

NAME :

# SOUTH SYDNEY HIGH SCHOOL



## YEAR 10 - MATHEMATICS (ADVANCED)

### GRADING TEST

#### INSTRUCTIONS:

- Place all answers in the spaces provided.
- Show all necessary working.
- Marks will be deducted for untidy work.
- All questions are NOT of equal value.
- You have 1 hour to complete this task.

**Question 1 (20 marks)**

(a) Use your calculator to find to 2 decimal places (2m)

(i)  $\frac{1.3+2.58}{\sqrt{7.6}}$

(ii)  $\sqrt[4]{(4.2)^3}$

(b) Express the following in increasing numerical order :

$0.\dot{5}$ ,  $0.45$ ,  $\frac{23}{50}$ ,  $\sqrt{\frac{6}{11}}$  (1m)

(c) The half-life of Uranium 235 is 5 470 000 000 years. Express this in scientific notation. (1m)

(d) Simplify  $\frac{x^3y^5}{x^{-1}y^7}$  (1m)

(e) Find the value of  $x$  if  $2^x = \sqrt{2}$  (1m)

(f) Expand and simplify the following:

(i)  $2(3x - y) - (y + 5x)$

(2m)

(ii)  $(3x - 4y)^2$

(2m)

(iii)  $\left(x^{\frac{1}{3}} - y^{\frac{1}{3}}\right)\left(x^{\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}}\right)$

(3m)

(g) If  $\frac{2x+4y}{2x-2y} = 5$ , find the value of  $\frac{x^2+2y^2}{xy}$

(3m)

(h) Make  $y$  the subject of the formula:  $\frac{1}{x} = \frac{1}{y} - \frac{1}{z}$

(2m)

(i) Given that  $V = kr^2$  and if  $V = 12$  when  $r = 2$ , find the value of  $V$  when  $r = 3$ .

(2m)

Question 2 (22 marks)

(a) Find the value of  $x$  in the following:

(i)  $3^{x+1} = 9$

(1m)

(ii)  $4^{2x+1} = \sqrt{32}$

(b) Evaluate the following:

(2m)

(i)  $\log_3 81$

(ii)  $\log_5 \frac{1}{125}$

(1m)

(iii)  $\log_{0.1} 100$

(2m)

(iv)  $\log_{\sqrt{2}} 64$

(2m)

(v)  $\log_3 27 + \log_3 81$

(2m)

(2m)

(vi)  $\log_5 \sqrt{25} - \log_5 \frac{1}{125}$

(2m)

(c) If  $\log_y (2x+1) = 4 \log_y 2$ ; find the value of  $x$

(2m)

(d) If  $\log_3 a = 1.38$  evaluate the following:

(i)  $\log_3 \sqrt{a}$

(1m)

(ii)  $\log_3 3a$

(2m)

(e) If  $\log p = \log q - 2 \log r$ , circle the correct statement below.

(i)  $p = q - 2r$

(ii)  $p = q - r^2$

(iii)  $p = \frac{q}{2r}$

(iv)  $p = \frac{q}{r^2}$

(1m)

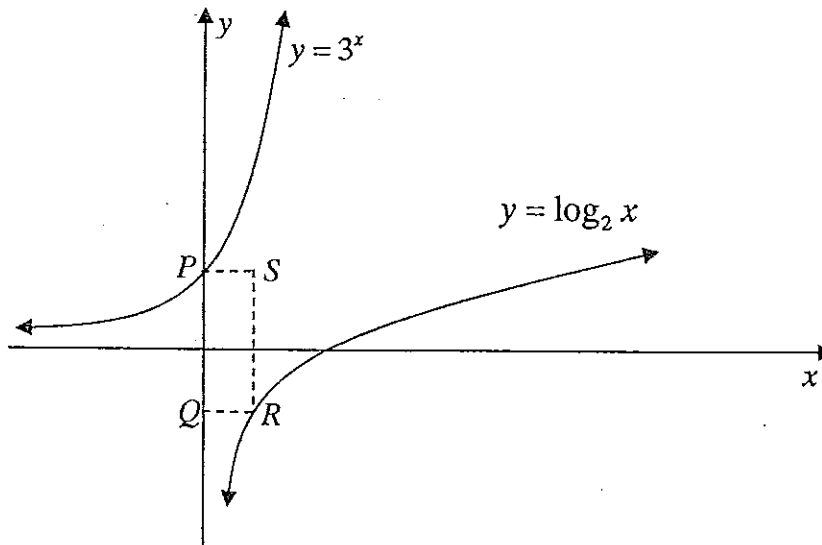
(f) If  $\log_b x = \frac{\log_{10} x}{\log_{10} b}$ , find  $y$  correct to 2 decimal places,

if  $5^y = 120$

(2m)

Question 3 (8 marks)

(a)



$PQRS$  is a rectangle. The coordinates of  $Q$  are  $(0, -1)$ .

