TEST 10 Volume and Surface Area

Marks: /60

Time: 1 hour 30 minutes

Name: .... Date: ....

INSTRUCTIONS TO CANDIDATES

Section A (30 marks) **Time: 45 minutes**

1. Answer **all** the questions in this section.
2. Calculators may **not** be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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1 A rectangular block of metal 20 cm by 21 cm by 22 cm is melted down and minted into cylindrical coins of diameter 14 cm and height 2 cm. Find the number of coins that can be made.

$\left[\text{Take } \pi \text{ to be } \frac{22}{7}.\right]$

Answer .... coins [2]

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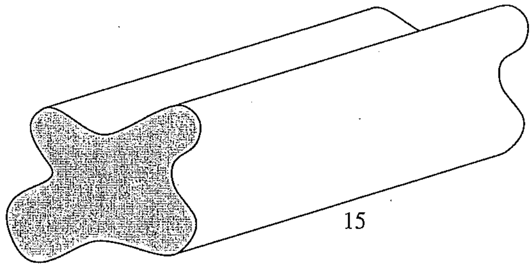
2 (a) A solid cube of side x cm weighs 160 g. If its density is 2.5 g/cm^3, find the value of x.
(b) The diagram shows a solid of length 15 cm and weighing 420 g. The area of the cross-section of the solid is 20 cm^2. Find the density of the solid.



Answer (a) x = ... [2]

(b) .... g/cm^3 [1]

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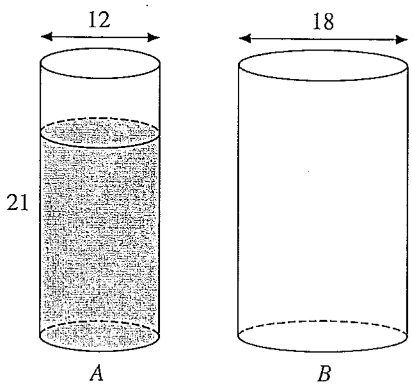
3 Two cylindrical jars A and B, have diameters 12 cm and 18 cm respectively. Initially, Cylinder A contains water to a depth of 21 cm and Cylinder B is empty. If all the water from Cylinder A is poured into Cylinder B, find the height of water in Cylinder B.



Answer .... cm [3]

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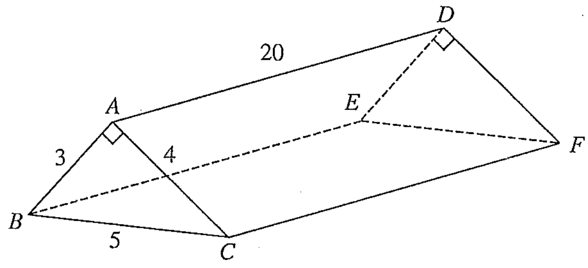
4 The diagram shows a triangular prism in which three of the faces are rectangular. Given that $AB = 3$ cm, $BC = 5$ cm, $AC = 4$ cm and $AD = 20$ cm, calculate

(a) the volume of the prism,

(b) the total surface area.



Answer (a) .... cm³ [2]

(b) .... cm² [2]

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5 The volume of water in a glass cylinder is 704 cm^3. Given that the height of water in the cylinder is 14 cm, calculate the diameter of the glass cylinder.

$\left[\text{Take } \pi \text{ to be } \frac{22}{7}.\right]$

Answer .... cm [2]

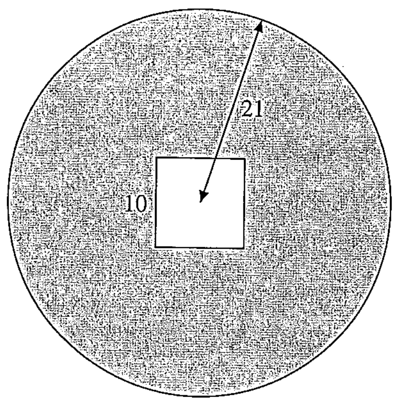
---

6 The diagram shows the cross-section of a circular metal disc of radius 21 mm. The central hole of the disc is a square of side 10 mm.

