

TEST 17

Trigonometrical Ratios

Marks: /80

Time: 1 hour 30 minutes

Name: Date:

INSTRUCTIONS TO CANDIDATES

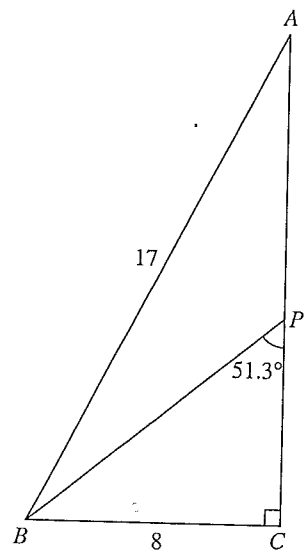
Section A (40 marks)

Time: 45 minutes

1. Answer **all** the questions in this section.
2. Calculators may **not** be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [] at the end of each question.

- 1 In the diagram, ABC is a right-angled triangle. $AB = 17$ cm, $BC = 8$ cm and $\widehat{BPC} = 51.3^\circ$. Using as much of the information given below as is necessary, find
- (a) $\sin \widehat{BAC}$,
 - (b) the length of AP .

$\sin 51.3^\circ = 0.78$
$\cos 51.3^\circ = 0.63$
$\tan 51.3^\circ = 1.25$

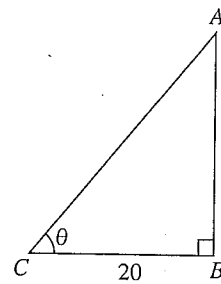


Answer (a) [1]

(b) $AP =$ cm [3]

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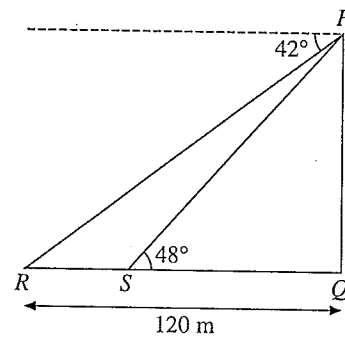
- 2 In the right-angled triangle ABC , $\cos \theta = \frac{5}{13}$ and $BC = 20$ cm. Calculate the length of AC .



Answer $AC = \dots\dots\dots$ cm [2]

- 3 Jack stands at P , the top of a vertical cliff PQ overlooking the sea. He sees a ship at R which is 120 m from Q . The angle of depression of R from P is 42° .
- (a) Using as much of the information given below as is necessary, calculate the height of the cliff.
- (b) A boat is anchored at S where QSR is a straight line. The angle of elevation of P from S is 48° . Calculate the distance RS .

$\sin 42^\circ = 0.669, \cos 42^\circ = 0.743, \tan 42^\circ = 0.900$



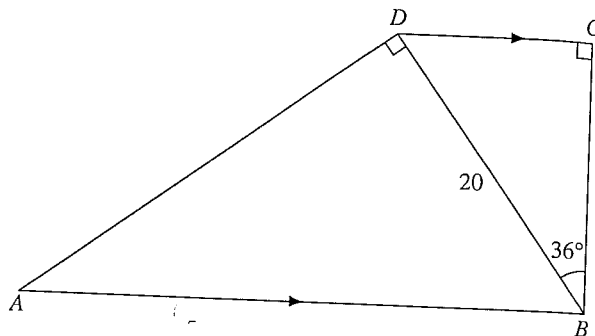
Answer (a) $\dots\dots\dots$ m [2]

(b) $RS = \dots\dots\dots$ m [2]

- 4 In the diagram, ABD and BCD are right-angled triangles. $BD = 20$ cm, $\hat{C}BD = 36^\circ$ and AB is parallel to DC . Using as much of the information given below as is necessary, find the lengths of

- (a) BC ,
 (b) AD .

$\sin 54^\circ = 0.809$
$\cos 54^\circ = 0.588$
$\tan 54^\circ = 1.376$

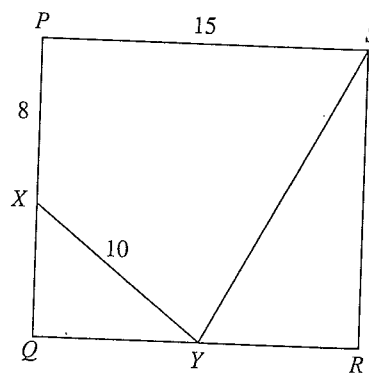


Answer (a) $BC = \dots\dots\dots$ cm [2]

(b) $AD = \dots\dots\dots$ cm [2]

- 5 $PQRS$ is a rectangle in which $PX = 8$ cm, $PS = 15$ cm and $XY = 10$ cm. Given that $\cos \hat{QXY} = \frac{3}{5}$, calculate

- (a) the length of QX ,
 (b) the length of QY ,
 (c) the value of $\tan \hat{YSR}$.



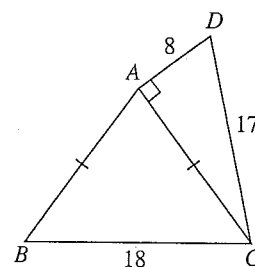
Answer (a) $QX = \dots\dots\dots$ cm [1]

(b) $QY = \dots\dots\dots$ cm [2]

(c) $\tan \hat{YSR} = \dots\dots\dots$ [3]

6 The diagram shows an isosceles triangle ABC and a right-angled triangle CAD . $AD = 8$ cm, $BC = 18$ cm and $CD = 17$ cm.

- (a) Find the length of AC .
- (b) Write down the value of $\cos \hat{ACD}$.
- (c) Calculate the area of triangle ABC .



