THE QUADRATIC FUNCTION

1) (a) Find the equation of the axis of symmetry of the parabola $y = x^2 - 4x + 1$.
(b) Find the minimum value of the parabola.
(c) Hence sketch the parabola on a number plane.
(d) State the domain and range of the parabola.

2) (a) Find the equation of the axis of symmetry of the parabola $y = 3 - 2x - x^2$.
(b) Find the maximum point on the parabola.
(c) Sketch the parabola.

3) Solve $x^2 - 6x + 8 > 0$

4) Solve $4 - x^2 \geq 0$

5) Find the values of x for which $x^2 - 5x + 6 < 0$.

6) (a) Sketch $y = 2x^2 - x + 3$ showing its axis of symmetry.
(b) Show that $y = 2x^2 - x + 3$ is a positive definite quadratic function.

7) Find the discriminant of $3x^2 + 2x - 4 = 0$. How would you describe the roots of this quadratic equation?

8) (a) Find the discriminant of $x^2 - x + 5 = 0$.
(b) How many real roots does this quadratic equation have?

9) Find the values of c in $x^2 + 5x + c = 0$ if the quadratic equation has real roots.

10) Find values of k in $3x^2 - kx + 3 = 0$ if the quadratic equation has no real roots.

11) Show that the line $y = 2x - 6$ cuts the circle $x^2 + y^2 = 16$ in two points..

12) Find values of a, b and c if $2x^2 + 3x - 5 = a(x + 1)^2 + b(x + 1) + c$

13) If α and β are the roots of the quadratic equation $x^2 + 3x - 5 = 0$, find
(a) $\alpha + \beta$
(b) $\alpha \beta$
(c) $\alpha \beta^2 + \alpha^2 \beta$
(d) $\alpha^2 + \beta^2$
(e) $\alpha^3 + \beta^3$

14) If one root of $2x^2 - px + 3 = 0$ is 4, find the value of p.

15) Find the quadratic equation that has roots -1 and 3.

16) Find the quadratic equation with roots $2 + \sqrt{5}$ and $2 - \sqrt{5}$

17) Find the values of k in the quadratic equation $x^2 - 5x + k - 1 = 0$ if
(a) one root is equal to 2
(b) the roots are real
(c) one root is two more than the other root.

18) Find the value of p if the quadratic equation $px^2 + 2x + 7 = 0$ has roots that are reciprocals.

19) The quadratic equation $2x^2 - bx + 1 = 0$ has one root twice the other root. Find the values of b.

20) Solve $(x + 3)^2 + (x + 3) - 2 = 0$

21) Solve $2^{2x} - 9.2^x + 8 = 0$.

22) Solve $m - \dfrac{5}{m} = 4$

23) Solve $(x^2 - 2x)^2 + 7(x^2 - 2x) + 6 = 0$.

24) Solve $4\cos^2 \alpha - 1 = 0$ for $0° \leq \alpha \leq 360°$.

25) Solve $\left(x+\dfrac{1}{x}\right)^2 + 2\left(x+\dfrac{1}{x}\right) - 3 = 0$ (Give exact answers)
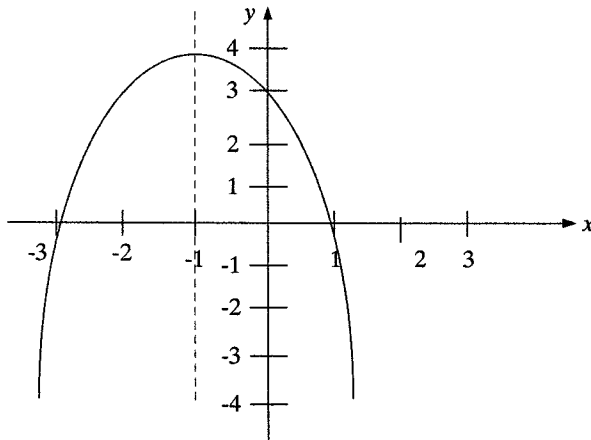
ANSWERS

1)   (a) $x = 2$
    (b) -3
    (c)
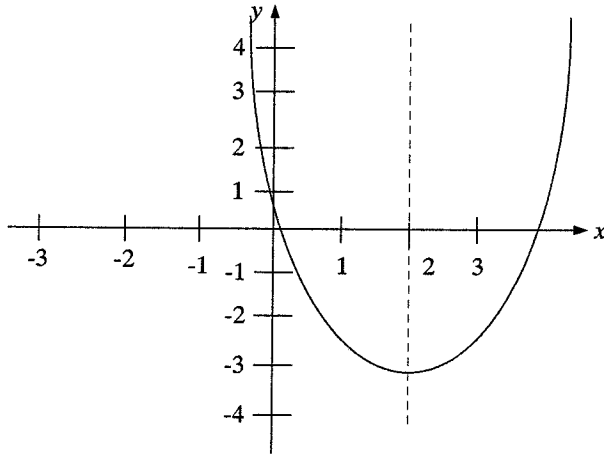
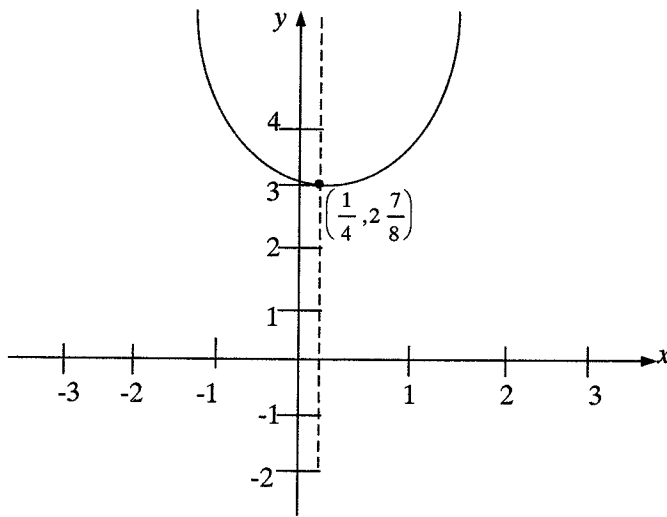   (d) Domain: all real x. Range: $y \geq -3$.

2)   (a) $x = -1$
    (b) (-1,4)
    (c)

3) $x < 2, x > 4$
4) $-2 \leq x \leq 2$
5) $2 < x < 3$

6)　(a)

(b) Since the graph is always above the x-axis, the quadratic function is positive definite.

7)　52. Real, different and irrational roots.

8)　(a) -19　(b) No real roots.

9)　$c \le 6\dfrac{1}{4}$

10)　$-6 < k < 6$

11)　Solving simultaneous equations, $5x^2 - 24x + 20 = 0$. Discriminant = 176 > 0. Therefore, there are 2 points of intersection.

12)　a = 2, b = -1, c = -6

13)　(a) -3　(b) -5　(c) 15　(d) 19　(e) -72

14)　$p = 8\dfrac{3}{4}$

15)　$x^2 - 2x - 3 = 0$

16)　$x^2 - 4x - 1 = 0$

17)　(a) k = 7　(b) k ≤ 7.25　(c) k = 6.25

18)　p = 7

19)　b = ±3

20)　$x = -5, -2$

21)　$x = 0, 3$

22)　m = -1, 5

23)　$x = 1$

24)　$\alpha = 60^0, 120^0, 240^0, 300^0$

25)　$x = \dfrac{-3 \pm \sqrt{5}}{2}$