TEST 10 Simultaneous Linear Equations

Marks: /80

Time: 1 hour 30 minutes

Name: .... Date: ....

INSTRUCTIONS TO CANDIDATES

Section A (40 marks)

Time: 45 minutes

1. Answer **all** the questions in this section.
2. Calculators may **not** be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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1 Solve the simultaneous equations

$$2x + 3y = 12,$$
$$x - 4y = -5.$$

Answer $x =$ ....

$y =$ .... [3]

---

2 Solve the simultaneous equations

$$4x - 3y = 18,$$
$$7x + 5y = 11.$$

Answer $x =$ ....

$y =$ .... [3]

---

3 Solve the simultaneous equations

$$2x - y = 11,$$
$$-2x + 5y + 7 = 0.$$

Answer $x =$ ....

$y =$ .... [3]

4 Solve the simultaneous equations

$$h - 5k = -3,$$
$$h + 3k = 1.$$

Answer $h =$ ....

$k =$ .... [3]

5 Solve the simultaneous equations

$$y = 2x - 3,$$
$$4x - 3y = 10.$$

Answer $x =$ ....

$y =$ .... [3]

6

7

Test

6 Solve the simultaneous equations

$$\frac{1}{2}q + 3p = 38,$$

$$\frac{1}{2}p - \frac{1}{4}q = -3.$$

Answer $p =$ ....

$q =$ .... [3]

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7 Solve the simultaneous equations

$$28 + 3y = 11x,$$

$$2y = 5x.$$

Answer $x =$ ....

$y =$ .... [3]

8 Solve the simultaneous equations

$$0.5x + 1.2y = 2.6,$$
$$0.8x - 0.3y = -2.5.$$

Answer $x =$ ....

$y =$ .... [3]

9 Solve the simultaneous equations

$$x = 2y + 13,$$
$$5x + 3y = 0.$$

Answer $x =$ ....

$y =$ .... [3]

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10 Solve the simultaneous equations

$$3(x - 1) - 5y = 1,$$
$$7x + 2(3 - y) = 25.$$

Answer $x =$ ....

$y =$ .... [4]

11 Solve the simultaneous equations

$$2px + qy = 20,$$
$$7y = \frac{1}{2}(2qx + pqy)$$

given that $p = 2$ and $q = 6$.

Answer $x =$ ....

$y =$ .... [4]

12 The ratio of two numbers is $\frac{3}{4}$. Seven times the smaller number is 17 more than five times the larger number. Find the two numbers.

Answer ...., ... [5]

INSTRUCTIONS TO CANDIDATES

Section B (40 marks)

Time: 45 minutes

1. Answer **all** the questions in this section.
2. Calculators may be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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13 **(a)** Solve the simultaneous equations

$$2x + y = 3x - 5y = 13.$$

(b) Five years ago, Steven was 8 times as old as Jason. In three years' time, Steven will be twice as old as Jason. How old are Steven and Jason now? (Give your answers in years and months.)

Answer *(a)* $x =$ ....

$y =$ .... [3]

(b) Steven: ....yrs .... mths

Jason: ....yrs .... mths [4]

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14 **(a)** Andy has 25 coins, made up of 10-cent and 50-cent coins. If the total value of his coins is \$9.30, how many coins of each value does he have?

(b) Mrs Tan bought a total of 143 apples and pears. Later she found that one third of the apples and one fifth of the pears were rotten. She counted and found that the number of rotten apples was twice the number of rotten pears. How many of the apples she bought were rotten?

Answer (a) 10-cent coins: ....

50-cent coins: .... [4]

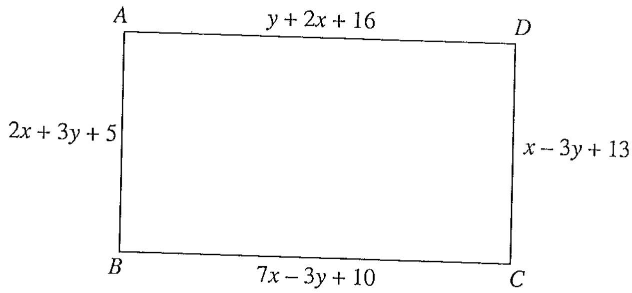
(b) .... apples [4]

15 **(a)** The sum of the digits of a two-digit number is 12. If the digits are reversed, the number will be $\frac{4}{7}$ of the original number. Find the number.

