The Ultimate Trigonometry Revision

1) In ΔPQR, $\angle Q$ is a right angle. If $\tan P = \frac{9}{40}$, find the value of sin P.

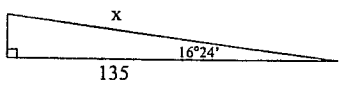
2) Find the value of θ if $\cos\theta = \sin 35°$.

3) Find the value of θ if $\sec 51 = \operatorname{cosec}(2\theta + 17)$.

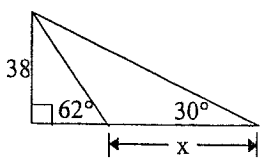
4) Find the value of cosec22° (correct to 4 d.p.).

5) Find the acute angle θ (to the nearest minute) if $\sec\theta = 1.2230$.

6) Find x (to 1 decimal place).

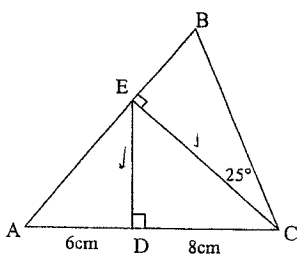


Find x (to 1 decimal place).

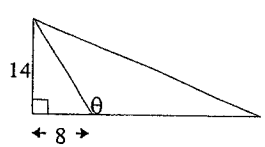


8) The larger diagonal of a rhombus of side 6cm is $6\sqrt{3}$ cm. Find the size of the acute angle formed by adjacent sides of the rhombus.

9) In the diagram below, find (to the nearest cm^2) the area of ΔABC, given that the area of ΔCDE is 20 cm^2.



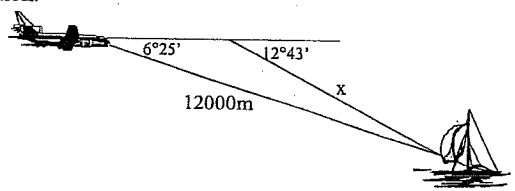
10) Find θ (to the nearest degree).



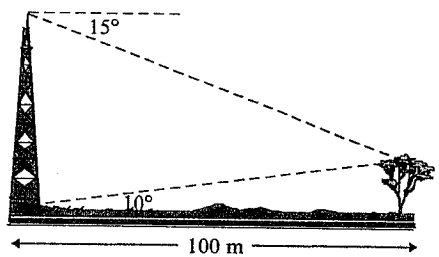
11) Find the height of a street light if it casts a shadow 11.8m long when the elevation of the sun is 36°14'. Round your answer off to 1 decimal place.

12) The bases of a tower and a building are on the same level. The distance between the two bases is twice the height of the tower and the angle of elevation of the top of the tower from the top of the building is 20°. Prove that the tower is (approximately) 3.7 times as high as the building.

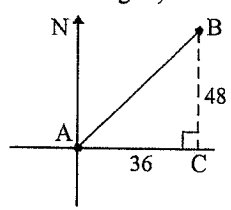
13) From a plane, the angle of depression of a boat 12 000m away (measured in a straight line) is 6°25'. After flying some time on the same course, the angle of depression of the boat is 12°43'. Find (to the nearest metre) the distance x metres of the plane from the ship (measured in a straight line) at the time of the second observation.



14) From the top of a tower, the angle of depression of the top of a tree is 15°. The angle of elevation of the top of the same tree observed from the base of the tower is 10°. If the distance between the base of the tree and the base of the tower is 100 m, find (to the nearest metre) the height of the tree and the height o the tower.



15) Find the bearing of B from A (to the nearest degree).



16) A ship is 3.6km from a lighthouse on a bearing of 215°. How far west (to the nearest tenth of a kilometre) is the ship from the lighthouse?

17) A patrol boat is sailing on a course bearing 030°, 2.3km due south of a tiny island on which survivors of a shipwreck are waiting to be rescued. They have built a small fire hoping to attract the attention of any passers-by. The light from the fire is visible for 1.2km. If the patrol boat maintains its present course, will the survivors attract the attention of the patrol boat? Give reasons.

18) Find the exact value of sec30°.

19) Find the exact value of $\frac{\cos 30°}{\operatorname{cosec} 30° + \cot 45°}$.

20) Prove that $\frac{\tan 60°}{2 \sin 45° \sec 45°} = \cos 30°$.