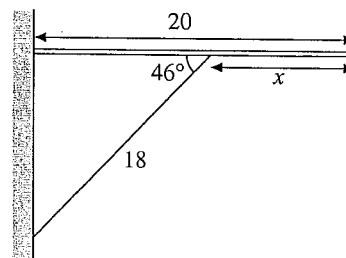
Answer (a) $AC = \dots\dots\dots$ cm [2]

(b) $\cos \hat{ACD} = \dots\dots\dots$ [1]

(c) $\dots\dots\dots$ cm² [2]

7 A bookshelf of width 20 cm is held in a horizontal position by a support of length 18 cm. The angle formed between the support and the shelf is 46° . Find the distance x , by which the shelf overhangs the support, using as much of the information given below as is necessary.

$\sin 46^\circ = 0.719$
 $\cos 46^\circ = 0.695$
 $\tan 46^\circ = 1.036$

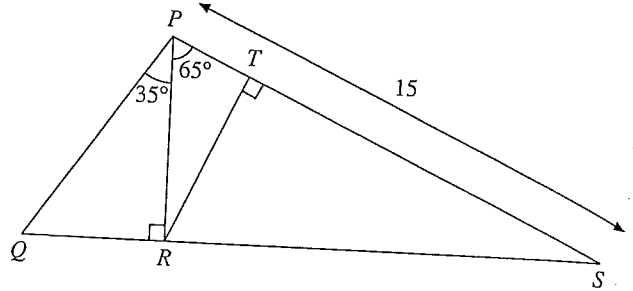


Answer $x = \dots\dots\dots$ cm [2]

- 8 In the diagram, $PS = 15$ cm, $\widehat{PRQ} = \widehat{RTS} = 90^\circ$, $\widehat{QPR} = 35^\circ$ and $\widehat{RPS} = 65^\circ$. PTS and QRS are straight lines. Using as much of the information given below as is necessary, calculate

- (a) QR ,
 (b) RT .

	sin	cos	tan
35°	0.57	0.82	0.70
65°	0.91	0.42	2.14



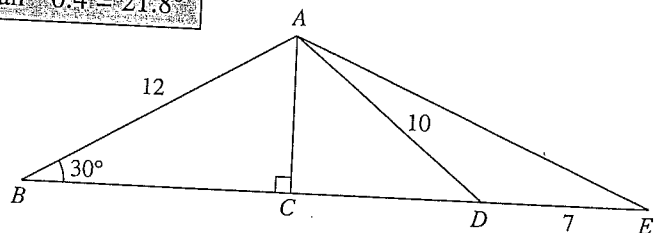
Answer (a) $QR = \dots\dots\dots$ cm [3]

(b) $RT = \dots\dots\dots$ cm [2]

- 9 In the diagram, $AB = 12$ cm, $AD = 10$ cm, $DE = 7$ cm, $\widehat{ABC} = 30^\circ$, $\widehat{ACB} = 90^\circ$ and $BCDE$ is a straight line. Using as much of the information given below as is necessary, find

- (a) CD ,
 (b) \widehat{AEC} .

$\sin 30^\circ = 0.5$, $\cos 30^\circ = 0.866$, $\tan 30^\circ = 0.577$
$\sin^{-1} 0.4 = 23.6^\circ$, $\cos^{-1} 0.4 = 66.4^\circ$, $\tan^{-1} 0.4 = 21.8^\circ$

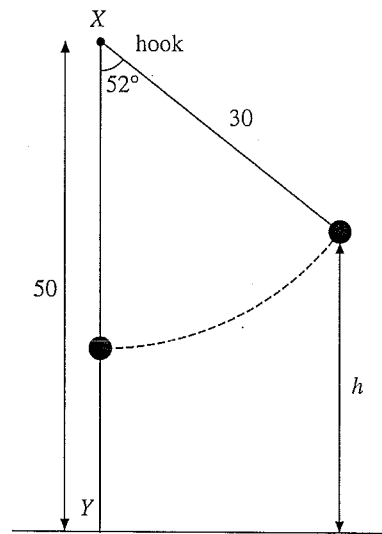


Answer (a) $CD = \dots\dots\dots$ cm [3]

(b) $\widehat{AEC} = \dots\dots\dots^\circ$ [2]

- 10 A pendulum of length 30 cm is attached to a hook on a vertical stand XY which is 50 cm high. Calculate the vertical height, h cm, of the pendulum above the stand when it makes an angle of 52° with XY , using as much of the information given below as is necessary.

$\sin 52^\circ = 0.788$
$\cos 52^\circ = 0.616$
$\tan 52^\circ = 1.280$



Answer $h = \dots\dots\dots$ [3]

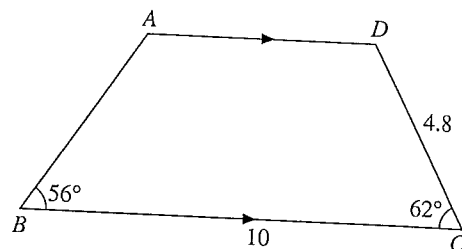
INSTRUCTIONS TO CANDIDATES

Section B (40 marks)

Time: 45 minutes

1. Answer **all** the questions in this section.
2. Calculators may be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [] at the end of each question.

- 11 (a) In the diagram, $ABCD$ is a trapezium. $BC = 10$ cm, $CD = 4.8$ cm, $\hat{A}BC = 56^\circ$ and $\hat{B}CD = 62^\circ$. Calculate
- (i) AD ,
 - (ii) the area of trapezium $ABCD$.



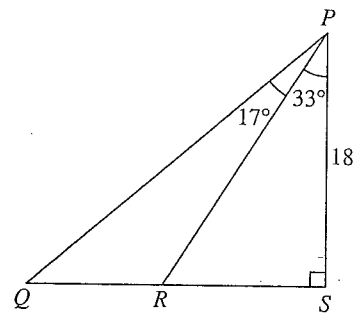
- (b) A ladder of length 12.5 m, rests against a vertical wall with its foot on a horizontal ground. It makes an angle of 46° with the horizontal. When the top of the ladder slides x metres down the wall, it makes an angle of 28° with the horizontal. Find the value of x .

