ASSIGNMENT 2 (TERM 4)

NAME:
Due:

Year 12 MATHEMATICS

* Answer on your own paper.
* Show ALL necessary working.

The Quadratic Polynomial

EXERCISE 15.6

1. Write down the sum and product of the roots of the following quadratic equations.
   (a) $x^2 - 2x + 5 = 0$ (c) $kx^2 - 2x + k = 0$ (e) $4x^2 = 5x - 1$
   (b) $3x^2 + x - 4 = 0$ (d) $4x^2 - kx + k - 1 = 0$ (f) $7x = 3 - 2x^2$

2. If α and β are the roots of the quadratic equation $x^2 - 5x + 2 = 0$ find the value of:
   (a) $\alpha + \beta$ (d) $\alpha^2 + \beta^2$ (g) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$
   (b) $\alpha\beta$ (e) $(\alpha - 2)(\beta - 2)$ (h) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$
   (c) $\frac{1}{\alpha} + \frac{1}{\beta}$ (f) $\alpha^2\beta + \alpha\beta^2$ (i) $(\alpha - \beta)^2$

3. If α and β are the roots of $2x^2 + 3x - 4 = 0$, find the value of:
   (a) $\alpha + \beta$ (c) $\alpha^2 + \alpha\beta + \beta^2$ (e) $\alpha\beta - 2\alpha - 2\beta + 1$
   (b) $\alpha\beta$ (d) $(\alpha - 1)(\beta - 1)$ (f) $\alpha^2\beta^3 + \alpha^3\beta^2$

4. Form the quadratic equation whose roots are:
   (a) 3 and 5 (c) 1 and -7 (e) -4 and $-\frac{1}{2}$ (g) $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$
   (b) 4 and -2 (d) $\frac{1}{2}$ and $1\frac{1}{2}$ (f) $\sqrt{3}$ and $-\sqrt{3}$ (h) $(5 - \sqrt{2})$ and $(5 + \sqrt{2})$

5. Given the quadratic equation $x^2 - (2 + k)x + 3k = 0$,
   (a) write down the sum and product of the roots in terms of k.
   (b) find the value of k if:
   (i) the sum of the roots is 5.
   (ii) the product of the roots is 12.
   (iii) the product of the roots is four times the sum of the roots.

6. For what value of n will one root of the equation $(n - 2)x^2 + (n + 2)x + 2n + 1 = 0$ be the reciprocal of the other?

7. For what value of k will the equation $x^2 - (k + 2)x + (k - 4) = 0$ have:
   (a) one root equal to zero?
   (b) one root equal to 4?
   (c) one root which is the reciprocal of the other?

8. If one root of $3x^2 - 8x + k = 0$ is three times the other root, find k.

9. Given that one root of $x^2 - 1{\cdot}3x - 8{\cdot}88 = 0$ is 3·7, find the other.

10. If α and β are the roots of $\frac{1}{x + m} = nx$, find in terms of m and n:
    (a) $\alpha + \beta$ (b) $\alpha\beta$ (c) $(\alpha - \beta)^2$

11. Find the value of k if the sum of the roots of the equation, $x^2 - (4 - k)x + (k - 2) = 0$, is equal to the reciprocal of the product of the roots.