(a) Calculate the shaded area of the cross-section of the disc.

(b) Given that the thickness of the disc is 5 mm, calculate the volume of metal needed to make 20 such discs.

$\left[\text{Take } \pi \text{ to be } \frac{22}{7}.\right]$



Answer (a) .... mm^2 [1]

(b) .... mm^3 [2]

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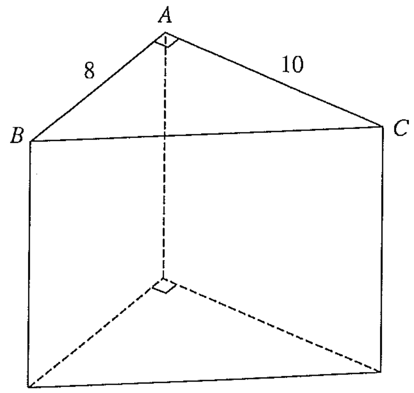
7 The diagram represents a prism where $AB = 8$ cm and $AC = 10$ cm. The prism is completely filled with 2.5 kg of a chemical solution. Given that the density of the solution is 12.5 g/cm^3, find the height of the prism.



Answer .... cm [2]

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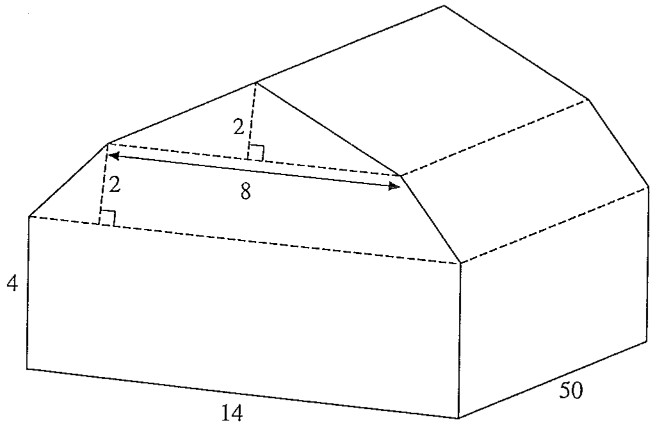
8 An exhibition hall is shown in the diagram below. The dimensions are given in the metres.

(a) Find the volume of air in the hall.

(b) If the density of air is approximately 1.26 kg/m^3, find the mass of air contained in the hall.



Answer (a) .... m^3 [2]

(b) .... kg [1]

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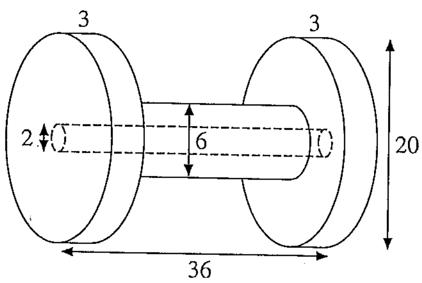
9 The diagram shows a wooden spindle made of two similar circular discs, each of diameter 20 cm and thickness 3 cm which are connected together by a solid wooden cylinder of diameter 6 cm and length 30 cm. A circular hole of diameter 2 cm is cored through the centre of the two discs and the cylinder. Calculate the volume of the spindle giving your answer in terms of π.



Answer .... cm^3 [3]

10 The diagram represents a solid wedge. The faces $ABFE$, $ADHE$, $DCGH$ and $BCGF$ are rectangles. $ABCD$ and $EFGH$ are trapeziums. $AB = 6$ cm, $BC = 12$ cm, $CD = 10$ cm, $AD = 4$ cm and $AE = 8$ cm.

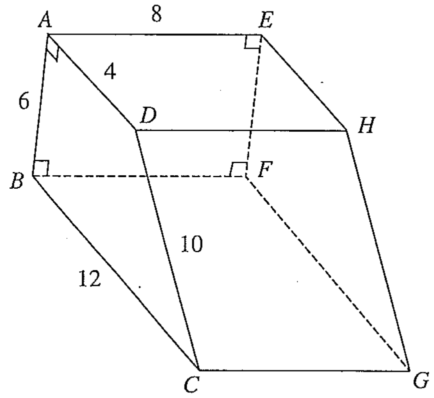
Calculate

(a) the area of $ABCD$,

(b) the volume of the solid,

(c) the total surface area of the solid,

(d) the mass of the solid given that its density is 7.5 g/cm^3.



