1. Differentiate:

   (a) $y = \log 4x$ (b) $y = \log(x^2 - 2x + 3)$ (c) $y = \log\sqrt{\dfrac{x+2}{x-2}}$

2. Use logarithms and implicit differentiation to find $\dfrac{dy}{dx}$ given $y = \dfrac{\sqrt{x}(x+7)}{x-3}$.

3. Determine:

   (a) $\displaystyle\int \frac{x-2}{x^2-4x+5}\,dx$ (b) $\displaystyle\int \frac{5+4x}{1+2x}\,dx$ [HINT: $\dfrac{2+4x}{1+2x} = 2$.]

4. The area between $y = \sqrt{\dfrac{x}{x^2+1}}$, the x-axis, $x = 1$ and $x = 3$ is rotated about the x-axis. Show the volume generated is $\frac{\pi}{2}\log 5$ cubic units.

5. (a) Differentiate $y = x^2 \log x$.

   (b) Hence or otherwise show $\displaystyle\int_1^e x \log x\, dx = \tfrac{1}{4}(e^2 + 1)$.

6. Consider the curve with equation $y = \dfrac{\log x}{x^2}$ for $0 < x < \infty$.

   (a) Find any x- or y-intercepts.

   (b) Show that $y = 0$ is an asymptote. Are there any other asymptotes?

   (c) Find y' and show that $y'' = x^{-4}(6\log x - 5)$.

   (d) Find the coordinates of any stationary point, and determine its nature.

   (e) Make a sketch showing these features.

   (f) Use your graph to explain why there must be an inflexion point. Do not find this point. You may not use the second derivative to justify your answer.

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$y = \log 4x$

$= \log 4 + \log x$

$y' = \frac{1}{x}$

b) $y = \log(x^2 - 2x + 3)$

$y' = \frac{2x-2}{x^2-2x+3}$

$\left[= \frac{2(x-1)}{(x-3)(x+1)} \right]$

c) $y = \log \sqrt{\frac{x+2}{x-2}}$

$= \frac{1}{2}\log(x+2) - \frac{1}{2}\log(x-2)$

$y' = \frac{1}{2(x+2)} - \frac{1}{2(x-2)}$

$\left[= \frac{-2}{(x+2)(x-2)} \right]$

2/ $y = \frac{\sqrt{x}\,(x+7)}{x-3}$

$\log y = \frac{1}{2}\log x + \log(x+7) - \log(x-3)$

differentiate with respect to x

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2x} + \frac{1}{x+7} - \frac{1}{x-3}$

so $\frac{dy}{dx} = y\left(\frac{1}{2x} + \frac{1}{x+7} - \frac{1}{x-3}\right)$

$= \frac{\sqrt{x}\,(x+7)}{x-3}\left(\frac{1}{2x} + \frac{1}{x+7} - \frac{1}{x-3}\right)$

3/ a) $\int \frac{x-2}{x^2-4x+5}\,dx = \frac{1}{2}\int \frac{2x-4}{x^2-4x+5}\,dx$

$= \frac{1}{2}\log(x^2-4x+5) + C$

b) $\int \frac{5+4x}{1+2x}\,dx = \int \frac{2+4x}{1+2x} + \frac{3}{1+2x}\,dx$

$= \int 2 + \frac{3}{2}\cdot\frac{2}{1+2x}\,dx$

$= 2x + \frac{3}{2}\log(1+2x) + C$

4/ $\text{Vol} = \pi \int_1^3 y^2\,dx$

$= \pi \int_1^3 \frac{x}{x^2+1}\,dx$

$= \frac{\pi}{2}\int_1^3 \frac{2x}{x^2+1}\,dx$

$= \frac{\pi}{2}\left[\log(x^2+1)\right]_1^3$

$= \frac{\pi}{2}(\log 10 - \log 2)$

$= \frac{\pi}{2}\log 5$.

5/ a) $y = x^2 \log x$

$\frac{dy}{dx} = 2x\log x + x$

b) integrate both sides between 1 & e.

$\left[x^2\log x\right]_1^e = 2\int_1^e x\log x\,dx + \int_1^e x\,dx$

so $2\int_1^e x\log x\,dx = \left[x^2\log x - \frac{x^2}{2}\right]_1^e$

$= \left(e^2 - \frac{e^2}{2}\right) - \left(-\frac{1}{2}\right)$

$= \frac{1}{2}(e^2+1)$

thus $\int_1^e x\log x\,dx = \frac{1}{4}(e^2+1)$

6/ a) only x-int at $x=1$ $y=0$

b) $\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{1}{x^2}\log x = 0$

so $y=0$ is an asymptote

$\lim_{x\to 0^+} y = \lim_{x\to 0^+} x^{-2}\log x \to -\infty$

so $x=0$ is a vertical asymptote

c) $y' = \frac{-2}{x^3}\log x + \frac{1}{x^2}\cdot\frac{1}{x}$

$= \frac{1}{x^3}(1 - 2\log x)$

$y'' = \frac{-3}{x^4}(1-2\log x) + \frac{1}{x^3}\cdot\frac{-2}{x}$

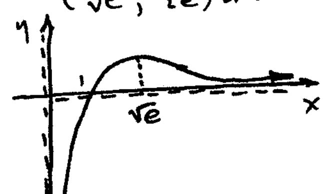
$= \frac{1}{x^4}(6\log x - 5)$

d) stat pt when $1 - 2\log x = 0$

so $x = \sqrt{e}$, $y = \frac{1}{2e}$

at $x = \sqrt{e}$ $y'' = -2/e^2$ so

$(\sqrt{e}, \frac{1}{2e})$ is a max. turning pt.

e) 

f) At $x=\sqrt{e}$ the curve is concave down. At some pt as the curve approaches the asymptote, y' must begin increasing and the curve is concave up. Hence there must be an inflex. pt. (change of concavity).