1. Solve the equation $x^6 + 7x^3 - 8 = 0$.

2. If the quadratic equation $2x^2 + 5x - 1 = 0$ has roots α and β, without finding these roots evaluate:

   (a) $\alpha + \beta$,

   (b) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$,

   (c) $2\alpha^2 + 2\beta^2$.

3. If (4,1) is the vertex of the parabola $y = kx^2 - 4mx + 3$,

   (a) show that $k = \frac{1}{8}$ and $m = \frac{1}{4}$.

   (b) Hence show that the parabola does not cut the x axis.

4. Find the value(s) of k for which the quadratic equation $x^2 - 2kx + (k + 2) = 0$ has

   (a) equal roots,

   (b) real roots,

   (c) roots whose sum is three times their product.

5. The quadratic equation $x^2 + Lx + M = 0$ has one root which is twice the other.

   (a) Prove that $2L^2 = 9M$.

   (b) Prove that the roots are rational whenever L is rational.

6. If x is real, prove that $\dfrac{x}{x^2 - 5x + 9}$ cannot have a value greater than 1 or less than $-\dfrac{1}{11}$.

The Quadratic Function

Qu1 Let $u = x^3$

$\therefore u^2 + 7u - 8 = 0$

$(u+8)(u-1) = 0$ ✓

$\therefore x^3 = -8$ or $x^3 = 1$

$x = -2, 1$. ✓

Qu2. (a) $-\frac{5}{2}$. ✓

(b) $\frac{\alpha + \beta}{\alpha\beta} = \frac{-5}{2} \times -2$

$= 5$. ✓

(c) $2(\alpha^2 + \beta^2) = 2((\alpha+\beta)^2 - 2\alpha\beta))$

$= 2(\frac{25}{4} + 1)$

$= 14\frac{1}{2}$ ✓

Qu3(a) $y = k(x^2 - \frac{4m}{k}x + \frac{3}{k})$

$= k((x - \frac{4m}{2k})^2 - \frac{16m^2}{4k^2} + \frac{3}{k})$

$= k((x - \frac{2m}{k})^2 - \frac{4m^2 + 3k}{k^2})$

$= k(x - \frac{2m}{k})^2 - \frac{4m^2 + 3k}{k}$

$\therefore \frac{2m}{k} = 4 \Rightarrow 2m = 4k$

$m = 2k$ — ①

$\frac{-4m^2 + 3k}{k} = 1$

$\therefore -4m^2 + 3k = k$

$-4m^2 = -2k$ ✓

$m^2 = \frac{k}{2}$ — ②

sub ① in ②: $4k^2 = \frac{k}{2}$

$8k^2 = k$.

$8k = 1$ ✓

$\therefore k = \frac{1}{8}$ sub in ①

$\therefore m = \frac{2}{8} = \frac{1}{4}$

$\therefore k = \frac{1}{8},\ m = \frac{1}{4}$ ✓

(b) $y = \frac{1}{8}x^2 - x + 3$

when $y = 0$, $\frac{1}{8}x^2 - x + 3 = 0$

$\Delta = 1 - 4 \cdot \frac{1}{8} \cdot 3$

$= 1 - \frac{3}{2} = -\frac{1}{2}$ ✓

$\therefore$ no real soln. ✓

$\therefore$ does not cross x-axis

Qu 4 $\Delta = 4k^2 - 4(k+2)$

$= 4k^2 - 4k - 8$. ✓

(a) equal roots when $\Delta = 0$,

$\therefore k^2 - k - 2 = 0$

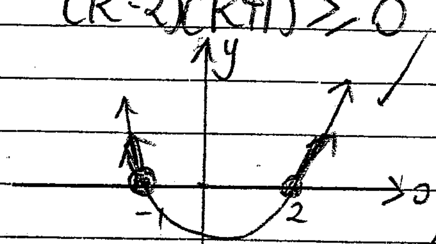
$(k-2)(k+1) = 0$

$k = -1, 2$ ✓

(b) real roots when $\Delta \geq 0$

$k^2 - k - 2 \geq 0$

$(k-2)(k+1) \geq 0$ ✓



$k \leq -1$ ✓

$k \geq 2$ ✓

(c) $\alpha + \beta = 3\alpha\beta$.

$2k = 3k + 6$ ✓

$-6 = k$. ✓

Ques 5

(a) let one root be β

$\therefore \beta = 2\beta$

$\beta + 2\beta = 3\beta = -L \therefore \beta = -\frac{L}{3}$

$2\beta^2 = M$. ✓

$\therefore \beta = \sqrt{\frac{M}{2}}$

$\therefore \frac{-L}{3} = \frac{\pm\sqrt{M}}{\sqrt{2}}$

$\therefore \frac{L^2}{9} = \frac{M}{2}$ $\therefore 2L^2 = 9M$ ✓

(b) $2L^2 = 9M \Rightarrow M = \frac{2L^2}{9}$

root = β

$\beta = -\frac{L}{3} = \sqrt{\frac{M}{2}}$

$\therefore$ When $\beta = -\frac{L}{3}$, when L is rational, root is rational

When $\beta = \pm\sqrt{\frac{M}{2}}$, $\beta = \frac{\pm\sqrt{9}}{\sqrt{4L^2}} = \frac{\pm 3}{2L}$ ✓

$\therefore$ when L is rational, root is rational

$\therefore$ roots rational whenever L is rational.

6. $x^{-1} - 5 + \frac{1}{9}x$

Let $u = x^{-1}$

$\therefore \frac{1}{9}u^2 + u - 5 = c$

$u^2 + 9u - 45 = 9c$

$u^2 + 9u - (45 + 9c) = 0$

$\Delta = 81u^2 - 4u^2 \cdot (45 + 9c)$

$= 81u^2 + 180u^2 + 36u^2c$

$= 261u^2 + 36u^2c$

for u to be real, $\Delta \geq 0$

$261u^2 + 36u^2c \geq 0$

$87u^2 + 12u^2c \geq 0$

$29u^2 + 4u^2c \geq 0$

$u^2(29 + 4c) \geq 0$

$x(29 + 4c) \geq 0$

Let $\frac{x}{x^2 - 5x + 9} = c$

$x = cx^2 - 5cx + 9c$ ✓

$cx^2 - (5c + 1)x + 9c = 0$

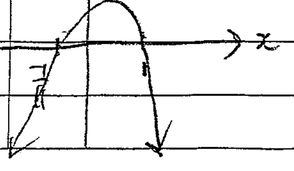
$\Delta = 25c^2 + 10c + 1 - 4c.9c$

$= -11c^2 + 10c + 1$ ✓

for x to be real, $\Delta \geq 0$

$-11c^2 + 10c + 1 \geq 0$

$(-11c - 1)(c - 1) \geq 0$ ✓



$\therefore -\frac{1}{11} \leq c \leq 1$ ✓