Answer (a) .... cm^2 [1]

(b) .... cm^3 [1]

(c) .... cm^2 [2]

(d) .... g [1]

INSTRUCTIONS TO CANDIDATES

Section B (30 marks)

Time: 45 minutes

1. Answer **all** the questions in this section.
2. Calculators may be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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11 A rectangular tank of length 3.5 m and breadth 1.6 m contains 4200 litres of liquid chemical.

(a) Calculate the height of the liquid chemical in the tank, giving your answer in metres.

(b) After 280 solid metal cubes were dropped into the tank, the liquid level rises by 6.2 cm. Calculate the volume of each metal cube, giving your answer in cubic centimetres.

(c) The density of the liquid chemical is 2.1 g/cm^3 and the density of the metal cubes is 8.5 g/cm^3. Find the total mass of the liquid chemical and the metal cubes, giving your answer in kilograms.

Answer (a) .... m [2]

(b) .... cm^3 [2]

(c) .... kg [2]

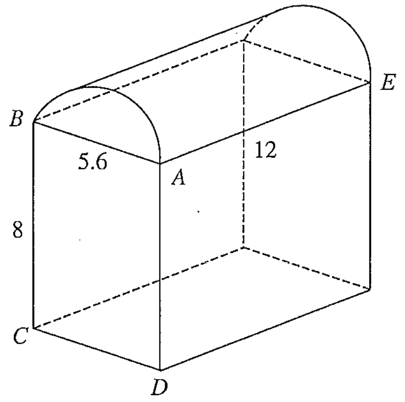
12 The diagram shows a closed metal storage container made up of a cuboid joined to half of a cylinder. $AB = 5.6$ m, $BC = 8$ m and $AE = 12$ m.

(a) Calculate the volume of the container, giving your answer in litres.

(b) The exterior surfaces of the container is to be painted. Find the total surface area to be painted.

(c) The paint is sold in 5-litre tins. One litre of paint covers 8 m^2. Find the number of tins that should be bought.

$\left[\text{Take } \pi \text{ to be } \frac{22}{7}.\right]$



Answer (a) .... l [2]

(b) .... m^2 [2]

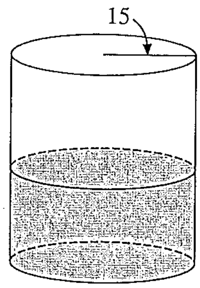
(c) .... tins [2]

13 **(a)** The diagram shows a cylindrical drum which is $\frac{1}{2}$ filled with liquid. The radius of the drum is 15 cm. After 4713 cm^3 of liquid is transferred into the drum, it became $\frac{2}{3}$ full.

(i) Calculate the height of the liquid in the drum after the transfer.

(ii) Find the surface area of the drum in contact with liquid after the transfer.

[Take π to be 3.142.]

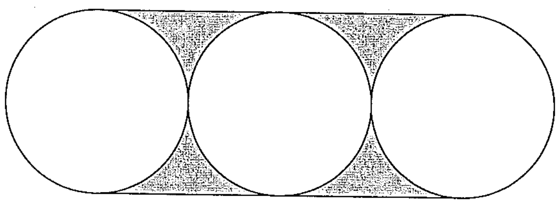


(b) Three circular cans are tied together with a piece of string. The radius of each can is 14 cm. The diagram shows the top view of the cans. Calculate

(i) the area of the shaded region,

(ii) the length of string needed to tie the three cans together.

$\left[\text{Take } \pi \text{ to be } \frac{22}{7}.\right]$



Answer *(a)* (i) .... cm [2]

(ii) .... cm^2 [2]

(b) (i) .... cm^2 [2]

(ii) .... cm [1]

14 The diagram shows the cross-sectional area of a metal pipe with an internal radius of 8 cm and a thickness of 1.5 cm.

(a) Calculate the area of the shaded region, giving your answer correct to the nearest square centimetres.