Answer (a) (i) $AD = \dots\dots\dots$ cm [4]

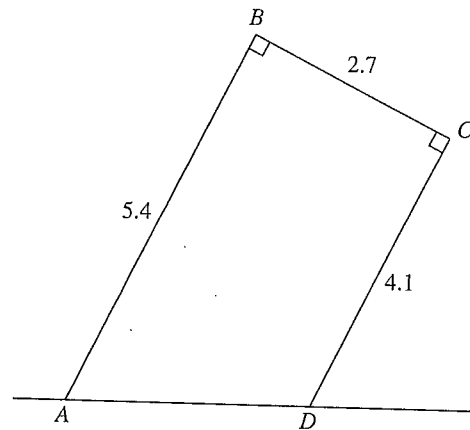
(ii) $\dots\dots\dots$ cm² [1]

(b) $x = \dots\dots\dots$ [3]

- 12 (a) In the diagram, PQS is a right-angle triangle. $\widehat{QPR} = 17^\circ$, $\widehat{RPS} = 33^\circ$ and $PS = 18$ cm.
Find
 (i) RS ,
 (ii) QR ,
 (iii) the area of triangle PQR .

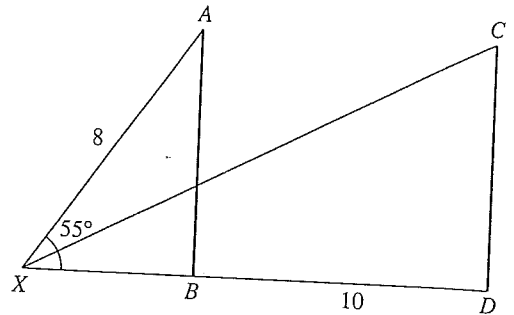


- (b) The diagram represents a signboard in the shape of a trapezium with its base AD on horizontal ground. Given that $AB = 5.4$ m, $BC = 2.7$ m, $CD = 4.1$ m and $\widehat{ABC} = \widehat{BCD} = 90^\circ$, calculate
 (i) \widehat{BAD} ,
 (ii) AD .



- Answer (a) (i) $RS = \dots\dots\dots$ cm [2]
 (ii) $QR = \dots\dots\dots$ cm [3]
 (iii) $\dots\dots\dots$ cm² [1]
 (b) (i) $\widehat{BAD} = \dots\dots\dots^\circ$ [3]
 (ii) $AD = \dots\dots\dots$ m [2]

- 13 The diagram shows two identical vertical poles AB and CD which are 10 metres apart standing on horizontal ground. Wires are then used to join the tops of each pole to a point X on the ground. Given that $AX = 8$ m and $\hat{AXB} = 55^\circ$, calculate
- the height of pole CD ,
 - the distance of CX ,
 - the value of $\sin \hat{XCD}$.

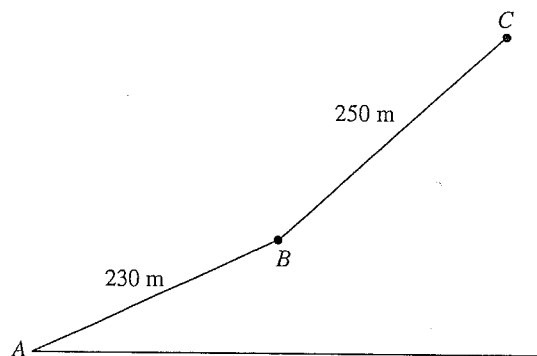


Answer (a) $CD = \dots\dots\dots$ m [2]

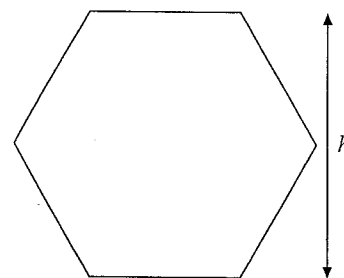
(b) $CX = \dots\dots\dots$ m [3]

(c) $\sin \hat{XCD} = \dots\dots\dots$ [1]

- 14 (a) A cable car ride is made up of two sections AB and BC . The lengths of each section of the cable are shown in the diagram. The angles of elevation of B from A is 22° and C from B is 24° . Calculate the vertical height of C above the ground.



- (b) The diagram shows a regular hexagon of sides 12 cm and height h cm. Find the value of h .



Answer (a) m [3]

(b) $h =$ [3]

- 15 (a) A statue was erected on top of a building. The angle of elevation of the top and bottom of the statue from a car parked 65 m away from the foot of the building was 58° and 52° respectively. Calculate
- (i) the height of the building,
 - (ii) the height of the statue.
- A van is parked 30 m from the foot of the building.
- (iii) Find the angle of depression of the van from the top of the statue.
- (b) A clock tower is 24 m tall. The angle of depression from the top of the tower to the foot of a tree nearby is 22.5° . The angle of elevation from the top of the tree to the top of the clock tower is 18.4° . Calculate the height of the tree.

Answer (a) (i) m [2]
 (ii) m [2]
 (iii) $^\circ$ [2]
 (b) m [3]

(d) Distance (km)	5	10	15	20	25
No. of students	6	4	12	8	10

(i) Mode = 15 km Choose the data with the highest frequency.

(ii) Median = 15 km The middle term is the mean of the 20th and 21st term which are both 15 km.

Median (middle no.)