21) Simplify $(\sec^2\theta - 1)\cot^2\theta$.

22) Simplify $\sec^2\theta\cos\theta - \tan^2\theta\cos\theta$.

23) Simplify $\sqrt{\sec\theta \operatorname{cosec}\theta \tan\theta - 1}$.

24) Simplify $(1 + \sin\theta)^2 + (1 + \cos\theta)^2 - 2(\sin\theta + \cos\theta)$.

25) Prove that $\frac{\cot\theta \operatorname{cosec}\theta}{1 + \cot^2\theta} = \cos\theta$.

26) Prove that $\left(\frac{\sin\theta + \cos\theta}{\tan\theta}\right)^2 = 2\cot\theta \cos^2\theta + \cot^2\theta$.

27) An angle of 482° lies in which quadrant?

28) An angle of -179° lies in which quadrant?

29) In which quadrants does θ lie if cosecθ is negative and secθ is negative?

30) In which quadrants does θ lie if cotθ and secθ have different signs and cosθ is positive?

31) Express tan265°24′ as a ratio of an acute angle.

32) Find the exact value of cosec315°.

33) Find the exact value of $\frac{\sin^2 120°}{(\cot 420° + \sec 330°)^2}$.

34) Find the value of cos125° (correct to 4 decimal places).

35) Find the value of $\frac{6.2 \tan 92°16'}{5.4}$ (correct to 2 decimal places).

36) Find the value of $\sqrt{\frac{\operatorname{cosec}^2 8°10' - \cot^2(-187)°}{-(\sin 540° + \sec 3°07')}}$ (correct to 2 decimal places).

37) Find all possible values of θ if $\cos\theta = \frac{1}{\sqrt{2}}$ and $0 \le \theta \le 360°$.

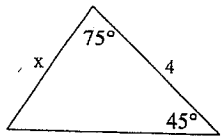
38) Find all possible values of θ if $\sec 3\theta = 2$ and $0 \le \theta \le 360°$.

39) Find θ, to the nearest degree, if $\sin\theta = 0.809$, tanθ is negative and $0 \le \theta \le 360°$.

40) Find all possible values of θ (to the nearest minute) if $3\sin^2\theta + 8\sin\theta - 3 = 0$ and $0 \le \theta \le 360°$.

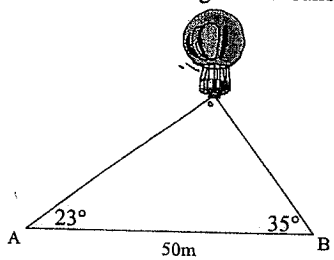
41) Solve $\cos\theta = -1$ if $0 \le \theta \le 360°$.

42) Find x. Leave your answer in simplest surd form.

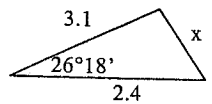


43) In ΔABC, ∠B = 12°, a = 19.2 and b = 23.7. Find ∠C (to the nearest degree).

44) A hot air balloon is tethered at two points A and B, 50m apart on the ground. The line from A to the balloon makes an angle of 23° with the ground, whilst the line from B to the balloon makes an angle of 35°. Calculate the length of the tethers and the height of the balloon (to the nearest metre).

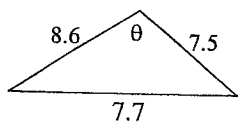


45) Find x (to 1 d.p.).



46) Find the smallest angle of a triangle with sides 85.9, 63.4 and 105.2. Answer to the nearest degree.

47) Find θ (to the nearest degree).



48) A hockey player is 7m from one goal post and 8.5m from the other. A hockey goal is 2m wide. Within what angle (to the nearest degree) must she "shoot" for goal to have a chance of scoring?

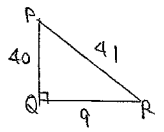
49) In ΔABC, ∠A = 60°, b = 20m and c = 12m. Find the exact area of the triangle.

50) Find the area of an equilateral triangle of side length 7.5cm. Give your answer correct to 1 d.p.

The Ultimate Trigonometry Revision.

Check corrections! Angelina Tjakravidjaja

Qu 1.



$\therefore \sin P = \frac{9}{41}$ ✓

Qu 2. $\cos(90-35) = \sin 35$.