(b) In the diagram, $ABCD$ is a rectangle. Given that $AB = (2x + 3y + 5)$ cm, $BC = (7x - 3y + 10)$ cm, $CD = (x - 3y + 13)$ cm and $AD = (y + 2x + 16)$ cm.

(i) By forming suitable equations, find the values of x and y.

(ii) Calculate the area of rectangle $ABCD$.



Answer *(a)* .... [4]

(b) (i) x = ...

y = ... [5]

(ii) .... cm² [2]

16 Belinda and Michelle each invested a sum of money into two different investment plans in a bank. Belinda's plan earns her an interest rate of 8% per annum while Michelle's plan earns her an interest rate of 6% per annum. After 3 years, both of them withdrew their money and found that their interest gained were equal. If the combined amount of the money they invested was \$10 500, find the sum of money invested by each of them.

Answer Belinda: \$ ....

Michelle: \$ .... [6]

17 **(a)** A chemist has Solution A that contains 18% of concentrated acid and Solution B that contains 45% concentrated acid. How many litres of each should he use to make 12 litres of a mixture containing 36% concentrated acid?

(b) Mr Lee received \$1200 for renting out two television sets last year. If he charged \$10 per month more on one than the other, find the monthly rental on each of these television sets if the more expensive set was not rented out for 2 months last year.

Answer *(a)* Solution A: l

Solution B: .... l [4]

(b) \$ .... ,

\$ .... [4]

(d) $x^2 + 46x - 200 = 0$
$(x - 4)(x + 50) = 0$
$\therefore x - 4 = 0$ or $x + 50 = 0$
$x = 4$ **or** $x = -50$

(e) $\therefore x = 4$ Reject $x = -50$.
No. of kilograms of durians sold
$= \frac{300}{4} - 3$
$= 72$

Test 10: Simultaneous Linear Equations

Section A

1. **Method 1: Elimination Method**

$$2x + 3y = 12 \quad \text{— (1)}$$
$$x - 4y = -5 \quad \text{— (2)}$$

(2) × 2: $2x - 8y = -10$ — (3)
(1) – (3): $3y - (-8y) = 12 - (-10)$
$3y + 8y = 12 + 10$
$11y = 22$
$y = \frac{22}{11} = 2$

To eliminate x, make the coefficients of x to be equal by multiplying (2) by 2.

Substitute $y = 2$ into (2):
$x - 4(2) = -5$
$x - 8 = -5$
$x = -5 + 8$
$= 3$

Substitute the value of y into any of the original equations to find the value of x.

$\therefore$ the solution set is $x = 3, y = 2$.

Teacher's Tip

1) In this method, we first eliminate one of the two unknowns. Either of the unknowns may be eliminated first. In the above question, it is easier to eliminate x.
2) Make a habit of checking your answer either mentally or by writing down.

Check: Substitute $x = 3$, $y = 2$ into (1):
LHS $= 2(3) + 3(2) = 6 + 6 = 12 =$ RHS

Method 2: Substitution Method

$$2x + 3y = 12 \quad \text{— (1)}$$
$$x - 4y = -5 \quad \text{— (2)}$$

From (2): $x = -5 + 4y$ — (3)

Make x the subject of the equation.

Substitute (3) into (1):
$2(-5 + 4y) + 3y = 12$
$-10 + 8y + 3y = 12$
$11y = 22$
$y = \frac{22}{11} = 2$

Substitute (3) into (1) to obtain an equation with only the y variable.

Substitute $y = 2$ into (3):
$x = -5 + 4(2)$
$= -5 + 8$
$= 3$

$\therefore$ the solution set is $x = 3, y = 2$.

Teacher's Tip

In this method, we solve one of the equations for one unknown and substitute this expression into the other equation to solve for the remaining unknown.

2.
$$4x - 3y = 18 \quad \text{— (1)}$$
$$7x + 5y = 11 \quad \text{— (2)}$$

(1) × 5: $20x - 15y = 90$ — (3)
(2) × 3: $21x + 15y = 33$ — (4)
(4) + (3): $41x = 123$
$x = \frac{123}{41} = 3$

To eliminate y, make the coefficients of y to be equal by multiplying (1) by 5 and (2) by 3.