15. (a) 30, 30, 45, 60, 120

(i) Mode = 30 min

(ii) Median = 45 min

$$\begin{aligned} \text{(iii) Mean} &= \frac{30 + 30 + 45 + 60 + 120}{5} \\ &= \frac{285}{5} \\ &= 57 \text{ min} \end{aligned}$$

(b) Total time spent watching TV on weekdays = 285 min

Total time spent watching TV last week = 1 h 30 min \times 7 = 90 min \times 7 = 630 min

Time spent watching TV on Saturday and Sunday = 630 min - 285 min = 345 min

Time spent watching TV on Sunday

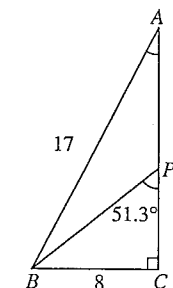
$$= \frac{12}{11 + 12} \times 345$$

$$= \frac{12}{23} \times 345$$

$$= 180 \text{ min}$$

$$= 3 \text{ h}$$

$$\begin{aligned} 180 \text{ min} &= \frac{180}{60} \text{ h} \\ &= 3 \text{ h} \end{aligned}$$



(b) Using Pythagoras' Theorem on $\triangle ABC$,

$$AB^2 = AC^2 + CB^2$$

$$17^2 = AC^2 + 8^2$$

$$AC^2 = 17^2 - 8^2 = 225$$

$$AC = \sqrt{225} = 15 \text{ cm}$$

From $\triangle BPC$,

$$\tan \hat{BPC} = \frac{BC}{PC}$$

$$\tan 51.3^\circ = \frac{8}{PC}$$

$$PC = \frac{8}{\tan 51.3^\circ}$$

$$= \frac{8}{1.25}$$

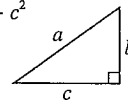
$$= 6.4 \text{ cm}$$

$$\therefore AP = AC - PC$$

$$= 15 - 6.4$$

$$= 8.6 \text{ cm}$$

Pythagoras' Theorem
 $a^2 = b^2 + c^2$



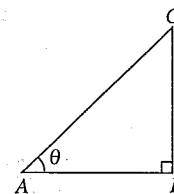
Teacher's Tip

For a right-angled $\triangle ABC$,

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AB}$$



2. $\cos \hat{ACB} = \frac{BC}{AC}$

$$\cos \theta = \frac{20}{AC}$$

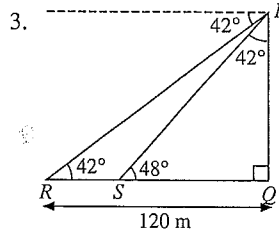
$$AC = \frac{20}{\cos \theta}$$

$$= \frac{20}{\frac{5}{13}}$$

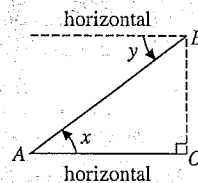
$$\cos \theta = \frac{5}{13} \text{ (Given)}$$

$$= 20 \times \frac{13}{5}$$

$$= 52 \text{ cm}$$



Teacher's Tip



$\angle x$ = angle of elevation of B from A.

$\angle y$ = angle of depression of A from B.

$$\angle x = \angle y$$

Test 17: Trigonometrical Ratios

Section A

1. (a) From $\triangle ABC$,

$$\begin{aligned} \sin \hat{BAC} &= \frac{BC}{AB} \\ &= \frac{8}{17} \end{aligned}$$

(a) From $\triangle PRQ$,

$$\tan \widehat{PRQ} = \frac{PQ}{RQ}$$

$$\tan 42^\circ = \frac{PQ}{120}$$

$$\begin{aligned} PQ &= 120 \times \tan 42^\circ \quad \tan 42^\circ = 0.900 \text{ (Given)} \\ &= 120 \times 0.900 \\ &= 108 \text{ m} \end{aligned}$$

(b) $\widehat{SPQ} = 180^\circ - 90^\circ - 48^\circ = 42^\circ$ \angle sum of \triangle
From $\triangle PSQ$,

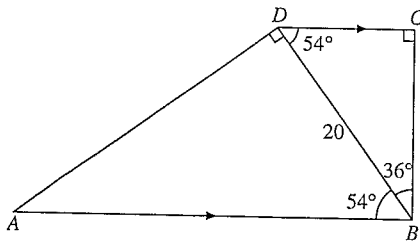
$$\tan \widehat{SPQ} = \frac{SQ}{PQ}$$

$$\tan 42^\circ = \frac{SQ}{108}$$

$$\begin{aligned} SQ &= 108 \times \tan 42^\circ \quad \tan 42^\circ = 0.900 \text{ (Given)} \\ &= 108 \times 0.900 \\ &= 97.2 \text{ m} \end{aligned}$$

$$\begin{aligned} RS &= RQ - SQ \\ &= 120 - 97.2 \\ &= 22.8 \text{ m} \end{aligned}$$

4. (a)



$$\begin{aligned} \widehat{BDC} &= 180^\circ - 90^\circ - 36^\circ \quad \angle \text{sum of } \triangle \\ &= 54^\circ \end{aligned}$$

From $\triangle BDC$,

$$\sin \widehat{BDC} = \frac{BC}{BD}$$

$$\sin 54^\circ = \frac{BC}{20}$$

$$\begin{aligned} BC &= 20 \times \sin 54^\circ \quad \sin 54^\circ = 0.809 \text{ (Given)} \\ &= 20 \times 0.809 \\ &= 16.18 \text{ cm} \end{aligned}$$

(b) $\widehat{ABD} = \widehat{BDC}$ alt. \angle s, $AB \parallel DC$
 $= 54^\circ$

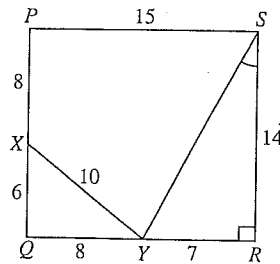
From $\triangle ABD$,

$$\tan \widehat{ABD} = \frac{AD}{BD}$$

$$\tan 54^\circ = \frac{AD}{20}$$

$$\begin{aligned} AD &= 20 \times \tan 54^\circ \quad \tan 54^\circ = 1.376 \text{ (Given)} \\ &= 20 \times 1.376 \\ &= 27.52 \text{ cm} \end{aligned}$$

5.



