11 \times \frac{1}{4}) + 8.5 - 6$

$= \frac{1}{4} - \frac{11}{4} + 8.5 - 6$

$= -\frac{10}{4} + 8.5 - 6$

$= 6 - 6$

$= 0$

$x^2 - 5x + 6$

$2x-1 \;\overline{)}\; 2x^3 - 11x^2 + 17x - 6$

$2x^3 - x^2$

$-10x^2 + 17x$

$-10x^2 + 5x$

$12x - 6$

$12x - 6$

0

$x^2 - 5x + 6$

$= (x-2)(x-3)$

$\therefore$ the factors are $(2x-1)(x-2)(x-3)$

(d) $x = -3$

$P(-3) = (-3)^3 + 12(-3)^2 + 47(-3) + 60$

$= -27 + 108 - 141 + 60$

$= 81 - 141 + 60$

$= -60 + 60$

$= 0$

$2x-3$

$x+3 \;\overline{)}\; x^3 + 12x + 47x + 60$

Page 7

Q12 (a) $x = -2$

$P(-2) = (-2)^3 + 6(-2)^2 + 11(-2) + 6$

$= -8 + 24 - 22 + 6$

$= 16 - 22 + 6$

$= -6 + 6$

$= 0$

$x^2 + 4x + 3$

$x+2 \;\overline{)}\; x^3 + 6x^2 + 11x + 6$

$x^3 + 2x^2$

$4x^2 + 11x$

$4x^2 + 8x$

$3x + 6$

$3x + 6$

0

$x^2 + 4x + 3$

$= (x+3)(x+1)$

$\therefore$ the factors are $(x+1)(x+2)(x+3)$

(b) $x = 2$

$P(2) = 2(2)^3 + 5(2)^2 - 23(2) + 10$

$= 2 \times 8 + 20 - 46 + 10$

$= 16 + 20 - 46 + 10$

$= -10 + 10$

$= 0$

$2x^2 + 9x - 5$

$x-2 \;\overline{)}\; 2x^3 + 5x^2 - 23x + 10$

$2x^3 - 4x^2$

$9x^2 - 23x$

$9x^2 - 18x$

$-5x + 10$

$-5x + 10$

0

continued...