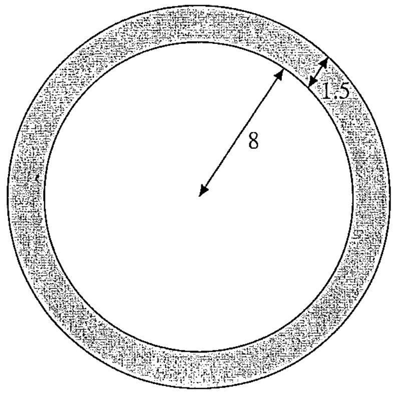
(b) If the pipe has a length of 12 m, calculate

(i) the volume of metal used in cubic centimetres,

(ii) the internal curved surface area in square centimetres.

(c) If the density of the metal is 4.25 g/cm^3, calculate the mass of pipe, giving your answer correct to the nearest kilogram.

[Take π to be 3.142.]



Answer (a) .... cm^2 [2]

(b) (i) .... cm^3 [1]

(ii) .... cm^2 [1]

(c) .... kg [2]

15 A carpenter made a table by carving out a prism from a rectangular block of wood. Diagram 1 shows the cross-sectional area of the table. The length of the table is 1.5 m.
[All dimensions in the diagrams are given in centimetres.]

(a) Calculate the volume of wood required to make the table, giving your answer in cubic centimetres.

(b) A customer requested for a slight change in the design of the table. He wanted a circular hole dug out below the table to provide more leg room. The cross-sectional area of the new design for the table is shown in Diagram 2. Calculate the volume of wood required to make this table.

(c) If the mass of the table in part (b) is 500 kg, find the density of wood used to make the table, giving your answer in g/cm^3, correct to 2 decimal places.

$\left[\text{Take } \pi \text{ to be } \frac{22}{7}.\right]$

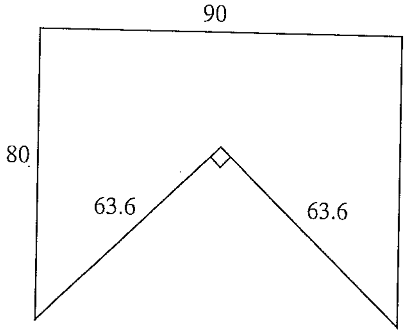


Diagram 1

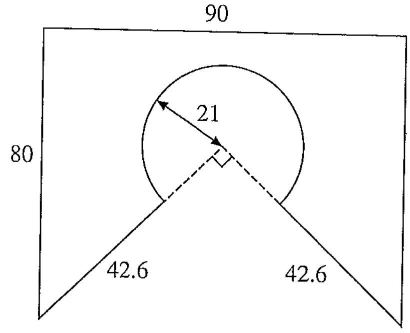


Diagram 2

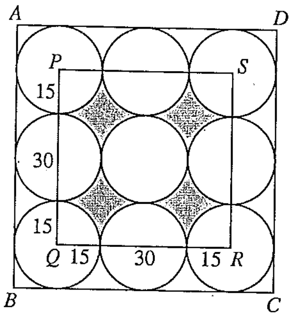
Answer (a) .... cm^3 [2]

(b) .... cm^3 [2]

(c) .... g/cm^3 [1]

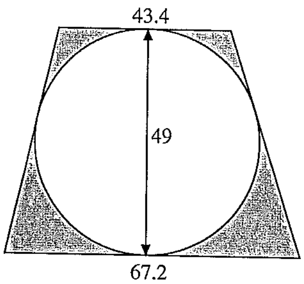
Area of rectangle $= x(3x)$
$= 3x^2$
$= 3(22)^2$
$= \mathbf{1452\ cm^2}$

(b)



Total area of shaded parts
= Area of square $PQRS$ – Area of 4 circles
$= 60^2 - 4 \times 3.142 \times 15^2$
$= \mathbf{772.2\ cm^2}$

15. (a)



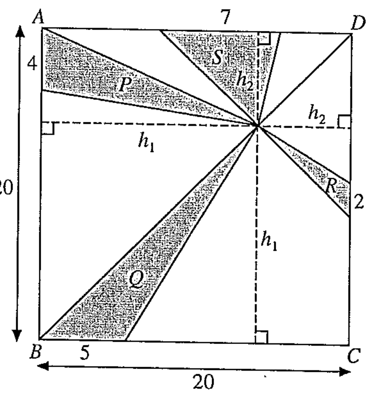
Teacher's Tip
The diameter of the circle is the height of the trapezium.