Substitute $x = 3$ into (1):
$4(3) - 3y = 18$
$12 - 3y = 18$
$-3y = 6$
$y = \frac{6}{-3} = -2$

$\therefore$ the solution set is $x = 3, y = -2$.

3.
$$2x - y = 11 \quad \text{— (1)}$$
$$-2x + 5y = -7 \quad \text{— (2)}$$

Rewrite $-2x + 5y + 7 = 0$ as $-2x + 5y = -7$.

(1) + (2): $4y = 4$
$y = \frac{4}{4} = 1$

Substitute $y = 1$ into (1):
$2x - 1 = 11$
$2x = 12$
$x = \frac{12}{2} = 6$

$\therefore$ the solution set is $x = 6, y = 1$.

Teacher's Tip

Note that the coefficients of x are equal but one is positive and the other is negative. To eliminate x, add the two equations.

4.
$$h - 5k = -3 \quad \text{— (1)}$$
$$h + 3k = 1 \quad \text{— (2)}$$

Since the coefficients of h are equal, subtract (1) from (2) to eliminate h.

(2) – (1): $3k - (-5k) = 1 - (-3)$
$3k + 5k = 1 + 3$
$8k = 4$
$k = \frac{4}{8} = \frac{1}{2}$

Substitute $k = \frac{1}{2}$ into (2):

$$h + 3\left(\frac{1}{2}\right) = 1$$

$$h + 1\frac{1}{2} = 1$$

$$h = -\frac{1}{2}$$

$\therefore$ the solution set is $\boldsymbol{h = -\frac{1}{2}, k = \frac{1}{2}}$.

5. $y = 2x - 3$ —— (1)
$4x - 3y = 10$ —— (2)

Substitute (1) into (2):

$$4x - 3(2x - 3) = 10$$
$$4x - 6x + 9 = 10$$
$$-2x = 1$$
$$x = -\frac{1}{2}$$

Use the substitution method when one of the variable y, has a coefficient of 1.

Substitute $x = -\frac{1}{2}$ into (1):

$$y = 2\left(-\frac{1}{2}\right) - 3$$
$$= -1 - 3$$
$$= -4$$

$\therefore$ the solution set is $\boldsymbol{x = -\frac{1}{2}, y = -4}$.

Teacher's Tip

The substitution method is chosen as it is more efficient because it requires less rewriting of the equations.

6. $3p + \frac{1}{2}q = 38$ —— (1)
$\frac{1}{2}p - \frac{1}{4}q = -3$ —— (2)

Rewrite $\frac{1}{2}q + 3p = 38$ as $3p + \frac{1}{2}q = 38$.

(2) × 2: $p - \frac{1}{2}q = -6$ —— (3)

(1) + (3): $4p = 32$

$$p = \frac{32}{4} = 8$$

Substitute $p = 8$ into (3):

$$8 - \frac{1}{2}q = -6$$
$$-\frac{1}{2}q = -14$$
$$q = -14 \times (-2)$$
$$= 28$$

$\therefore$ the solution set is $\boldsymbol{p = 8, q = 28}$.

7. $11x - 3y = 28$ —— (1)
$5x - 2y = 0$ —— (2)

Rewrite $28 + 3y = 11x$ as $11x - 3y = 28$ and $2y = 5x$ as $5x - 2y = 0$.

(1) × 2: $22x - 6y = 56$ —— (3)
(2) × 3: $15x - 6y = 0$ —— (4)
(3) – (4): $7x = 56$

$$x = \frac{56}{7} = 8$$

Substitute $x = 8$ into (2):

$$5(8) - 2y = 0$$
$$2y = 40$$
$$y = \frac{40}{2} = 20$$

$\therefore$ the solution set is $\boldsymbol{x = 8, y = 20}$.