(a) From $\triangle XQY$,

$$\cos \widehat{QXY} = \frac{QX}{XY}$$

$$\frac{3}{5} = \frac{QX}{10}$$

$$\begin{aligned} QX &= \frac{3}{5} \times 10 \\ &= 6 \text{ cm} \end{aligned}$$

(b) Using Pythagoras' Theorem on $\triangle XQY$,

$$XY^2 = XQ^2 + QY^2$$

$$10^2 = 6^2 + QY^2$$

$$QY^2 = 10^2 - 6^2 = 64$$

$$QY = \sqrt{64} = 8 \text{ cm}$$

(c) $YR = QR - QY$

$$= 15 - 8$$

$$= 7 \text{ cm}$$

$$SR = PX + XQ$$

$$= 8 + 6$$

$$= 14 \text{ cm}$$

From $\triangle SYR$,

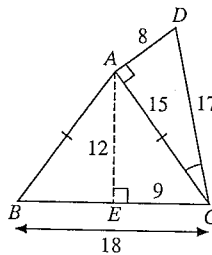
$$\tan \widehat{YSR} = \frac{YR}{SR}$$

$$= \frac{7}{14}$$

$$= \frac{1}{2}$$

$QR = PS = 15 \text{ cm}$
since $PQRS$ is a rectangle.

6.



(a) Using Pythagoras' Theorem on $\triangle DAC$,

$$DC^2 = DA^2 + AC^2$$

$$17^2 = 8^2 + AC^2$$

$$AC^2 = 17^2 - 8^2 = 225$$

$$AC = \sqrt{225} = 15 \text{ cm}$$

(b) From $\triangle DAC$,

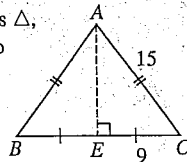
$$\cos \widehat{ACD} = \frac{AC}{DC}$$

$$= \frac{15}{17}$$

(c) **Teacher's Tip**



Since $\triangle ABC$ is an isosceles \triangle , the perpendicular from A to BC bisects BC at E .



$$EC = 18 \div 2 = 9 \text{ cm}$$

Using Pythagoras' Theorem on $\triangle AEC$,

$$AC^2 = AE^2 + EC^2$$

$$15^2 = AE^2 + 9^2$$

$$AE^2 = 15^2 - 9^2 = 144$$

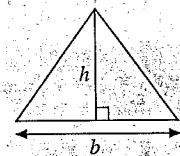
$$AE = \sqrt{144} = 12 \text{ cm}$$

Area of $\triangle ABC$

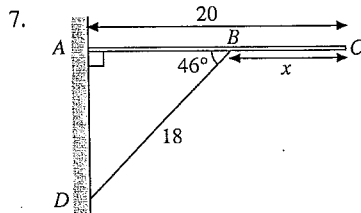
$$= \frac{1}{2} \times 18 \times 12$$

$$= 108 \text{ cm}^2$$

$$\text{Area of } \triangle = \frac{1}{2} \times b \times h$$



b = base, h = height



From $\triangle DAB$,

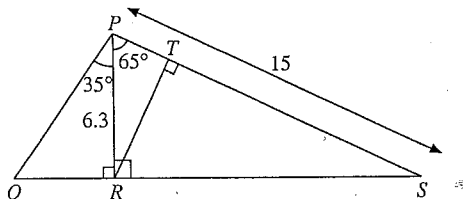
$$\cos \hat{A}BD = \frac{AB}{BD}$$

$$\cos 46^\circ = \frac{AB}{18}$$

$$\begin{aligned} AB &= 18 \times \cos 46^\circ \\ &= 18 \times 0.695 \quad \cos 46^\circ = 0.695 \text{ (Given)} \\ &= 12.51 \end{aligned}$$

$$\begin{aligned} x &= AC - AB \\ &= 20 - 12.51 \\ &= 7.49 \text{ cm} \end{aligned}$$

8.



(a) From $\triangle PRS$,

$$\cos \hat{S}PR = \frac{PR}{PS}$$

$$\cos 65^\circ = \frac{PR}{15}$$

$$\begin{aligned} PR &= 15 \times \cos 65^\circ \quad \cos 65^\circ = 0.42 \text{ (Given)} \\ &= 15 \times 0.42 \\ &= 6.3 \text{ cm} \end{aligned}$$

From $\triangle PQR$,

$$\tan \hat{Q}PR = \frac{QR}{PR}$$

$$\tan 35^\circ = \frac{QR}{6.3}$$

$$\begin{aligned} QR &= 6.3 \times \tan 35^\circ \\ &= 6.3 \times 0.70 \quad \tan 35^\circ = 0.70 \text{ (Given)} \\ &= 4.41 \text{ cm} \end{aligned}$$

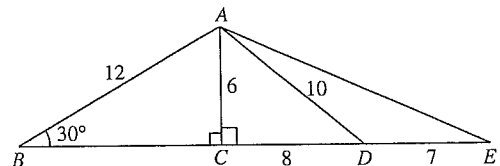
(b) From $\triangle PRT$,

$$\sin \hat{R}PT = \frac{RT}{PR}$$

$$\sin 65^\circ = \frac{RT}{6.3}$$

$$\begin{aligned} RT &= 6.3 \times \sin 65^\circ \quad \sin 65^\circ = 0.91 \text{ (Given)} \\ &= 6.3 \times 0.91 \\ &= 5.733 \text{ cm} \end{aligned}$$

9.



(a) From $\triangle ABC$,

$$\sin \hat{A}BC = \frac{AC}{AB}$$

$$\sin 30^\circ = \frac{AC}{12}$$

$$\begin{aligned} AC &= 12 \times \sin 30^\circ \quad \sin 30^\circ = 0.5 \text{ (Given)} \\ &= 12 \times 0.5 \\ &= 6 \text{ cm} \end{aligned}$$

Using Pythagoras' Theorem on $\triangle ACD$,

$$AD^2 = AC^2 + CD^2$$

$$10^2 = 6^2 + CD^2$$

$$CD^2 = 10^2 - 6^2 = 64$$

$$CD = \sqrt{64} = 8 \text{ cm}$$

(b) From $\triangle ACE$,

$$\tan \hat{A}EC = \frac{AC}{CE}$$

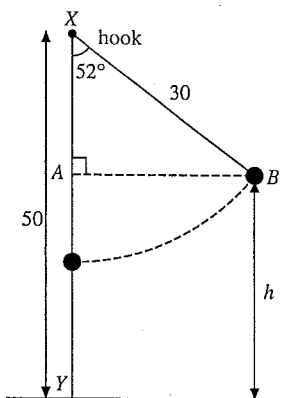
$$= \frac{6}{8 + 7}$$

$$= \frac{6}{15}$$

$$= 0.4$$

$$\begin{aligned} \hat{A}EC &= \tan^{-1} 0.4 \quad \tan^{-1} 0.4 = 21.8^\circ \text{ (Given)} \\ &= 21.8^\circ \end{aligned}$$

10.



