

## THE ABSOLUTE VALUE — BASIC QUESTIONS

Two numbers with the same *magnitude* but opposite sign are said to have the same *absolute value*. Thus the absolute value of +6 is 6 and the absolute value of -6 is 6.

The absolute value of a number  $a$  is indicated by the symbol  $|a|$ .

Thus  $|+7| = 7$        $|-3| = 3$        $|0| = 0$ .

The absolute value of a number is sometimes called the *modulus* of the number and hence the symbol  $|a|$  may be read as *mod a*.

Algebraically absolute value is defined as follows:

If  $a > 0$ , then  $|a| = a$     i.e.  $|6| = 6$  or  $|+8| = 8$   
 If  $a = 0$ , then  $|a| = 0$     i.e.  $|0| = 0$   
 If  $a < 0$ , then  $|a| = -a$     i.e.  $|-6| = -(-6) = 6$

Example (i):

$$|8| + |-4| = 8 + 4 = 12$$

Example (ii):

$$2|-6| - |3|^2 = 12 - 9 = 3$$

### EXERCISE 4.3

1. From the definition, write the absolute value of:

(a) -8

(c) 6

(e) -5

(g) 5.6

(b) +4

(d) 0

(f)  $+2\frac{1}{2}$

(h)  $-\frac{3}{2}$

2. Evaluate

(a)  $|6| - |-4|$

(c)  $|-2| \times |3|^2$

(e)  $|+7|^2 - |-2|^2$

(g)  $|-2|^3 - |6|$

(b)  $3|5| + 2|-3|$

(d)  $4|-3| \div |-2|$

(f)  $4|-5| + 3|4|$

(h)  $|+6|^2 - 3|-6|$

3. If  $a = 6$ ,  $b = -4$ ,  $c = 0$ ,  $d = -3$ , evaluate:

(a)  $|-b|$

(d)  $|a| - |d|$

(g)  $|d - 5|$

(j)  $|b| \times |d|$

(b)  $|c|$

(e)  $|ab|$

(h)  $|a + b| - |ab|$

(k)  $|b| + |d|$

(c)  $|a - d|$

(f)  $|b - d|$

(i)  $|a^2 - b^2|$

(l)  $|b + d|$

### SOME PROPERTIES OF ABSOLUTE VALUES

(i)  $|ab| = |a| \times |b|$

Since  $|a| \times |b| =$  magnitude of  $a \times$  magnitude of  $b$   
 $=$  magnitude of  $ab$   
 $= |ab|$

Check the truth of property (i) in the following:

(a)  $|7 \times 3| = |7| \times |3|$

(c)  $|(-6) \times 4| = |-6| \times |4|$

(b)  $|(-4) \times (-5)| = |-4| \times |-5|$

(d)  $|3 \times (-5)| = |3| \times |-5|$

### EXERCISE 4.4

Solve these equations:

1.  $|x| = 6$

6.  $|2x + 6| = |x + 10|$

11.  $|2x + 1| = |x - 2|$

2.  $|4x| = 20$

7.  $|3x - 12| = 0$

12.  $2|x + 8| = 3|x + 5|$

3.  $|x - 5| = 8$

8.  $|2x - 5| = |x + 2|$

13.  $5|x - 7| = |9x + 1|$

4.  $|2x - 3| = 17$

9.  $|7x - 4| = |3x + 16|$

14.  $6|x + 3| - 2|x + 1| = 0$

5.  $|5x + 2| = 12$

10.  $|9x + 2| = |3x - 4|$

15.  $|7x - 3| = 4|x + 6|$

Solve the following equations and check the validity of each solution.

16.  $|3x + 1| = 2x + 4$

18.  $|2x| = 9 - x$

20.  $|6x - 5| = 5x + 27$

17.  $|4x - 1| = 2x + 7$

19.  $|2x + 5| = 3x + 9$

21.  $|4 - 2x| = x - 2$

## ANSWERS

### EXERCISE 4.3 (Page 45)

- |          |                    |         |                    |
|----------|--------------------|---------|--------------------|
| 1. (a) 8 | (b) 4              | (c) 6   | (d) 0              |
| (e) 5    | (f) $2\frac{1}{2}$ | (g) 5.6 | (h) $1\frac{1}{2}$ |
| 2. (a) 2 | (b) 21             | (c) 18  | (d) 6              |
| (e) 45   | (f) 32             | (g) 2   | (h) 18             |
| 3. (a) 4 | (b) 0              | (c) 9   | (d) 3              |
| (e) 24   | (f) 1              | (g) 8   | (h) -22            |
| (i) 20   | (j) 12             | (k) 7   | (l) 7              |

### EXERCISE 4.4 (Page 47)

- |                                  |                                 |
|----------------------------------|---------------------------------|
| 1. $x = \pm 6$                   | 2. $x = \pm 5$                  |
| 3. $x = 13$ or $-3$              | 4. $x = 10$ or $-7$             |
| 5. $x = 2$ or $-2\frac{2}{3}$    | 6. $x = 4$ or $-5\frac{1}{3}$   |
| 7. $x = 4$                       | 8. $x = 7$ or $1$               |
| 9. $x = 5$ or $-1\frac{1}{3}$    | 10. $x = -1$ or $\frac{1}{6}$   |
| 11. $x = \frac{1}{3}$ or $-3$    | 12. $x = 1$ or $-6\frac{1}{3}$  |
| 13. $x = -9$ or $2\frac{3}{7}$   | 14. $x = -4$ or $-2\frac{1}{2}$ |
| 15. $x = 9$ or $-1\frac{10}{11}$ | 16. $x = -1$ or $3$             |
| 17. $x = -1$ or $4$              | 18. $x = 3$ or $-9$             |
| 19. $x = -2\frac{4}{5}$          | 20. $x = 32$ or $-2$            |
| 21. $x = 2$                      |                                 |