8. $0.5x + 1.2y = 2.6$ —— (1)
$0.8x - 0.3y = -2.5$ —— (2)

(2) × 4: $3.2x - 1.2y = -10$ —— (3)
(1) + (3): $3.7x = -7.4$

$$x = \frac{-7.4}{3.7} = -2$$

Substitute $x = -2$ into (1):

$$0.5(-2) + 1.2y = 2.6$$
$$-1 + 1.2y = 2.6$$
$$1.2y = 3.6$$
$$y = \frac{3.6}{1.2} = 3$$

$\therefore$ the solution set is $\boldsymbol{x = -2, y = 3}$.

9. $x = 2y + 13$ —— (1)
$5x + 3y = 0$ —— (2)

Substitute (1) into (2):

$$5(2y + 13) + 3y = 0$$
$$10y + 65 + 3y = 0$$
$$13y = -65$$
$$y = \frac{-65}{13} = -5$$

Substitute $y = -5$ into (1):

$$x = 2(-5) + 13$$
$$= -10 + 13$$
$$= 3$$

$\therefore$ the solution set is $\boldsymbol{x = 3, y = -5}$.

10. $3(x - 1) - 5y = 1$ —— (1)
$7x + 2(3 - y) = 25$ —— (2)

Simplify (1) and (2) first.

From (1): $3x - 3 - 5y = 1$
$3x - 5y = 4$ —— (3)

From (2): $7x + 6 - 2y = 25$
$7x - 2y = 19$ —— (4)

(3) × 2: $6x - 10y = 8$ —— (5)
(4) × 5: $35x - 10y = 95$ —— (6)

(6) – (5): $29x = 87$

$$x = \frac{87}{29} = 3$$

Substitute $x = 3$ into (3):

$$3(3) - 5y = 4$$
$$9 - 5y = 4$$
$$-5y = -5$$
$$y = \frac{-5}{-5} = 1$$

∴ the solution set is $\boldsymbol{x = 3, y = 1}$.

11. When $p = 2$, $q = 6$,

$$2px + qy = 20 \Rightarrow 2(2)x + 6y = 20$$
$$4x + 6y = 20 \text{ —— (1)}$$
$$7y = \frac{1}{2}(2qx + pqy) \Rightarrow 7y = \frac{1}{2}[2(6)x + (2)(6)y]$$
$$7y = \frac{1}{2}(12x + 12y)$$
$$7y = 6x + 6y$$
$$y = 6x \text{ —— (2)}$$

Substitute (2) into (1):

$$4x + 6(6x) = 20$$
$$4x + 36x = 20$$
$$40x = 20$$
$$x = \frac{20}{40} = \frac{1}{2}$$

Substitute $x = \frac{1}{2}$ into (2):

$$y = 6\left(\frac{1}{2}\right) = 3$$

∴ the solution set is $\boldsymbol{x = \frac{1}{2}, y = 3}$.

12. Let the smaller number be x and the larger number be y.

$$\frac{x}{y} = \frac{3}{4}$$ ← The ratio of the two numbers is $\frac{3}{4}$.

$$4x = 3y$$
$$4x - 3y = 0 \text{ —— (1)}$$
$$7x = 17 + 5y$$ ← 7 times the smaller number is 17 more than 5 times the larger number.

$$7x - 5y = 17 \text{ —— (2)}$$

(1) × 5: $20x - 15y = 0$ —— (3)

(2) × 3: $21x - 15y = 51$ —— (4)

(4) – (3): $x = 51$

Substitute $x = 51$ into (1):

$$4(51) - 3y = 0$$
$$3y = 204$$
$$y = \frac{204}{3} = 68$$

∴ the two numbers are **51** and **68**.

Section B

13. (a) $2x + y = 3x - 5y = 13$ — Rewrite as two separate equations.

$$2x + y = 13 \text{ —— (1)}$$
$$3x - 5y = 13 \text{ —— (2)}$$

(1) × 5: $10x + 5y = 65$ —— (3)

(2) + (3): $13x = 78$

$$x = \frac{78}{13} = 6$$

Substitute $x = 6$ into (1):

$$2(6) + y = 13$$
$$12 + y = 13$$
$$y = 1$$

∴ the solution set is $\boldsymbol{x = 6, y = 1}$.