From $\triangle XAB$,

$$\cos \hat{AXB} = \frac{XA}{XB}$$

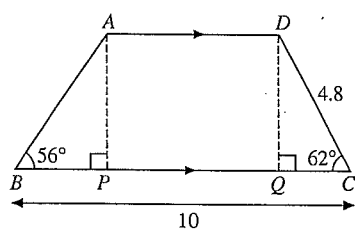
$$\cos 52^\circ = \frac{XA}{30}$$

$$\begin{aligned} XA &= 30 \times \cos 52^\circ && \cos 52^\circ = 0.616 \text{ (Given)} \\ &= 30 \times 0.616 \\ &= 18.48 \text{ cm} \end{aligned}$$

$$\begin{aligned} h &= XY - XA \\ &= 50 - 18.48 \\ &= 31.52 \text{ cm} \end{aligned}$$

Section B

11. (a)



(i) From $\triangle DQC$,

$$\cos \hat{QCD} = \frac{QC}{CD}$$

$$\cos 62^\circ = \frac{QC}{4.8}$$

$$\begin{aligned} QC &= 4.8 \times \cos 62^\circ \\ &= 2.253 \text{ cm} \end{aligned}$$

Using Pythagoras' Theorem on $\triangle DQC$,

$$DC^2 = DQ^2 + QC^2$$

$$4.8^2 = DQ^2 + 2.253^2$$

$$DQ^2 = 4.8^2 - 2.253^2$$

$$= 17.96$$

$$DQ = \sqrt{17.96}$$

$$= 4.238 \text{ cm}$$

$$AP = DQ = 4.238 \text{ cm}$$

From $\triangle ABP$,

$$\tan \hat{ABP} = \frac{AP}{BP}$$

$$\tan 56^\circ = \frac{4.238}{BP}$$

$$\begin{aligned} BP &= \frac{4.238}{\tan 56^\circ} \\ &= 2.859 \text{ cm} \end{aligned}$$

$$\begin{aligned} AD &= BC - BP - QC \\ &= 10 - 2.859 - 2.253 \\ &= 4.888 \\ &\approx 4.89 \text{ cm (correct to 3 sig. fig.)} \end{aligned}$$



Teacher's Tip

If the degree of accuracy is not stated in the question and if the answer is not exact, the answer should be given to three significant figures. This means that all working should be done in 4 significant figures.

Answers in degrees should be given to 1 decimal place.

(ii) Area of trapezium $ABCD$

$$= \frac{1}{2} \times (AD + BC) \times AP$$

$$= \frac{1}{2} \times (4.888 + 10) \times 4.238$$

$$\approx 31.5 \text{ cm}^2 \text{ (correct to 3 sig. fig.)}$$



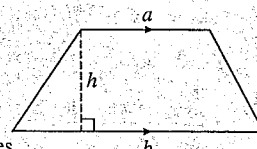
Teacher's Tip

Area of trapezium

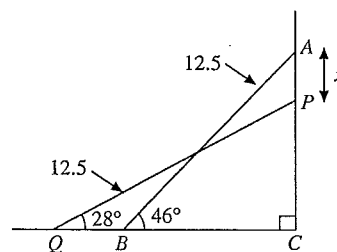
$$= \frac{1}{2} (a + b)h$$

a, b = length of parallel sides

h = height



(b)



In the diagram, AB and PQ represent the two positions of the ladder. APC represents the vertical wall.

From $\triangle ABC$,

$$\sin \hat{ABC} = \frac{AC}{AB}$$

$$\sin 46^\circ = \frac{AC}{12.5}$$

$$\begin{aligned} AC &= 12.5 \times \sin 46^\circ \\ &= 8.992 \text{ m} \end{aligned}$$

From $\triangle PQC$,

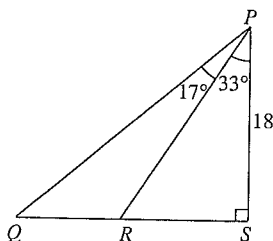
$$\sin \widehat{PQC} = \frac{PC}{PQ}$$

$$\sin 28^\circ = \frac{PC}{12.5}$$

$$PC = 12.5 \times \sin 28^\circ \\ = 5.868 \text{ m}$$

$$x = AC - PC \\ = 8.992 - 5.868 \\ = 3.124 \\ \approx \mathbf{3.12 \text{ m}} \text{ (correct to 3 sig. fig.)}$$

12. (a)



(i) From $\triangle PRS$,

$$\tan \widehat{RPS} = \frac{RS}{PS}$$

$$\tan 33^\circ = \frac{RS}{18}$$

$$RS = 18 \times \tan 33^\circ \\ = 11.69$$

$$\approx \mathbf{11.7 \text{ cm}} \text{ (correct to 3 sig. fig.)}$$

(ii) From $\triangle PQS$,

$$\tan \widehat{QPS} = \frac{QS}{PS}$$

$$\tan (17^\circ + 33^\circ) = \frac{QS}{18}$$

$$QS = 18 \times \tan 50^\circ \\ = 21.45 \text{ cm}$$

$$QR = QS - RS \\ = 21.45 - 11.69$$

$$= \mathbf{9.76 \text{ cm}} \text{ (correct to 3 sig. fig.)}$$

(iii) Area of $\triangle PQR$

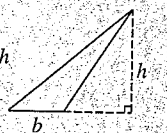
$$= \frac{1}{2} \times QR \times PS$$

$$= \frac{1}{2} \times 9.76 \times 18$$

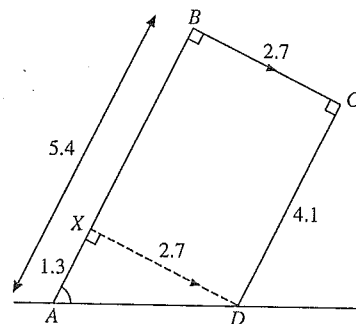
$$= \mathbf{87.8 \text{ cm}^2} \text{ (correct to 3 sig. fig.)}$$