(i) Find the coordinates of  $R$ .

(1m)

(ii) Find the coordinates of  $P$ .

(1m)

(iii) Find the area of rectangle  $PQRS$ .

(2m)

(b) (i) Evaluate  $\log_4(\log_2 16)$

(2m)

(ii) Simplify

$$\log_x(x^2 - 3x) - \log_x(x - 3)$$

(2m)

**SOLUTIONS**

Question 1 (20 marks)

(a) Use your calculator to find to 2 decimal places

(i)  $\frac{1.3+2.58}{\sqrt{7.6}} = 1.04182289$

(2m)

(ii)  $\sqrt[4]{(4.2)^3} = 2.92(2246)$

(b) Express the following in increasing numerical order:

0.5, 0.45,  $\frac{23}{50}$ ,  $\frac{23}{111}$ ,  $\sqrt[6]{6}$ ,  $\sqrt[11]{6}$

0.45,  $\frac{23}{50}$ , 0.5,  $\sqrt[11]{6}$ ,  $\sqrt[6]{6}$

(1m)

(c) The half-life of Uranium 235 is 5,470,000,000 years. Express this in scientific notation. =  $5.47 \times 10^9$

(1m)

(d) Simplify  $\frac{x^3y^5}{x^{-1}y^7} = x^{+4}y^{-2} = \frac{x^4}{y^2}$

(1m)

(e) Find the value of x if  $2^x = \sqrt{2}$   
 $2^x = 2^{\frac{1}{2}} \Rightarrow x = \frac{1}{2}$

(1m)

(f) Expand and simplify the following:

(i)  $2(3x-y)-(y+5x)$   
 $= 6x - 2y - y - 5x$   
 $= x - 3y$

(2m)

(ii)  $(3x-4y)^2$   
 $= 9x^2 - 24xy + 16y^2$

(2m)

(iii)  $(x^{\frac{1}{2}} - y^{\frac{1}{2}})^2$   
 $= x^{\frac{1}{2} \cdot \frac{1}{2}} - 2x^{\frac{1}{2}}y^{\frac{1}{2}} + y^{\frac{1}{2} \cdot \frac{1}{2}}$   
 $= x - 2\sqrt{xy} + y$

(2m)

(3m)

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(g) If  $\frac{2x+4y}{2x-2y} = 5$ , find the value of  $\frac{x^2+2y^2}{xy}$

$\frac{2(x+2y)}{2(x-y)} = 5$   
 $\frac{x+2y}{x-y} = 5$   
 $x+2y = 5(x-y)$   
 $x+2y = 5x-5y$   
 $x+7y = 5x$   
 $7y = 4x$   
 $y = \frac{4x}{7}$

$\frac{x^2+2y^2}{xy} = \frac{x^2+2(\frac{4x}{7})^2}{x(\frac{4x}{7})}$   
 $= \frac{x^2+2(\frac{16x^2}{49})}{\frac{4x^2}{7}}$   
 $= \frac{49x^2+32x^2}{4x^2}$   
 $= \frac{81x^2}{4x^2} = \frac{81}{4}$

(3m)

(h) Make y the subject of the formula:  $\frac{1}{x} = \frac{1}{y} - \frac{1}{z}$

\*  $y^2 = xz - zy$   
 $yz + xy = xz$   
 $y(z+x) = xz$   
 $\therefore y = \frac{xz}{z+x}$

(2m)

(i) Given that  $V = kr^2$  and if  $V = 12$  when  $r = 2$ , find the value of  $V$  when  $r = 3$ .

$12 = 4k$   
 $\therefore k = 3$   
 $V = 3^2k$   
 $V = 9k$   
 $\therefore V = 27$

(2m)

**Question 2 (22 marks)**

(a) Find the value of  $x$  in the following:

(i)  $3^{x+1} = 9$   
 $3^{x+1} = 3^2$   
 $x+1 = 2$   
 $\therefore x = 1$  (1m)

(ii)  $4^{2x+1} = \sqrt{52}$   
 $4^{2x+1} = 32^{\frac{1}{2}}$   
 $(2^2)^{2x+1} = (2^5)^{\frac{1}{2}}$   
 $2^{4x+2} = 2^{\frac{5}{2}}$   
 $4x+2 = \frac{5}{2}$   
 $8x+4 = 5$   
 $8x = 1$   
 $\therefore x = \frac{1}{8}$  (1m)

(b) Evaluate the following:

(i)  $\log_3 81$   
 $= \log_3 3^4$   
 $= 4$  (1m)