12. Find the value of k if the roots of the equation, $4x^2 - 20x + k = 0$, differ by 2.

The Quadratic Polynomial

Exercise 15.6

$$\alpha+\beta = \frac{-b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

1-a) $x^2-2x+5=0$

$\alpha+\beta =$ ~~4~~

Sum of roots = ~~4~~ 2

$\alpha\beta = 5$

product of roots = 5

b) $3x^2+x-4=0$

$\alpha+\beta = -1/3$

$\alpha\beta = -4/3 = -1\frac{1}{3}$

c) $kx^2-2x+k=0$

$\alpha+\beta = \frac{2}{k}$

$\alpha\beta = \frac{k}{k} = 1$

d) $4x^2-kx+k-1=0$

$\alpha+\beta = \frac{k}{4}$

$\alpha\beta = \frac{k-1}{4}$

e) $4x^2-5x+1=0$

$\alpha+\beta = 5/4 = 1\frac{1}{4}$

$\alpha\beta = 1/4$

f) $2x^2-3+7x=0$

$2x^2+7x-3=0$

$\alpha+\beta = -7/2 = -3\frac{1}{2}$

$\alpha\beta = -3/2 = -1\frac{1}{2}$

2) a) $x^2-5x+2=0$

$\alpha+\beta = 5$

b) $\alpha\beta$

$= 2$

c) $\frac{1}{\alpha}+\frac{1}{\beta}$

$= \frac{\beta+\alpha}{\alpha\beta} = 5/2 = 2\frac{1}{2}$

d) $\alpha^2+\beta^2$

$= (\alpha+\beta)^2$

$= \alpha^2+2\alpha\beta+\beta^2$

$\therefore = (\alpha+\beta)^2-2\alpha\beta$

$= (5)^2-4$

$= 21$

e) $(\alpha-2)(\beta-2)$

$= \alpha\beta-2\alpha-2\beta+4$

$= 2-2\alpha-2\beta+4$

$= -2\alpha-2\beta+6$

$= -2(\alpha+\beta)+6$

$= -10+6$

$= -4$

f) $\alpha^2\beta+\alpha\beta^2$

$= \alpha\beta(\alpha+\beta)$

$= 5(2)$

$= 10$

g) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$= \frac{\beta^2+\alpha^2}{\alpha^2\beta^2}$

$= \frac{(\beta+\alpha)^2-2\alpha\beta}{(\alpha\beta)^2}$

$= \frac{21}{4} = 5\frac{1}{4}$

h) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$= \frac{\alpha^2+\beta^2}{\beta\alpha}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\beta\alpha}$

$= \frac{21}{2} = 10\frac{1}{2}$

i) $(\alpha-\beta)^2$

$= \alpha^2 - 2\alpha\beta + \beta^2$

$= (\alpha+\beta)^2 + 2\alpha\beta$

$= (5)^2 + 4$

$= 29$ ON BACK PG

3) $2x^2+3x-1=0$

a) $\alpha+\beta = -\frac{3}{2} = -1\frac{1}{2}$

b) $\alpha\beta = -\frac{1}{2} = -2$

c) $\alpha^2+\alpha\beta+\beta^2$

$= (\alpha+\beta)^2 - \alpha\beta$

$= (-1.5)^2 + 2$

$= 4\frac{1}{4}$

d) $(\alpha-1)(\beta-1)$

$= \alpha\beta - \alpha - \beta + 1$

$= -1 - \alpha - \beta$

$= -1 - (\alpha+\beta)$

$= -1 - (-1\frac{1}{2})$

$= \frac{1}{2}$

e) $\alpha\beta - 2\alpha - 2\beta + 1$

$= -2 - 2(\alpha+\beta) + 1$

$= -1 - 2(-1\frac{1}{2})$

$= -1 + 3$

$= 2$

f) $\alpha^2\beta^3 + \alpha^3\beta^2$

$= \alpha^2\beta^2(\beta+\alpha)$

$= (\alpha\beta)^2(\beta+\alpha)$

$= 4 \times -1.5$

$= -6$

4) a) $(x+3)(x-5)=0$

$= x^2 - 5x - 3x + 15 = 0$

$= x^2 - 8x + 15 = 0$

b) $(x-4)(x+2)=0$

$x^2 - 2x - 4x - 8 = 0$

$= x^2 - 6x - 8 = 0$

c) $(x-1)(x+7)=0$

$x^2 + 7x - x - 7 = 0$

$x^2 + 6x - 7 = 0$

d) $(x-\frac{1}{2})(x-1\frac{1}{2})=0$

$= x^2 - 1.5x - 0.5x + \frac{3}{4}$

$= x^2 - 2x + \frac{3}{4} = 0$

e) $(x+4)(x+\frac{1}{2})$

$= x^2 + \frac{1}{2}x + 4x + 2$

$= x^2 + 4\frac{1}{2}x + 2 = 0$

f) $(x-\sqrt{3})(x+\sqrt{3})$

$= x^2 + \sqrt{3}x - \sqrt{3}x - 3$

$= x^2 - 3$

g) $(x-2-\sqrt{3})(x+2-\sqrt{3})$

$= x^2 + 2x - \sqrt{3}x - 2x - 4 + 2\sqrt{3} - \sqrt{3}x - 2\sqrt{3} + 3$

$= x^2 + 4x - 1 = 0$

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4) $(5-\sqrt{2})(5+\sqrt{2})$

$(x-5+\sqrt{2})(x-5-\sqrt{2})$

$= x^2-5x-\sqrt{2}x-5x+25+5\sqrt{2}+\sqrt{2}x-5\sqrt{2}+2$

$= x^2-10x+27=0$

5-a) $x^2-(2+k)x+3k=0$

$a=1$

$b=2+k$

$c=3$

$\alpha+\beta = \frac{2+k}{1} = 2+k$

$\alpha\beta = 3k$

b) i) $2+k=5$

$k=3$

ii) $3k=12$

$k=4$

iii) $\alpha\beta = 4(\alpha+\beta)$

$3k = 4(2+k)$

$3k = 8+4k$

$-5 = k$

6) Let roots $= \alpha$ and $\frac{1}{\alpha}$

$\therefore \alpha+\frac{1}{\alpha} = \frac{-n-2}{n-2}$

$\frac{\alpha^2+1}{\alpha} = \frac{-n-2}{n-2}$

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$\alpha\beta = \frac{c}{a}$

$1 = \frac{2n}{n-2} \rightarrow 2n+1$?