Area of \triangle

$$= \frac{1}{2} \times b \times h$$



(b)



(i) Let X be a point on AB such that XD is parallel to BC .

$$XD = BC = 2.7 \text{ m}$$

$$AX = AB - DC$$

$$= 5.4 - 4.1$$

$$= 1.3 \text{ m}$$

From $\triangle XAD$,

$$\tan \widehat{XAD} = \frac{DX}{XA}$$

$$\tan \widehat{XAD} = \frac{2.7}{1.3} = 2.077$$

$$\widehat{XAD} = \tan^{-1} 2.077$$

$$= 64.29^\circ$$

$$\approx 64.3^\circ \text{ (correct to 1 d.p.)}$$

$$\widehat{BAD} = \widehat{XAD} = \mathbf{64.3^\circ}$$

(ii) Using Pythagoras' Theorem on $\triangle XAD$,

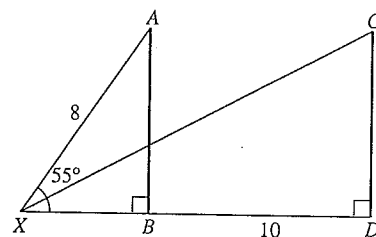
$$AD^2 = AX^2 + XD^2$$

$$= 1.3^2 + 2.7^2$$

$$= 8.98$$

$$AD = \sqrt{8.98} \approx \mathbf{3.00 \text{ m}} \text{ (correct to 3 sig. fig.)}$$

13.



(a) From $\triangle AXB$,

$$\sin \widehat{AXB} = \frac{AB}{AX}$$

$$\sin 55^\circ = \frac{AB}{8}$$

$$AB = 8 \times \sin 55^\circ$$

$$= 6.553 \text{ m}$$

$$CD = AB \approx \mathbf{6.55 \text{ m}} \text{ (correct to 3 sig. fig.)}$$

(b) Using Pythagoras' Theorem on $\triangle AXB$,

$$AX^2 = AB^2 + BX^2$$

$$8^2 = 6.553^2 + BX^2$$

$$BX^2 = 8^2 - 6.553^2$$

$$= 21.058$$

$$BX = \sqrt{21.058}$$

$$= 4.589 \text{ m}$$

$$XD = XB + BD$$

$$= 4.589 + 10$$

$$= 14.589 \text{ m}$$

Using Pythagoras' Theorem on $\triangle CXD$,

$$CX^2 = CD^2 + DX^2$$

$$= 6.553^2 + 14.589^2$$

$$= 255.78$$

$$CX = \sqrt{255.78}$$

$$= 15.99$$

$$\approx 16.0 \text{ m (correct to 3 sig. fig.)}$$

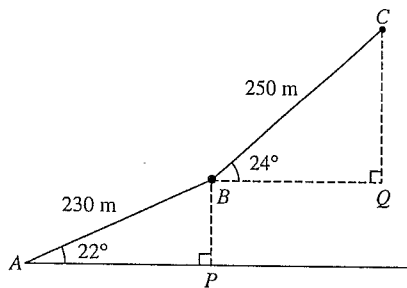
(c) From $\triangle CXD$,

$$\sin \widehat{XCD} = \frac{XD}{XC}$$

$$= \frac{14.589}{15.99}$$

$$\approx 0.912 \text{ (correct to 3 sig. fig.)}$$

14. (a)



From $\triangle BAP$,

$$\sin \widehat{BAP} = \frac{BP}{AB}$$

$$\sin 22^\circ = \frac{BP}{230}$$

$$BP = 230 \times \sin 22^\circ$$

$$= 86.16 \text{ m}$$

From $\triangle CBQ$,

$$\sin \widehat{CBQ} = \frac{CQ}{BC}$$

$$\sin 24^\circ = \frac{CQ}{250}$$

$$CQ = 250 \times \sin 24^\circ$$

$$= 101.7 \text{ m}$$

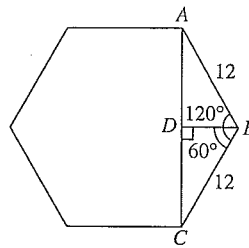
The vertical height of C

$$= BP + CQ$$

$$= 86.16 + 101.7$$

$$\approx 188 \text{ m (correct to 3 sig. fig.)}$$

(b)



Each interior angle of a regular hexagon

$$= \frac{(6-2) \times 180^\circ}{6}$$

$$= 120^\circ$$

For a n -sided polygon, each interior angle

$$= \frac{(n-2) \times 180^\circ}{n}$$

For a hexagon, $n = 6$.

$$\widehat{CBD} = 120^\circ \div 2 = 60^\circ$$

From $\triangle BCD$,

Since $\triangle ABC$ is isosceles, BD bisects \widehat{ABC} .

$$\sin \widehat{CBD} = \frac{CD}{BC}$$

$$\sin 60^\circ = \frac{CD}{12}$$

$$CD = 12 \times \sin 60^\circ$$

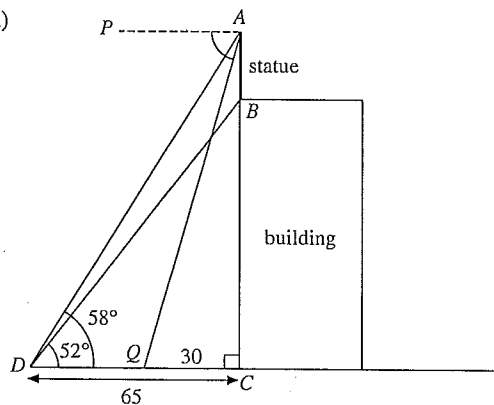
$$= 10.39 \text{ cm}$$

$$h = 2 \times CD$$

$$= 2 \times 10.39$$

$$\approx 20.8 \text{ (correct to 3 sig. fig.)}$$

15. (a)



In the diagram, AB represents the statue and BC the building.