(ii)  $\log_5 \frac{1}{125}$   
 $= \log_5 5^{-3}$   
 $= -3$  (1m)

(iii)  $\log_{0.1} 100$   
 $= \log_{10^{-1}} 10^2$   
 $= 2 \log_{10^{-1}} 10$   
 $= 2 \cdot (-1)$   
 $= -2$  (1m)

(iv)  $\log_{\sqrt{2}} 64$   
 $= \log_{2^{\frac{1}{2}}} 2^6$   
 $= \frac{\log_2 2^6}{\log_2 2^{\frac{1}{2}}}$   
 $= \frac{6}{\frac{1}{2}}$   
 $= 6 \div \frac{1}{2}$   
 $= 12$  (2m)

(v)  $\log_3 27 + \log_3 81$   
 $\log_3 3^3 + \log_3 3^4$   
 $= 3 + 4$   
 $= 7$  (2m)

(vi)  $\log_5 \sqrt{25} - \log_5 \frac{1}{125}$   
 $\log_5 5^{\frac{1}{2}} - (\log_5 5^{-3})$   
 $= \log_5 5^{\frac{1}{2}} - \log_5 5^{-3}$   
 $= \frac{1}{2} - (-3)$   
 $= 3\frac{1}{2}$  (2m)

(c) If  $\log_3 (2x+1) = 4 \log_3 2$ , find the value of  $x$   
 $\log_3 (2x+1) = \log_3 2^4$   
 $2x+1 = 16$   
 $2x = 15$   
 $\therefore x = \frac{15}{2}$  (2m)

(d) If  $\log_3 a = 1.38$  evaluate the following:

(i)  $\log_3 \sqrt{a}$   
 $= \frac{1}{2} \log_3 a$   
 $= \frac{1}{2} \times 1.38$   
 $= 0.69$  (1m)

(ii)  $\log_3 3a$   
 $= \log_3 3 + \log_3 a$   
 $= 1 + 1.38$   
 $= 2.38$  (2m)

(e) If  $\log p = \log q - 2 \log r$ , circle the correct statement below.

- (i)  $p = q - 2r$  (1m)
- (ii)  $p = \frac{q}{r^2}$  (1m)
- (iii)  $p = \frac{q}{2r}$  (1m)
- (iv)  $p = \frac{q}{r^2}$  (1m)

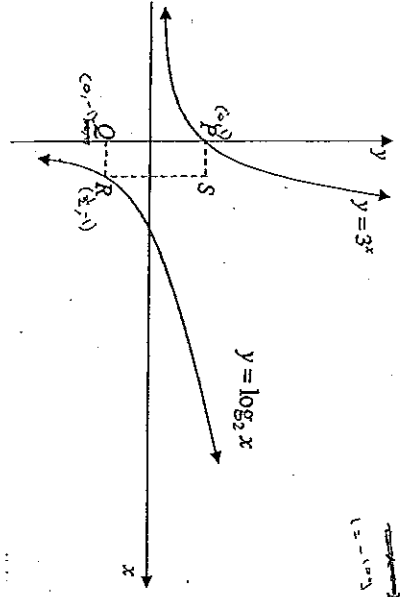
(f) If  $\log_3 x = \frac{\log_{10} x}{\log_{10} 3}$ , find  $y$  correct to 2 decimal places, if  $5^y = 120$

$\log_{10} 5^y = \log_{10} 120$   
 $y \log_{10} 5 = \log_{10} 120$   
 $y = \frac{\log_{10} 120}{\log_{10} 5}$   
 $\therefore y = 2.97$  (2 d.p.) (2m)



Question 3 (8 marks)

(a)



$PQRS$  is a rectangle. The coordinates of  $Q$  are  $(0, -1)$ .

(i) Find the coordinates of  $R$ .

$$y = 3^x \quad x = 2^{-1} \Rightarrow R\left(\frac{1}{2}, -1\right)$$

(1m)

(ii) Find the coordinates of  $P$ .

$$y = 3^x \quad y = 3^0 \Rightarrow P(0, 1)$$

(1m)

(iii) Find the area of rectangle  $PQRS$ .

$$A = \frac{1}{2} \times 2 = 1$$

$$\therefore A = 1 \text{ m}^2$$

(2m)

(b) (i) Evaluate  $\log_4(\log_2 16)$

$$= \log_4(4) = 1$$

(2m)

(ii) Simplify

$$\log_x(x^2 - 3x) = \log_x(x-3)$$

$$\log_x x^2 - \log_x 3x = \log_x x - \log_x 3$$

$$= 2 - \log_x 3x = 1 - \log_x 3$$

$$= 1 - \log_x 3x + \log_x 3$$

(2m)