$\cos 55^\circ = \sin 35^\circ$

$\therefore \theta = 55$ ✓

Qu 3 $\frac{1}{\cos 51} = \frac{1}{\sin(2\theta+17)}$

$\therefore \cos 51 = \sin(2\theta+17)$

$= \sin(90-51)$

$= \sin 39$

$\therefore 39 = 2\theta + 17$ ✓

$2\theta = 22 \quad \therefore \theta = 11$ ✓

Qu 4 $\operatorname{cosec} 22^\circ = \frac{1}{\sin 22^\circ}$.

$= 2.6695$ (to 4 d.p.) ✓

Qu 5 $\frac{1}{\cos\theta} = 1.2230$. ✓

$\cos\theta = \frac{1}{1.2230}$

$\theta = 35^\circ 9'$ ✓

Qu 6. $\cos 16^\circ 24 = \frac{135}{x}$ ✓

$\therefore x = \frac{135}{\cos 16^\circ 24}$

$= 140.7$ units ✓ (to 1dp)

Qu 7 $\frac{y}{38} = \cos 62$ Wrong ratio, try again $x = 45.64$ (to 1dp)

$\therefore y = 38\cos 62 = 17.84$ (to 2dp)

$\cos 30 = \frac{x+y}{38}$

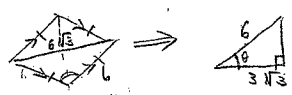
$\therefore 38\cos 30 = x + y$

$\therefore x = 38\cos 30 - 17.84$

$= 15.07$

$= 15.1$ units (to 1dp)

Qu 8



$\therefore \cos\theta = \frac{3\sqrt{3}}{6}$

$\theta = 30$ ✓ $\therefore 2\theta = 60$ ✓

$\therefore$ acute angle is 60°

Qu 9 $A\Delta CDE = 20\text{cm}^2$.

$\therefore \frac{1}{2} ED \times DC = 20$.

$8ED = 40$

$\therefore ED = 5$cm ✓

Using Pythagoras' Theorem.

$64 + 29 = EC^2$

$EC = \sqrt{89}$ cm

$A\Delta ADE = \frac{1}{2} \times 5 \times 6$

$= 15\text{cm}^2$ ✓

$\tan 25^\circ = \frac{EB}{EC}$

$EB = \sqrt{89} \times \tan 25^\circ$

$= 4.40$ cm

$\therefore A\Delta BCE = \frac{1}{2} \times 4.4 \times \sqrt{89}$

$= 20.75\text{cm}^2$

$\therefore \Delta ABC = \Delta CDE + \Delta ADE + \Delta BCE$

Qu 10 $\tan\alpha = \frac{14}{8}$

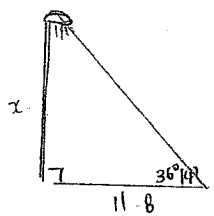
$\therefore \alpha = 60^\circ 15'$ ✓

since α & θ are supplementary $\angle$s:

$180 - \alpha = \theta$

$\therefore \theta = 120^\circ$ ✓ (to nearest degree)

Qu 11

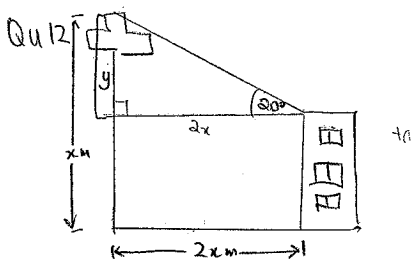


$\tan 36^\circ 14' = \frac{x}{11.8}$ ✓

$\therefore x = 8.65$m

$\therefore$ 8.7m tall ✓ (to 1dp)

Qu 12



$\therefore \tan 20 = \frac{y}{2x}$

$\therefore y = 2x\tan 20$ ✓

$\approx 0.73x$ (to 2dp)

Height of

$\therefore$: $x - 0.73x = 0.27x$ (to 2dp).

$\therefore \frac{\text{Height of Tower}}{\text{Height of building}} = \frac{x}{0.27x} = 3.7$ ✓ (to 1dp)

$\therefore$ The tower is approximately 3.7 times as high as the building. ✓

Qu 13 $180^\circ - 12^\circ 43' = 167^\circ 17'$ (supplementary $\angle$s)

$\therefore \frac{12000}{\sin 167^\circ 17'} = \frac{x}{\sin 6^\circ 25'}$ ✓

$\therefore x = \frac{12000 \sin 6^\circ 25'}{\sin 167^\circ 17'}$

$= 6092.29$ ✓

$\therefore$ 6092m ✓ (to the nearest m) is the distance of the plane from the ship at the time of 2nd observation.