Area of shaded region
= Area of trapezium – Area of circle

$$= \left[\frac{1}{2} \times 49 \times (43.4 + 67.2)\right] - \frac{22}{7} \times \left(\frac{49}{2}\right)^2$$

$= 2709.7 - 1886.5$
$= \mathbf{823.2\ cm^2}$

(b)



Area of shaded region
= Area of $\triangle P$ + Area of $\triangle Q$ +
Area of $\triangle R$ + Area of $\triangle S$

$$= \left(\frac{1}{2} \times 4 \times h_1\right) + \left(\frac{1}{2} \times 5 \times h_1\right) + \left(\frac{1}{2} \times 2 \times h_2\right) + \left(\frac{1}{2} \times 7 \times h_2\right)$$

$$= 2h_1 + \frac{5}{2}h_1 + h_2 + \frac{7}{2}h_2$$

$$= \frac{9}{2}h_1 + \frac{9}{2}h_2$$

$$= \frac{9}{2}(h_1 + h_2)$$

$$= \frac{9}{2}(20) \quad \boxed{h_1 + h_2 = 20\text{ cm}}$$

$= \mathbf{90\ cm^2}$

Test 10: Volume and Surface Area

Section A

1. Volume of rectangular block
$= 20 \times 21 \times 22$
$= 9240\text{ cm}^3$

Volume of each cylindrical coin

$$= \frac{22}{7} \times 7 \times 7 \times 2$$

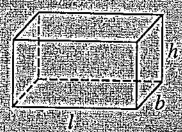
$= 308\text{ cm}^3$

No. of coins

$$= \frac{9240}{308}$$

$= \mathbf{30}$

Teacher's Tip
Volume of cuboid, $V = lbh$
where l = length,
b = breadth and h = height.



Volume of cylinder, $V = \pi r^2 h$
where r = radius and h = height.

2. (a) Density $= \dfrac{\text{Mass}}{\text{Volume}}$

$\therefore$ Volume $= \dfrac{\text{Mass}}{\text{Density}}$

$= \dfrac{160\text{ g}}{2.5\text{ g/cm}^3}$

$= 64\text{ cm}^3$

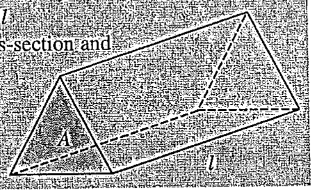
Volume of cube $= 64\text{ cm}^3$
$x^3 = 64$
$x = \sqrt[3]{64} = \mathbf{4}$

(b) Volume of solid = Area of cross-section × Length
$= 20 \times 15$
$= 300 \text{ cm}^3$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{420 \text{ g}}{300 \text{ cm}^3} = \mathbf{1.4 \text{ g/cm}^3}$$

Teacher's Tip

Volume of prism, $V = Al$
where A = area of cross-section and
l = length.



3. Let the height of water in Cylinder B be h cm.

$$\begin{pmatrix}\text{Volume of water}\\ \text{in Cylinder } A\end{pmatrix} = \begin{pmatrix}\text{Volume of water}\\ \text{in Cylinder } B\end{pmatrix}$$

$$\cancel{\pi} \times 6 \times 6 \times 21 = \cancel{\pi} \times 9 \times 9 \times h$$

$$h = \frac{6 \times 6 \times 21}{9 \times 9} = 9\frac{1}{3}$$

∴ the height of water in Cylinder B is $\mathbf{9\frac{1}{3}}$ **cm.**

4. (a) Area of cross-section $= \frac{1}{2} \times 3 \times 4$
$= 6 \text{ cm}^2$

Volume of prism = Area of cross-section × Length
$= 6 \times 20$
$= \mathbf{120 \text{ cm}^3}$

