NAME: Due:

Year 12 MATHEMATICS

* Answer on your own paper.

The Quadratic Polynomial

* Show ALL necessary working.

EXERCISE 15.6

1. Write down the sum and product of the roots of the following quadratic equations.

(a)
$$x^2 - 2x + 5 = 0$$

(c)
$$kx^2 - 2x + k = 0$$

(e)
$$4x^2 = 5x - 1$$

(b)
$$3x^2 + x - 4 = 0$$

(a)
$$x^2 - 2x + 5 = 0$$

(b) $3x^2 + x - 4 = 0$
(c) $kx^2 - 2x + k = 0$
(d) $4x^2 = 5x - 1$
(e) $4x^2 = 5x - 1$
(f) $7x = 3 - 2x^2$

(f)
$$7x = 3 - 2x^2$$

2. If α and β are the roots of the quadratic equation $x^2 - 5x + 2 = 0$ find the value of:

(a)
$$\alpha + \beta$$

(d)
$$\alpha^2 + \beta^2$$

$$(g) \ \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

(e)
$$(\alpha - 2)(\beta - 2)^{-1}$$

(h)
$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

(c)
$$\frac{1}{\alpha} + \frac{1}{\beta}$$

(f)
$$\alpha^2\beta + \alpha\beta^2$$

(i)
$$(\alpha - \beta)^2$$

3. If α and β are the roots of $2x^2 + 3x - 4 = 0$, find the value of:

(a)
$$\alpha + \beta$$

(c)
$$\alpha^2 + \alpha\beta + \beta^2$$

(e)
$$\alpha\beta - 2\alpha - 2\beta + 1$$

(f) $\alpha^2\beta^3 + \alpha^3\beta^2$

(b)
$$\alpha\beta$$

(d)
$$(\alpha - 1)(\beta - 1)$$

(f)
$$\alpha^2 \beta^3 + \alpha^3 \beta^2$$

4. Form the quadratic equation whose roots are:

(c) 1 and
$$-7$$

(e)
$$-4$$
 and $-\frac{1}{3}$

(e)
$$-4$$
 and $-\frac{1}{2}$ (g) $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ (f) $\sqrt{3}$ and $-\sqrt{3}$ (h) $(5 - \sqrt{2})$ and $(5 + \sqrt{2})$

(b)
$$4$$
 and -2

(d)
$$\frac{1}{2}$$
 and $1\frac{1}{2}$

(f)
$$\sqrt{3}$$
 and $-\sqrt{3}$

$$=$$
 (h) (5 = $\sqrt{2}$) and (5 + $\sqrt{2}$

5. Given the quadratic equation $x^2 - (2 + k)x + 3k = 0$,

- (a) write down the sum and product of the roots in terms of k.
- (b) find the value of k if:
 - (i) the sum of the roots is 5.
 - (ii) the product of the roots is 12.
 - (iii) the product of the roots is four times the sum of the roots.

For what value of n will one root of the equation $(n-2)x^2 + (n+2)x + 2n + 1 = 0$ be the reciprocal of the other?"

7. For what value of k will the equation $x^2 - (k+2)x + (k-4) = 0$ have:

- (a) one root equal to zero?
- (b) one root equal to 4?
- (c) one root which is the reciprocal of the other?

8. If one root of $3x^2 - 8x + k = 0$ is three times the other root, find k.

9. Given that one root of $x^2 - 1.3x - 8.88 = 0$ is 3.7, find the other.

0. If α and β are the roots of $\frac{1}{x+m} = nx$, find in terms of m and n:

(a) $\alpha + \beta$ (b) $\alpha\beta$ (c)

(a)
$$\alpha + \beta$$

(c)
$$(\alpha - \beta)$$

(11.) Find the value of k if the sum of the roots of the equation, $x^2 - (4 - k)x + (k - 2) = 0$, is equal to the reciprocal of the product of the roots.

12. Find the value of k if the roots of the equation, $4x^2 - 20x + k = 0$, differ by 2.

The Ornidratic Polynomial Excercise 15.6

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = 5$$
product of roots = 5

b)
$$3x^2 + x - 4 = 0$$

 $x + \beta = -\frac{1}{3}$

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e)
$$4x^2 - 5x + 1 = 0$$

2)a)
$$3(^{2}-5x+2=0)$$

$$x+\beta=5$$

c)
$$\frac{1}{\alpha} + \frac{1}{\beta}$$

$$= \frac{\beta + \lambda}{\alpha \beta} = \frac{5}{2} = \frac{2}{2}$$

d)
$$x^2+\beta^2$$

$$= (\alpha + \beta)^2$$

$$= (\lambda + \beta)^2 - 2\lambda \beta$$

$$= (5)^2 - 4$$

e)
$$(x-2)(3-2)$$

$$= -2(\alpha + \beta) + 6$$

$$= 5(2)$$

$$= 10$$

9)
$$\frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$= \frac{(\beta + \alpha)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= (\mathcal{A}\mathcal{B})^2$$

h)
$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$= \frac{\alpha^2 + \beta^2}{\beta \alpha}$$