(b) Let Steven's present age be x years and Jason's present age be y years.

$$x - 5 = 8(y - 5)$$ ← Five years ago, Steven was 8 times as old as Jason.

$$x - 5 = 8y - 40$$
$$x - 8y = -35 \text{ —— (1)}$$
$$x + 3 = 2(y + 3)$$ ← In three years' time, Steven will be twice as old as Jason.

$$x + 3 = 2y + 6$$
$$x - 2y = 3 \text{ —— (2)}$$

(2) – (1): $-2y - (-8y) = 3 - (-35)$

$$-2y + 8y = 3 + 35$$
$$6y = 38$$
$$y = \frac{38}{6} = 6\frac{1}{3} \text{ years}$$
$$= 6 \text{ years } 4 \text{ months}$$

Substitute $y = 6\frac{1}{3}$ into (2):

$$x - 2\left(6\frac{1}{3}\right) = 3$$
$$x = 3 + 12\frac{2}{3}$$
$$= 15\frac{2}{3} \text{ years}$$
$$= 15 \text{ years } 8 \text{ months}$$

∴ Jason is 6 years 4 months old now and Steven is 15 years 8 months old now.

Teacher's Tip

To solve word problems involving simultaneous equations:

1) Assign variables to the two unknown quantities to be found.
2) Use the information given to write two equations in two unknowns.
3) Solve the simultaneous equations to find the unknowns.
4) Find the solution to the problem.

14. (a) Let the number of 10-cent coins be x and the number of 50-cent coins be y.

$x + y = 25$ —— (1) ← There are 25 coins altogether.

$10x + 50y = 930$ —— (2) ← The total value of the coins is $9.30. Express the amount in cents.

From (1): $x = 25 - y$ —— (3)

Substitute (3) into (2):

$$10(25 - y) + 50y = 930$$
$$250 - 10y + 50y = 930$$
$$40y = 680$$
$$y = \frac{680}{40} = 17$$

Substitute $y = 17$ into (3):

$$x = 25 - 17 = 8$$

∴ Andy has 8 ten-cent coins and 17 fifty-cent coins.

(b) Let x represent the number of rotten apples and y represent the number of rotten pears.

$x = 2y$ —— (1) ← The number of rotten apples was twice the number of rotten pears.

$3x + 5y = 143$ —— (2) ← $\frac{1}{3}$ of no. of apples $= x$; No. of apples $= 3x$; $\frac{1}{5}$ of no. of pears $= y$; No. of pears $= 5y$; The total no. of fruits = 143

Substitute (1) into (2):

$$3(2y) + 5y = 143$$
$$6y + 5y = 143$$
$$11y = 143$$
$$y = \frac{143}{11} = 13$$

Substitute $y = 13$ into (1):

$$x = 2(13) = 26$$

∴ 26 of the apples bought were rotten.

15. (a) Let the two digit number be $10x + y$.

$x + y = 12$ ← The sum of the digits is 12.

$x = 12 - y$ —— (1)

$10y + x = \frac{4}{7}(10x + y)$ ← If the digits are reversed, the number is $\frac{4}{7}$ of the original number.

$$7(10y + x) = 4(10x + y)$$
$$70y + 7x = 40x + 4y$$
$$66y - 33x = 0 \text{ —— (2)}$$

Substitute (1) and (2):

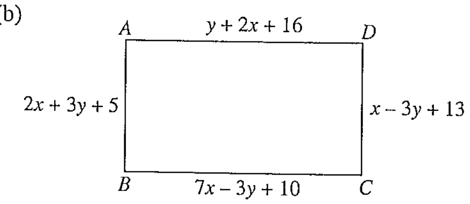
$$66y - 33(12 - y) = 0$$
$$66y - 396 + 33y = 0$$
$$99y = 396$$
$$y = \frac{396}{99} = 4$$

Substitute $y = 4$ into (1):

$$x = 12 - 4 = 8$$

∴ the number is $10(8) + 4 =$ **84**.