(b) Perimeter of cross-section $= 3 + 4 + 5$
$= 12$ cm

Total surface area

$$= \begin{pmatrix}\text{Perimeter of}\\ \text{cross-section}\end{pmatrix} \times (\text{Length}) + 2\begin{pmatrix}\text{Area of}\\ \text{cross-section}\end{pmatrix}$$

$= 12 \times 20 + 2(6)$
$= 240 + 12$
$= \mathbf{252 \text{ cm}^2}$

5. Let the radius of the cylinder be r cm.
Volume of water in cylinder $= 704 \text{ cm}^3$

$$\frac{22}{\cancel{7}_1} \times r^2 \times \cancel{14}^2 = 704$$

$$r^2 = \frac{704}{44} = 16$$

$$r = \sqrt{16} = 4$$

∴ the diameter of the glass cylinder is **8 cm.**

6. (a) Shaded area $= \left(\frac{22}{7} \times 21 \times 21\right) - 10^2$
$= 1386 - 100$
$= \mathbf{1286 \text{ cm}^2}$

(b) Volume of 20 discs
$= 20 \times (1286 \times 5)$
$= \mathbf{128\ 600 \text{ cm}^3}$

7. Let h cm be the height of the prism.

Volume of prism $= \left(\frac{1}{2} \times 8 \times 10\right) \times h$
$= 40h \text{ cm}^3$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

$$40h = \frac{\cancel{2500}^{200}}{\cancel{12.5}_1}$$

Change 2.5 kg to 2500 g

$$40h = 200$$

$$h = \frac{200}{40} = 5$$

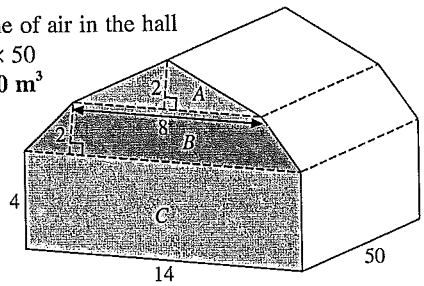
∴ the height of the prism is **5 cm.**

8. (a) Area of cross-section
= Area of A + Area of B + Area of C

$$= \left(\frac{1}{2} \times 8 \times 2\right) + \left[\frac{1}{2} \times 2 \times (8 + 14)\right] + (4 \times 14)$$

$= 8 + 22 + 56$
$= 86 \text{ m}^2$

Volume of air in the hall
$= 86 \times 50$
$= \mathbf{4300 \text{ m}^3}$



(b) Density $= \frac{\text{Mass}}{\text{Volume}}$

Mass = Density × Volume
$= 1.26 \text{ kg/m}^3 \times 4300 \text{ m}^3$
$= \mathbf{5418 \text{ kg}}$

9. Volume of spindle

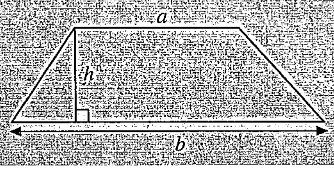
$$= \begin{pmatrix}\text{Volume of}\\ \text{2 circular}\\ \text{discs}\end{pmatrix} + \begin{pmatrix}\text{Volume of}\\ \text{cylinder}\end{pmatrix} - \begin{pmatrix}\text{Volume}\\ \text{of circular}\\ \text{hole}\end{pmatrix}$$

$= 2(\pi \times 10^2 \times 3) + (\pi \times 3^2 \times 30) - (\pi \times 1^2 \times 36)$
$= 600\pi + 270\pi - 36\pi$
$= \mathbf{834\pi \text{ cm}^3}$

10. (a) Area of $ABCD = \frac{1}{2} \times 6 \times (4 + 12)$
$= \mathbf{48\ cm^2}$

Teacher's Tip

Area of trapezium $= \frac{1}{2} \times$ height $\times$ sum of parallel sides
$= \frac{1}{2}h(a + b)$



(b) Volume of solid $= \left(\begin{array}{c}\text{Area of}\\ \text{cross-section}\end{array}\right) \times$ Length
$=$ Area of $ABCD \times AE$
$= 48 \times 8$
$= \mathbf{384\ cm^3}$