(b) $2x^2 + 9x - 5$

$2x^2 + 10x - x - 5$

$2x(x+5) - (x+5)$

$(x+5)(2x-1)$

$\therefore$ the factors are $(x-2)(x+5)(2x-1)$

(c) $2x + 3 = 0$

$2x = -3$

$x = -\frac{3}{2}$

$P(-\frac{3}{2}) = 2(-\frac{3}{2})^3 + 5(-\frac{3}{2})^2 + 7(-\frac{3}{2}) + 6$

$= -3.375 \times 2 + 11.25 + -10.5 + 6$

$= -6.75 + 11.25 - 10.5 + 6$

$= 4.5 - 10.5 + 6$

$= -6 + 6$

$= 0$

$x^2 + x + 2$

$2x+3 \;\overline{)}\; 2x^3 + 5x^2 + 7x + 6$

$2x^3 + 3x^2$

$2x^2 + 7x$

$2x^2 + 3x$

$4x + 6$

$4x + 6$

0

$\therefore$ the factors are $(2x+3)(x^2+x+2)$

(d) $2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

$P(\frac{3}{2}) = 2(\frac{3}{2})^3 - 3(\frac{3}{2})^2 - 2(\frac{3}{2}) + 3$

$= 6.75 - 6.75 - 3 + 3$

$= 0$

$x^2 - 0 - 1$

$2x-3 \;\overline{)}\; 2x^3 - 3x^2 - 2x + 3$

$2x^3 - 3x^2$

$-2x + 3$

$-2x + 3$

0

Qn3 (a) $5 \rightarrow \pm 1, \pm 5$

$P(1) = 4(1)^3 - 17(1)^2 - 16(1) + 5$

$= 4 - 17 - 16 + 5$

$= -24$

$P(-1) = 4(-1)^3 - 17(-1)^2 - 16(-1) + 5$

$= -4 - 17 + 16 + 5$

$= -21 + 21$

$= 0$

$(x+1)$) $4x^3 - 17x^2 - 16x + 5$

$4x^3 + 4x^2$

$-21x^2 - 16x$

$-21x^2 - 21x$

$5x + 5$

$5x + 5$

0

$4x^2 - 21x + 5$

$= 4x^2 - 20x - x + 5$

$= 4x(x-5) - (x-5)$

$= (x-5)(4x-1)$

$\therefore (x-5)(4x-1)(x+1) = 4x^3 - 17x^2 - 16x + 5$

(b) $-2 \rightarrow \pm 1, \pm 2$

$P(2) = 3(2)^3 + 4(2)^2 - 5(2) - 2$

$= 24 + 16 - 10 - 2$

$= 40 - 10 - 2$

$= 28$

$P(-2) = 3(-2)^3 + 4(-2)^2 - 5(-2) - 2$

$= -24 - 16 + 10 - 2$

$= -40 + 10 - 2$

$= -32$

continued...

(b) $P(1) = 3(1)^3 + 4(1)^2 - 5(1) - 2$

$= 3 + 4 - 5 - 2$

$= 7 - 7$

$= 0$

$3x^2 + 7x + 2$

$(x-1)$) $3x^3 + 4x^2 - 5x - 2$

$3x^3 - 3x^2$

$7x^2 - 5x$

$7x^2 - 7x$

$2x - 2$

$2x - 2$

0

$3x^2 + 7x + 2 = 3x^2 + 6x + x + 2$

$= 3x(x+2) + (x+2)$

$= (x+2)(3x+1)$

$\therefore$ the factors are $(x+2)(3x+1)(x-1)$

(c) $-3 \rightarrow \pm 3, \pm 1$

$P(3) = 2(3)^3 + 7(3)^2 + 2(3) - 3$

$= 54 + 63 + 6 - 3$

$= 117 + 6 - 3$

$= 120$

$P(-3) = 2(-3)^3 + 7(-3)^2 + 2(-3) - 3$

$= -54 + 63 - 6 - 3$

$= 9 - 9$

$= 0$

$2x^2 + x - 1$

$(x+3)$) $2x^3 + 7x^2 + 2x - 3$

$2x^3 + 6x^2$

$x^2 + 2x$

$x^2 + 3x$

$-x - 3$

$-x - 3$

0

$2x^2 + x - 1$

$= 2x^2 + 2x - x - 1$

$= 2x(x+1) - (x+1)$

$= (x+1)(2x-1)$

$\therefore$ the factors are $(x+3)(x+1)(2x-1)$

Qn3 (d) $10 \rightarrow \pm 10, \pm 5, \pm 2, \pm 1$

$P(10) = 4(10)^3 - 27(10)^2 + 33(10) + 10$

$= 4000 - 2700 + 330 + 10$

$= 1640$

$P(-10) = 4(-10)^3 - 27(-10)^2 + 33(-10) + 10$

$= -4000 - 2700 - 330 + 10$

$= 7020$

$P(5) = 4(5)^3 - 27(5)^2 + 33(5) + 10$

$= 500 - 675 + 165 + 10$

$= 0$!!!

$4x^2 - 7x - 2$

$x-5$) $4x^3 - 27x^2 + 33x + 10$

$4x^3 - 20x^2$

$-7x^2 + 33x$

$-7x^2 + 35x$

$-2x + 10$

$-2x + 10$

0

$4x^2 - 7x - 2$

$= 4x^2 - 8x + x - 2$

$= 4x(x-2) + (x-2)$

$= (x-2)(4x+1)$

$\therefore$ the factors are $(x-5)(x-2)(4x+1)$

Qn4 (a) $P(1) = 1^3 - 1^2 + p + 2$

$= 1 - 1 + p + 2$

$= p + 2$

$p + 2 = 0$

$p = -2$