Qu 14 Height of tree:

(Let the height of the tree be x)

$\therefore \tan 10 = \frac{x}{100}$

$x = 100 \tan 10$ ✓

$= 17.63$

$\therefore$ the tree is 18m tall ✓ (nearest m)

$BC^2 = 100^2 + (18^2)^2 = 10324$ (should use $(17.63)^2$)

$BC = \sqrt{10324} = 101.61$ m ✓

$\angle CAB = 90 - 15$ (complementary $\angle$s)

$= 75^\circ$ ✓

$\angle ACB = 15 + 10$ (Using parallel lines & alt $\angle$s)

$= 25^\circ$ ✓

$\therefore \frac{BC}{\sin\angle CAB} = \frac{AB}{\sin\angle ACB}$

$\therefore \frac{101.61}{\sin 75} = \frac{AB}{\sin 25}$ ✓

$\therefore AB = 44.46$ ✓

$\therefore$ Height of tower is 44m ✓.

) $\tan A = \frac{48}{36}$

$A = 53°8'$ ✓

$\therefore$ B is 037°T from A ✓

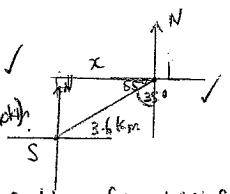
Qu16.

Let the distance be x

$\cos 55 = \frac{x}{3.6}$

$x = 2.1$ km ✓

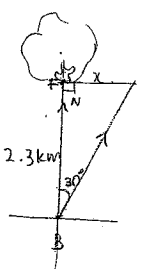
(to nearest $\frac{1}{10}$th)



✓

$\therefore$ the ship is 2.1km far west from the lighthouse.

Qu17



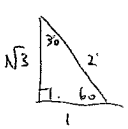
$\sin 30 = \frac{x}{2.3}$

$x = 2.3 \sin 30$

$= 1.15$ km ✓

$\therefore$ The survivors will attract the attention of the boat as the boat will come within ✓ 1.2km East of the fire.

18. $\sec 30° = \frac{1}{\cos 30°} = \frac{1}{\frac{\sqrt{3}}{2}}$



$\therefore \sec 30° = \frac{2}{\sqrt{3}}$ ✓

Qu19. $\frac{\cos 30}{\operatorname{cosec} 30 \cot 45°}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\operatorname{cosec} 30° = 2$

$\cot 45° = 1$

$\therefore \frac{\frac{\sqrt{3}}{2}}{3} = \frac{\sqrt{3}}{2} \times \frac{1}{3}$ ✓

$\therefore \frac{\cos 30}{\operatorname{cosec} 30 \cot 45} = \frac{\sqrt{3}}{6}$ ✓

Qu20. RHS $= \frac{\sqrt{3}}{2}$

LHS $= \tan 60 = \sqrt{3}$

$\sin 45 = \frac{1}{\sqrt{2}}$ ✓

$\sec 45 = \sqrt{2}$

$\therefore \frac{\sqrt{3}}{2(\frac{1}{\sqrt{2}} \times \sqrt{2})} = \frac{\sqrt{3}}{2}$ ✓ = RHS

$\therefore$ LHS = RHS.

Qu21. $(\sec^2\theta - 1)\cot^2\theta$

$= \tan^2\theta \times \cot^2\theta$

$= (\tan\theta)^2 \times \frac{1}{(\tan\theta)^2}$ ✓

$= 1$ ✓

Qu22. $\sec^2\theta \cos\theta - \tan^2\theta \cos\theta$ ✓

$= \cos\theta (\sec^2\theta - \tan^2\theta)$

$= \cos\theta$ ✓

Qu23 $\sqrt{\frac{1}{\sin\theta} \times \frac{1}{\cos\theta} \times \tan\theta - 1}$

$= \sqrt{\frac{1}{\sin\theta\cos\theta} \times \frac{\sin\theta}{\cos\theta} - 1}$

$= \sqrt{\frac{1-\cos^2\theta}{\cos^2\theta}} = \sqrt{\frac{\sin^2\theta}{\cos^2\theta}}$ ✓