$n-2 = 2n$

$-2 = n$ SOB

$\therefore \alpha+\beta = \frac{0}{4} = 0$

7) a) $x^2-(k+2)x+(k-4)=0$

If 0 is a root then $x=0$

$= 0-(k+2)0+(k-4)=0$

$= k-4=0$

$k=4$

b) $x=4$

$= 16-4k-8+k-4=0$

$-3k=0$

$k=0$

$k=\frac{4}{3}$

c) Roots are α and $\frac{1}{\alpha}$

$\therefore \alpha+\beta = -k+2$

$\alpha\beta = 1$

$k-4=1$

$k=5$

8) $3x^2 - 8x + k = 0$

Root α & 3α

$\alpha + \beta = \frac{8}{3} = 2\frac{2}{3}$

$3\alpha + \alpha = 2\frac{2}{3}$

$4\alpha = 2\frac{2}{3}$

$\alpha = \frac{2}{3}$

$\alpha\beta = \frac{k}{3}$

~~$\frac{2}{3}\beta =$~~

$3\alpha(\alpha) = \frac{k}{3}$

$= 1\frac{1}{3} = \frac{k}{3}$

$k = 4$

9) $x^2 - 1.3x - 8.88 = 0$

$x = 3.7$

$(3.7)^2 - (1.3)(3.7) - 8.88 = 0$

$= 13.69 - 4.81 - 8.88 = 0$

$= 0$

10) $nx^2 + nxm - 1 = 0$

a) $\alpha + \beta = -\frac{nxm}{n}$

b) $\alpha\beta = -\frac{1}{n}$

c) $(\alpha+\beta)^2 + 2\alpha\beta$

$= \left(-\frac{nxm}{n}\right)^2 + 2\left(-\frac{1}{n}\right)$

10)

$\frac{1}{x+m} = nx$

$nx^2 + nmx - 1$

a) $\alpha + \beta = -b/a$

$= \frac{-nm}{n}$

$= -m$

b) $\alpha\beta = \frac{c}{a}$

$= -1/n$

c) $(\alpha-\beta)^2$

$= (\alpha+\beta)^2 - 4\alpha\beta$

$= m^2 + \frac{4}{n}$

$= m^2 + \frac{4}{n}$

(9) $\alpha + 3.7 = 1.3$

$\therefore \underline{\alpha = -2.4}$ is the other root.

4

g) $(2+\sqrt{3})(2-\sqrt{3})$

$\alpha+\beta = 2+\sqrt{3}+2-\sqrt{3}$
$= 4$
$\alpha\beta = (2+\sqrt{3})(2-\sqrt{3})$
$= 4-2\sqrt{3}+2\sqrt{3}-3$
$= 1$
$x^2-(\alpha+\beta)x+\alpha\beta=0$
$\therefore \quad x^2-4x+1=0$

h) $(5-\sqrt{2})(5+\sqrt{2})$

$\alpha+\beta = 5-\sqrt{2}+5+\sqrt{2}$
$= 10$
$\alpha\beta = (5-\sqrt{2})(5+\sqrt{2})$
$= 25+5\sqrt{2}-5\sqrt{2}-2$
$= 23$

$x^2-10x+23=0$

2) i) $(\alpha+\beta)^2-4\alpha\beta$
$= 25-8$
$= 17$

6) $\alpha\beta = 1$
$\frac{2n+1}{n-2} = 1$

$n = -3$

7) c) $\frac{k+2}{k} = 1$
$k = -1$

11) Let roots be α, β

$\alpha+\beta = -b/a$
$= 4-k$

$\alpha\beta = \frac{c}{a} = \frac{a}{c}$
$= k-2$

$\frac{1}{\alpha\beta} = \frac{1}{k-2}$

$\alpha+\beta = \frac{1}{\alpha\beta}$

$4-k = \frac{1}{k-2}$

$(4-k)(k-2) = 1$

$6k-k^2-9=0$
$k^2-6k+9=0$
$(k-3)(k-3)=0$
$\therefore \quad k=3$

(B)

12) Let roots be $\alpha, \alpha-2$

$\alpha+\alpha-2=5$
$2\alpha = 7$
$\alpha = 7/2$

$\therefore$ Roots $= 7/2 = 3\frac{1}{2}$

$= 1\frac{1}{2}$

$\therefore 4\left(\frac{3}{2}\right)^2 - 20\left(\frac{3}{2}\right) + k = 0$

$9 - 30 + k = 0$

$\therefore \underline{k = 21}$