(c) Total surface area
$= \left(\begin{array}{c}\text{Perimeter of}\\ \text{cross-section}\end{array}\right) \times (\text{Length}) + 2\left(\begin{array}{c}\text{Area of}\\ \text{cross-section}\end{array}\right)$
$= (6 + 12 + 10 + 4) \times 8 + 2(48)$
$= 256 + 96$
$= \mathbf{352\ cm^2}$

(d) Density $= \frac{\text{Mass}}{\text{Volume}}$
Mass $=$ Density $\times$ Volume
$= 7.5\ \text{g/cm}^3 \times 384\ \text{cm}^3$
$= \mathbf{2880\ g}$

Section B

11. (a) Let h cm be the height of liquid chemical in the tank.
Volume of liquid chemical $= 4200\ l$
$350 \times 160 \times h = 4\ 200\ 000\ \text{cm}^3$
$h = \frac{4\ 200\ 000}{350 \times 160}$
$= 75$ cm
$= \frac{75}{100}$ m
$= 0.75$ m
$\therefore$ the height of the liquid chemical is **0.75 m.**

Alternative method:
Let h m be the height of liquid chemical in the tank.
Volume of chemical $= 4200\ l$
$3.5 \times 1.6 \times h = \frac{4200}{1000}\ \text{m}^3$
$h = \frac{4.2}{3.5 \times 1.6}$
$= 0.75$
$\therefore$ the height of the liquid chemical is **0.75 m.**

Teacher's Tip

1 litre (l) $= 1000\ \text{cm}^3$
$1000\ l = 1\ \text{m}^3$
$1\ \text{m} = 100\ \text{cm}$

(b) Volume of 250 solid metal cubes
$= 350 \times 160 \times 6.2$
$= 347\ 200$
Volume of each metal cube
$= \frac{347\ 200}{280}$
$= \mathbf{1240\ cm^3}$

(c) Mass $=$ Density $\times$ Volume
Mass of liquid chemical
$= 2.1\ \text{g/cm}^3 \times 4\ 200\ 000\ \text{cm}^3$
$= 8\ 820\ 000$ g
$= 8820$ kg

Mass of metal cubes $= 8.5\ \text{g/cm}^3 \times 347\ 200\ \text{cm}^3$
$= 2\ 951\ 200$ g
$= 2951.2$ kg

Total mass of liquid chemical and metal cubes
$= 8820 + 2951.2$
$= \mathbf{11\ 771.2\ kg}$

12. (a) Volume of container
$= \left(\begin{array}{c}\text{Volume of}\\ \text{cuboid}\end{array}\right) + \left(\begin{array}{c}\text{Volume of}\\ \text{half a cylinder}\end{array}\right)$
$= (12 \times 5.6 \times 8) + \frac{1}{2}\left[\frac{22}{7} \times \left(\frac{5.6}{2}\right)^2 \times 12\right]$
$= 537.6 + 147.84$
$= 685.44\ \text{m}^3$
$= \mathbf{685\ 440\ \mathit{l}}$ $\quad$ $1\ \text{m}^3 = 1000\ l$

(b) Total surface area
$= \left(\begin{array}{c}\text{Perimeter of}\\ \text{cross-section}\end{array} \times \text{Length}\right) + 2\left(\begin{array}{c}\text{Area of}\\ \text{cross-section}\end{array}\right)$
$= \left[\left(8 + 5.6 + 8 + \frac{1}{2} \times \frac{22}{7} \times 5.6\right) \times 12\right]$
$+ 2\left[8 \times 5.6 + \frac{1}{2} \times \frac{22}{7} \times \left(\frac{5.6}{2}\right)^2\right]$
$= 364.8 + 114.24$
$= \mathbf{479.04\ m^2}$

(c) Area that can be covered by 1 tin
$= 5 \times 8$
$= 40\ \text{m}^2$ $\quad$ Given each tin contains $5\ l$ and each litre can paint $8\ \text{m}^2$.
$\therefore$ no. of tins required
$= \frac{479.04}{40} = 11.976$
$\therefore$ no. of tins bought $=$ **12**

13. (a) (i) $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

Let the height of the drum be h cm.