$= \tan\theta$ ✓

Qu24 $(1+\sin\theta)^2 + (1+\cos\theta)^2 - 2(\sin\theta + \cos\theta)$

$= 1 + 2\sin\theta + \sin^2\theta + 1 + 2\cos\theta + \cos^2\theta - 2\sin\theta - 2\cos\theta$ ✓

$= 1 + 1 + 1 = 3$ ✓

Qu25 $\frac{\cot\theta \operatorname{cosec}\theta}{1+\cot^2\theta} = \cos\theta$

LHS: $\frac{\frac{\cos\theta}{\sin\theta} \times \frac{1}{\sin\theta}}{\operatorname{cosec}^2\theta} = \frac{\frac{\cos\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}}$ ✓

$= \frac{\cos\theta}{\sin^2\theta} \times \frac{\sin^2\theta}{1}$

$= \cos\theta$ = RHS ✓

$\therefore$ LHS = RHS

Qu26. $\left(\frac{\sin\theta + \cos\theta}{\tan\theta}\right)^2 = 2\cot\theta\cos^2\theta + \cot^2\theta$

LHS $= \frac{(\sin\theta+\cos\theta)^2}{\tan^2\theta} = \frac{\sin^2\theta + 2\sin\cos\theta + \cos^2\theta}{\tan^2\theta}$ ✓

$= 1 + 2\sin\cos\theta \times \frac{\cos^2\theta}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta} + \frac{2\cos^3\theta}{\sin\theta} = \cot^2\theta + 2\cot\theta\cos^2\theta$ ✓

$=$ RHS

$\therefore$ LHS = RHS

Qu27 $482 - 360 = 122$. ✓

$\therefore$ 2nd Quadrant

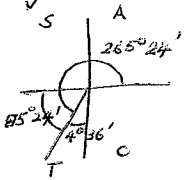
Qu28 $-179° =$ 3rd Quadrant ✓

Qu29. ~~2nd~~ and 3rd Quadrants only

Qu30. 4th Quadrant ✓

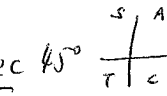
Qu31 ☆ $\tan 265°24'$

$= \tan 85°24'$



Qu32 $\operatorname{cosec} 315° = -\operatorname{cosec} 45°$ ✓

$= \frac{1}{\sin 315°} = -\sqrt{2}$



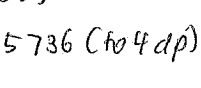
Qu33 $\frac{\sin^2 120}{(\cot 420° + \sec 330)^2}$

$= \frac{\sin^2 60}{(\cot 60 + \sec 30)^2} = \frac{(\frac{\sqrt{3}}{2})^2}{(\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}})^2}$ ✓

$= \frac{3}{4} \div \left(\frac{3\sqrt{3}}{3}\right)^2 = \frac{3}{4} \times \frac{9}{27}$

$= \frac{1}{4}$ ✓

Qu34. $\cos 125 = -\cos 55$



$= -0.5736$ (to 4dp)

Qu35 $\frac{6.2 \tan 92°16'}{5.4}$

$= 99.01$ (to 2dp)

36 $\sqrt{\dfrac{\operatorname{cosec}^2 8°10' - \cot^2 173}{-(\sin 180 + \sec 3°07')}}$

$= \sqrt{\dfrac{49.5563 - 66.3304}{-1.00148}}$ ✓

$= \sqrt{16.74931} = 4.09$ (to 2d.p.) ✓

Qu 37 $\cos\theta = \frac{1}{\sqrt{2}}$ for $0° \leq \theta \leq 360°$

$\theta = 45°, 315°$ ✓ ✓ for $0° \leq \theta \leq 360°$

Qu 38 $\sec 3\theta = 2 \quad 0° \leq \theta \leq 360°$

let α be 3θ

$\therefore \cos\alpha = \frac{1}{2}$ for $0° \leq 3\theta \leq 1080°$

$\alpha = 60°, 300°, 420°, 660°, 780°, 1020°$

$\therefore 3\theta = 60°, 300°, 420°, 660°,$

$\theta = 20°, 100°, 140°, 220°, 260°, 340°$

Qu 39 $\sin\theta = 0.809 \quad \theta = 54°$.