(b)



(i) **Teacher's Tip**

Since $ABCD$ is a rectangle, $AB = CD$ and $BC = AD$.

$$AB = CD$$
$$2x + 3y + 5 = x - 3y + 13$$
$$x + 6y = 8$$
$$x = 8 - 6y \text{ —— (1)}$$

$$BC = AD$$
$$7x - 3y + 10 = y + 2x + 16$$
$$5x - 4y = 6 \text{ —— (2)}$$

Substitute (1) into (2):

$$5(8 - 6y) - 4y = 6$$
$$40 - 30y - 4y = 6$$
$$-34y = -34$$
$$y = \frac{-34}{-34} = 1$$

Substitute $y = 1$ into (1):

$$x = 8 - 6(1) = 8 - 6 = 2$$

∴ $x = 2$ and $y = 1$.

(ii) $AB = 2(2) + 3(1) + 5 = 12$ cm

$AD = 1 + 2(2) + 16 = 21$ cm

Area of rectangle $ABCD = 12 \times 21$
$= \mathbf{252\ cm^2}$

16. Let the sum of money invested by Belinda be $\$x$ and Michelle be $\$y$.

$x + y = 10\,500$ —— (1) ← The combined amount invested as $10 500.

Interest gained by Belinda after 3 years

$= \$\left(\frac{x \times 8 \times 3}{100}\right)$ ← Use $I = \frac{PRT}{100}$ where I = Simple interest, P = Principal, R = Rate (per annum) and T = Time (years).

$= \$\left(\frac{6}{25}x\right)$

Interest gained by Michelle after 3 years

$= \$\left(\frac{y \times 6 \times 3}{100}\right)$

$= \$\left(\frac{9}{50}y\right)$

$\$\left(\frac{6}{25}x\right) = \$\left(\frac{9}{50}y\right)$ Their interest gained were equal after 3 years.

$x = \frac{9}{50}y \times \frac{25}{6}$

$x = \frac{3}{4}y$ —— (2)

Substitute (2) into (1):

$\frac{3}{4}y + y = 10\ 500$

$\frac{7}{4}y = 10\ 500$

$y = 10\ 500 \times \frac{4}{7}$

$= 6000$

Substitute $y = 6000$ into (2):

$x = \frac{3}{4} \times 6000$

$= 4500$

∴ Belinda invested \$4500 and Michelle invested \$6000.

17. (a) Let x be the number of litres from Solution A and y be the number of litres from Solution B.

$x + y = 12$ —— (1) 12 l of mixture is to be produced.

$0.18x + 0.45y = 0.36\,(12)$

$0.18x + 0.45y = 4.32$ —— (2)

0.18 of Solution A is concentrated acid while 0.45 of Solution B is concentrated acid. The mixture (12 l) produced contains 0.36 of concentrated acid.

(1) × 0.18: $0.18x + 0.18y = 2.16$ —— (3)
(2) − (3): $0.27y = 2.16$

$y = \frac{2.16}{0.27} = 8$

Substitute $y = 8$ into (1):

$x + 8 = 12$

$x = 4$

∴ 4 l of Solution A and 8 l of Solution B are needed.

(b) Let \$$x$ be the monthly rental of the more expensive television set and \$$y$ be the monthly rental of the less expensive television set.

$x = y + 10$
$x - y = 10$ —— (1)

Given that the monthly rental differs by \$10.

$10x + 12y = 1200$
$5x + 6y = 600$ —— (2)
(1) × 5: $5x - 5y = 50$ —— (3)
(2) − (3): $11y = 550$

$y = \frac{550}{11} = 50$

The more expensive set was rented for 10 months and the other for 12 months. The total annual income was \$1200.

Substitute $y = 50$ into (1):

$x - 50 = 10$

$x = 60$

∴ the monthly rentals are \$60 and \$50 respectively.

Mid-Year Examination Specimen Paper A: Part 1

1. (a) $0.065498 \approx$ **0.0655** (correct to 3 sig. fig.)

↑ Zeros preceding the first non-zero digit are not significant.

(b) $8189 \approx$ **8190** (correct to 3 sig. fig.)

2. Profit = (12% of \$250) + (\$285 − \$250)

$= \left(\frac{12}{100} \times \$250\right) + \$35$

$= \$30 + \$35$

$= \$65$

Percentage profit $= \frac{\$65}{(\$250 + \$250)} \times 100\%$

$= \frac{65}{500} \times 100\%$

= **13%**

Teacher's Tip

Percentage profit $= \frac{\text{Profit}}{\text{Cost price}} \times 100\%$