$\therefore \frac{1}{6} \times 3.142 \times 15^2 \times h = 4713$

$h = \frac{4713 \times 6}{3.142 \times 15^2}$

$= 40$

Height of liquid in drum after the transfer

$= \frac{2}{3} \times 40$

$= \mathbf{26\frac{2}{3}\ cm}$

(ii) Surface area of drum in contact with the liquid

$= (3.142 \times 15^2) + \left(2 \times 3.142 \times 15 \times 26\frac{2}{3}\right)$

$= 706.95 + 2513.6$

$= \mathbf{3220.55\ cm^2}$

Teacher's Tip

Surface area of a closed cylinder $= 2\pi r^2 + 2\pi rh$

$= 2\pi r(r + h)$

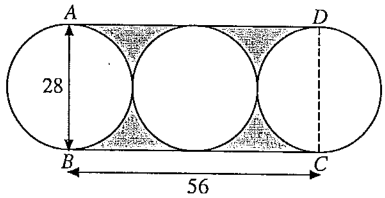
Surface area of cylinder with an opened top

$= \pi r^2 + 2\pi rh$

$= \pi r(r + 2h)$

where r = radius and h = height.

(b) (i)



Area of shaded region

= Area of rectangle $ABCD$ – 2(Area of circle)

$= (28 \times 56) - 2\left(\frac{22}{7} \times 14^2\right)$

$= 1568 - 1232$

$= \mathbf{336\ cm^2}$

(ii) Length of string needed

$= AD + BC$ + Circumference of circle

$= 56 + 56 + 2 \times \frac{22}{7} \times 14$

$= \mathbf{200\ cm}$

Teacher's Tip

The arcs of the two semicircles at each end forms a complete circle.

14. (a) Area of shaded region

$= 3.142(8 + 1.5)^2 - 3.142(8)^2$

$= 3.142[9.5^2 - 8^2]$

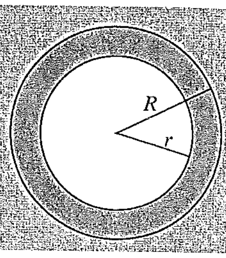
$= 82.4775$

$\approx \mathbf{82\ cm^2}$ (correct to the nearest cm^2)

Teacher's Tip

Area of annulus $= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$



(b) (i) Volume of metal

$= 82.4775 \times 1200$

$= \mathbf{98\ 973\ cm^3}$

12 m = 12 × 100 = 1200 cm

(ii) Internal curved surface area

$= 2 \times 3.142 \times 8 \times 1200$

$= \mathbf{60\ 326.4\ cm^2}$

Use the formula $2\pi rh$.

(c) Mass = Density × Volume

$= 4.25\ g/cm^3 \times 98\ 973\ cm^3$

$= 420\ 635.25$ g

$= 420.63525$ kg

$\approx$ **421 kg** (correct to the nearest kg)

15. (a) Cross-sectional area of table

= Area of rectangle – Area of triangle

$= 80 \times 90 - \frac{1}{2} \times 63.6 \times 63.6$

$= 5177.52\ cm^2$

Volume of wood required

$= 5177.52 \times 150$

$= \mathbf{776\ 628\ cm^3}$

1.5 m = 150 cm

(b) Volume of wood dug out

$= \left(\begin{array}{c}\text{Area of}\\ \text{cross-section}\end{array}\right) \times \text{Length}$

$= \left(\frac{3}{4} \times \frac{22}{7} \times 21^2\right) \times 150$

$= 155\ 925\ cm^3$

Volume of wood required

$= 776\ 628 - 155\ 925$

$= \mathbf{620\ 703\ cm^3}$

(c) Density $= \frac{\text{Mass}}{\text{Volume}}$

$= \frac{500\ \text{kg}}{620\ 703\ \text{cm}^3}$

$= \frac{500\ 000\ \text{g}}{620\ 703\ \text{cm}^3}$

$\approx \mathbf{0.81\ g/cm^3}$ (correct to 2 d.p.)