$\tan\theta < 0$

$0 \leq \theta \leq 360°$

$\therefore$ 2nd Quadrant & 4th Quadrant

$\therefore 180 - 54 = 126°$

$\therefore \theta = 126°, 306°$

Qu 40 $3\sin^2\theta + 8\sin\theta - 3 = 0$ for $0 \leq \theta \leq 360°$

$\therefore (3\sin\theta - 1)(\sin\theta + 3) = 0$ ✓

$\therefore \sin\theta = \frac{1}{3}$

$\theta = 19°28', 160°31'$ ✓ or,

$\sin\theta = -3$

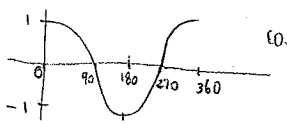
but $\sin\theta \neq -1$ ✓

$\therefore$ no solution

$\therefore \theta = 19°28', 160°31'$ ✓

for $0° < \theta < 360°$

Qu 41 $\cos\theta = -1$ if $0 \leq \theta \leq 360°$



$\therefore \theta = 180°$ ✓

Qu 42 $\frac{x}{\sin 45} = \frac{4}{\sin 60}$

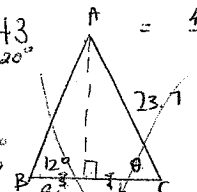
$x = \frac{4\sin 45}{\sin 60} = \frac{4}{\sqrt{2}} \times \frac{2}{\sqrt{3}}$ ✓

$= \frac{8}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$= \frac{4\sqrt{6}}{3}$ units.

$\therefore x = \left(\frac{8}{\sqrt{6}}\right)$ units.

Qu 43



Use sine rule first to find $\angle A$, then angle sum of Δ to find $\angle C$, try again

$\cos\theta = \frac{9.6}{23.7}$

$\theta = 66°6'$

$= 66°$ ✗

Qu 44 $\frac{AX}{\sin 35} = \frac{50}{\sin 122}$ ✓

$AX = 33.82$ m

$= 34$ m (nearest m) ✓

$\frac{BX}{\sin 23} = \frac{50}{\sin 122}$

$BX = 23$ m. (nearest m) ✓

Let height of balloon be y m

$\therefore \frac{y}{\sin 23} = \frac{34}{\sin 90}$

$\therefore y = 13$ m ✓ (nearest m)

Qu 45 $x^2 = 3.1^2 + 2.4^2 - 2(3.1)(2.4)\cos 26°18'$ ✓

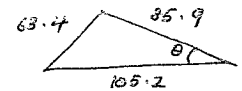
$= 1.4$ (to 1 dp) ✓

Qu 46 $\cos A = \dfrac{63.4^2 + 105.2^2 - 85.9^2}{2(63.4)(105.2)}$

$= 0.5778$.

$\therefore A = 85°$ (to nearest degree) ✓

Smallest angle is opposite the shortest side



$63.4^2 = 85.9^2 + 105.2^2 - 2(85.9)(105.2)\cos\theta$

$\therefore \cos\theta = \dfrac{85.9^2 + 105.2^2 - 63.4^2}{2 \times 85.9 \times 105.2}$

$\theta = 37°$ (to the nearest deg).

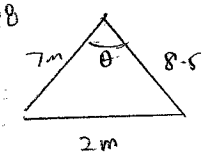
Qu 47 $\cos A = \dfrac{8.6^2 + 7.5^2 - 7.7^2}{2(8.6)(7.5)}$ ✓

$= 0.54977$

$\therefore A = 56°38'$.

$= 57°$ ✓ (to nearest degree)

Qu 48

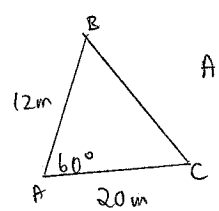


$\cos\theta = \dfrac{49 + (8.5)^2 - 4}{14 \times 8.5}$ ✓

$= 0.98529$.

$\therefore \theta = 10°$ ✓ (to nearest degree)

Qu 49

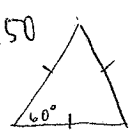


$A = \frac{1}{2} \times 12 \times 20 \times \sin 60$

$= 120 \times \frac{\sqrt{3}}{2}$ ✓

$= \left(\frac{120\sqrt{3}}{2}\right) m^2$ ✓ $= 60\sqrt{3}\ m^2$

Qu 50



$A = \frac{1}{2} \times 7.5 \times 7.5 \times \sin 60$ ✓

$= 24.4\ cm^2$ (to 1dp) ✓