$$= \frac{(\alpha + \beta)^2 + 2\alpha\beta}{\beta \alpha}$$

$$=\frac{21}{2}=10^{1/2}$$

3)
$$2x^2+3x-t=0$$

a) $x+\beta=-3/2=-1/2$

c)
$$\chi^{2} + \chi \beta + \beta^{2}$$

= $(\chi + \beta)^{2} - \chi \beta$
= $(-1.5)^{2} + 2$
= $4 \frac{1}{4}$

d)
$$(x-1)(\beta-1)$$

= $x\beta-x-\beta+1$
= $-1-x-\beta$
= $-1-(x+\beta)$
= $-1-(-1/2)$

e)
$$\angle \beta - 2\alpha - 2\beta + 1$$

= $-2 - 2(\alpha + \beta) + 1$
= $-1 - 2(-1/2)$
= $-1 + 3$
= $-1 + 2$

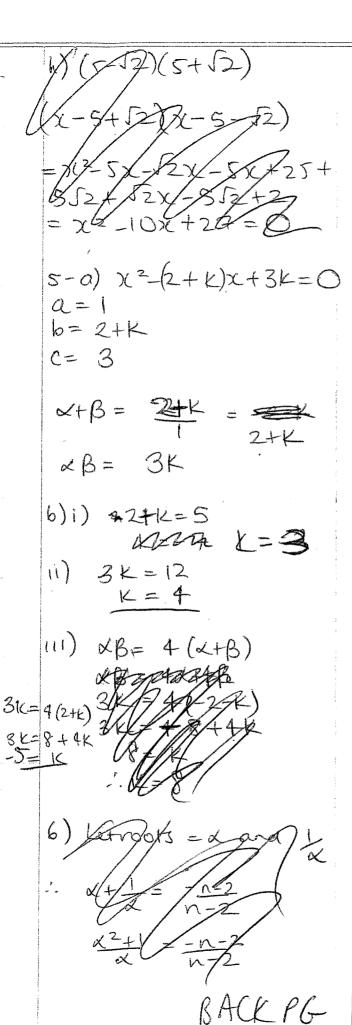
f)
$$\chi^{2}\beta^{3} + \chi^{3}\beta^{2}$$

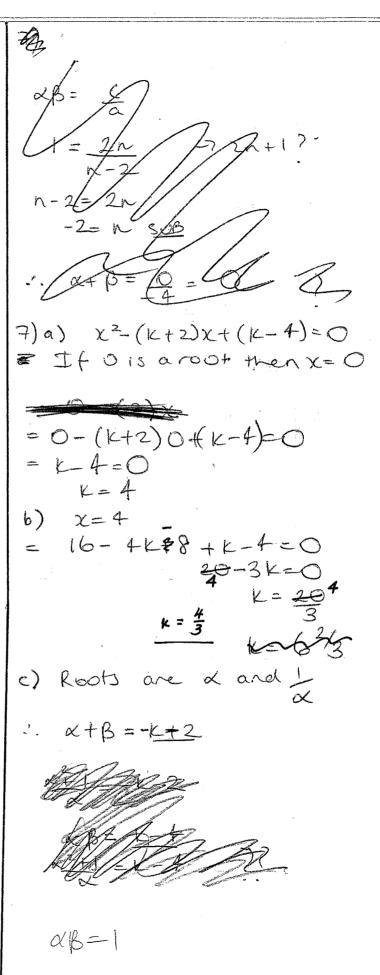
= $\chi^{2}\beta^{2}(\beta + \chi)$
= $(\chi\beta)^{2}(\beta + \chi)$
= 4×-1.5
= -6

4) a)
$$(x+3)(x-5)=0$$

= $x^2-5x-3x+15=0$
= $x^2-8x+15=0$
b) $(x-4)(x+2)=0$
 $x^2-2x-4x-8=0$
= $x^2-6x-8=0$
c) $(x-1)(x+7)=0$
 $x^2+7x-x-7=0$
 $(x-1)(x-1/2)=0$
= $x^2-1.5x+0.5x+34$
= $(x+4)(x+1/2)$
= $x^2+1/2x+4x+2$
= $x^2+1/2x+4x+2$
= $x^2+1/2x+4x+2$
= $x^2+1/2x+1/2=0$
f) $(x-1/3)(x+1/3)$
= x^2-3
 $(x-3)(x+3)$

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k-4=1

K = 5

$$\alpha = \frac{2}{3}$$

$$3\alpha(\lambda) = \frac{K}{3}$$

9)
$$\chi^2 - 1.3 \chi - 8.85 = 0$$

$$(3.7)^{2}$$
 $(1.3)(3.7)-8.88=0$

$$\frac{1}{x+x}=nx$$

$$nv^2 + nmx -$$

$$=$$
 $=$ n

$$=-m$$

c)
$$(\alpha - \beta)^2$$

$$= (\alpha + \beta)^{2} - 4\alpha\beta$$

$$= m^{2} + \frac{4}{3}$$

$$(2+53)(2-53)$$

$$a+B = 2+53+2-53$$

= 4

$$23 = (3.+53)$$

$$4 - 2\sqrt{3} + 2\sqrt{3} - 3$$

$$\chi^2 - (\chi + B)\chi + \chi B = 0$$

$$\chi^2 + \chi + 1 = 0$$

$$= 25 + 5\sqrt{2} - 1/2 - 2$$

= 23

$$(2)i)$$
 $(x+B)^2 - 4xB$
= 25-8
= 17

6)
$$\alpha \beta = 1$$

 $\frac{2n+1}{n-2} = 1$

$$n = -3$$

7) c)
$$\frac{k+2}{k-1}$$

$$X+B=-b/a$$

$$= K-2$$

$$4-K=1$$
 $K-2$

$$(K-3)(K-3)=0$$

$$2x = 7$$

$$x = 7/2$$

$$4\left(\frac{3}{2}\right)^2 - 20\left(\frac{3}{2}\right) + k = 0$$