(i) From $\triangle BDC$,

$$\tan \widehat{BDC} = \frac{BC}{DC}$$

$$\tan 58^\circ = \frac{BC}{65}$$

$$BC = 65 \times \tan 52^\circ$$

$$= 83.20$$

$$\approx 83.2 \text{ m (correct to 3 sig. fig.)}$$

\therefore the height of the building is **83.2 m**.

(ii) From $\triangle ADC$,

$$\tan \hat{ADC} = \frac{AC}{DC}$$

$$\tan 58^\circ = \frac{AC}{65}$$

$$AC = 65 \times \tan 58^\circ = 104.0 \text{ m}$$

$$\begin{aligned} \text{Height of statue} &= AC - BC \\ &= 104.0 - 83.20 \\ &\approx 20.8 \text{ m (correct to 3 sig. fig.)} \end{aligned}$$

(iii) From $\triangle AQC$,

$$\tan \hat{AQC} = \frac{AC}{QC}$$

$$= \frac{104.0}{30}$$

$$= 3.467$$

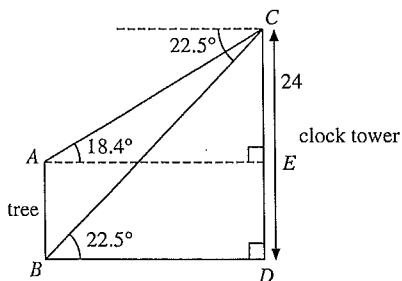
$$\hat{AQC} = \tan^{-1} 3.467$$

$$\approx 73.9^\circ \text{ (correct to 1 d.p.)}$$

$$\hat{PAQ} = \hat{AQC} = 73.9^\circ \text{ angle of depression} = \text{angle of elevation}$$

\therefore the angle of depression of the van from the top of the statue is 73.9° .

(b)



In the diagram, AB represents the tree and CD the clock tower.

From $\triangle CBD$,

$$\tan \hat{CBD} = \frac{CD}{BD}$$

$$\tan 22.5^\circ = \frac{24}{BD}$$

$$BD = \frac{24}{\tan 22.5^\circ} = 57.94 \text{ m}$$

$$AE = BD = 57.94 \text{ m}$$

From $\triangle CAE$,

$$\tan \hat{CAE} = \frac{CE}{AE}$$

$$\tan 18.4^\circ = \frac{CE}{57.94}$$

$$CE = 57.94 \times \tan 18.4^\circ = 19.27 \text{ m}$$

$$\begin{aligned} \text{Height of tree} &= CD - CE \\ &= 24 - 19.27 \\ &\approx 4.73 \text{ m (correct to 3 sig. fig.)} \end{aligned}$$

Final Examination Specimen Paper A: Part 1

1. (a) $8x = 3x^2$

$$3x^2 - 8x = 0$$

$$x(3x - 8) = 0$$

$$x = 0 \text{ or } 3x - 8 = 0$$

$$x = 2\frac{2}{3}$$

(b) $x^3 - 4x - 15x - 30$

$$= x(x^2 - 4) - 15(x + 2)$$

$$= x(x + 2)(x - 2) - 15(x + 2)$$

$$= (x + 2)[x(x - 2) - 15]$$

$$= (x + 2)(x^2 - 2x - 15)$$

$$= (x + 2)(x + 3)(x - 5)$$

Use $a^2 - b^2$

$$= (a + b)(a - b)$$

on $x^2 - 4$.

$$x^2 - 4 = x^2 - 2^2$$

$$= (x + 2)(x - 2)$$

2. (a) $\frac{3x^3y}{x^2 - 9} \div \frac{xy^3}{2x - 6}$

Use $a^2 - b^2$

$$= (a + b)(a - b)$$

on $x^2 - 9$.

$$= \frac{3x^3y}{(x + 3)(x - 3)} \div \frac{xy^3}{2(x - 3)}$$

$$x^2 - 9 = x^2 - 3^2$$

$$= (x + 3)(x - 3)$$

$$= \frac{3x^3y}{(x + 3)(x - 3)} \times \frac{2(x - 3)}{xy^3}$$

Change \div to \times by

inverting the divisor.

$$= \frac{6x^2}{y^2(x + 3)}$$

(b) $\frac{4}{2x - 5} - \frac{1}{2x^2 - 7x + 5}$

$$= \frac{4}{2x - 5} - \frac{1}{(2x - 5)(x - 1)}$$

$$= \frac{4(x - 1) - 1}{(2x - 5)(x - 1)}$$

$$= \frac{4x - 4 - 1}{(2x - 5)(x - 1)}$$

$$= \frac{4x - 5}{(2x - 5)(x - 1)}$$

3. Area on map

$$= \frac{1}{2}(3 + 8)(3)$$

$$= \frac{1}{2}(11)(3)$$

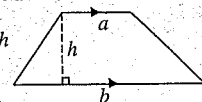
$$= 16.5 \text{ cm}^2$$



Teacher's Tip

Area of trapezium

$$= \frac{1}{2}(a + b)h$$



a, b = length of parallel sides

h = height

1 cm represents 3 km.

1 cm² represents $(3 \text{ km})^2 = 9 \text{ km}^2$.

\therefore 16.5 cm² represents $16.5 \times 9 = 148.5 \text{ km}^2$.

\therefore the actual area of the plot of land is **148.5 km²**.

two middle numbers

4. 0, 1, 2, 2, 2, 3, 3, 4, 7, 8

Arrange the data in ascending order first.