Centre Number

Student Number

CSSA
CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NSW

2013
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Morning Session
Monday, 5 August 2013

General Instructions

- Reading time – 5 mins
- Working time – 3 hours
- Write using blue or black pen
  Black pen is preferred
- Use Multiple Choice Answer Sheet provided
- Board-approved calculators may be used
- A table of standard integrals is provided on a SEPARATE sheet
- In Questions 11-16, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks – 100

Section I Pages 2-8

10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II Pages 9-16

90 marks
- Attempt Questions 11-16
- Allow about 2 hours and 45 minutes for this section

6200-1

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x > 0$

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 In exact form, which is the solution to the equation $2x^2 - 5x - 2 = 0$?

(A) $x = \dfrac{-5 \pm 3}{4}$

(B) $x = \dfrac{5 \pm 3}{4}$

(C) $x = \dfrac{-5 \pm \sqrt{41}}{4}$

(D) $x = \dfrac{5 \pm \sqrt{41}}{4}$

2 What is the focus of $(x-3)^2 = 8y$?

(A) (0,3)

(B) (3,2)

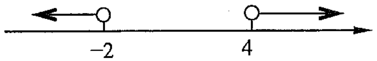
(C) (2,3)

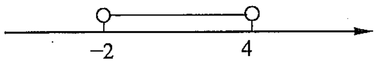
(D) (3,0)

3 Which of the following represents the solution to $|x-1| \geq 3$?

(A) 

(B) 

(C) 

(D) 

4 A raffle consists of twenty tickets in which there are two prizes. Darren buys five tickets. First prize is two movie vouchers and second prize is one movie voucher.

The probability that Darren wins at least one movie voucher is

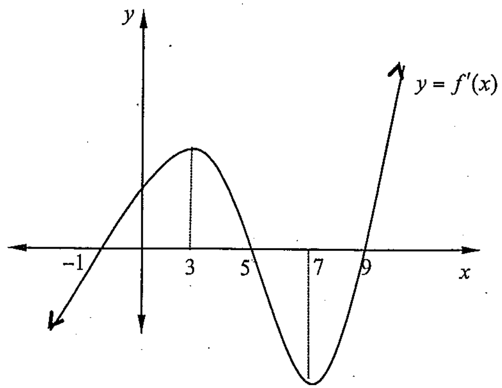
(A) $\dfrac{5}{20}$

(B) $\dfrac{27}{76}$

(C) $\dfrac{7}{16}$

(D) $\dfrac{17}{38}$

5 The graph of the derivative $y = f'(x)$ is drawn below.



A maximum turning point on $y = f(x)$ occurs at

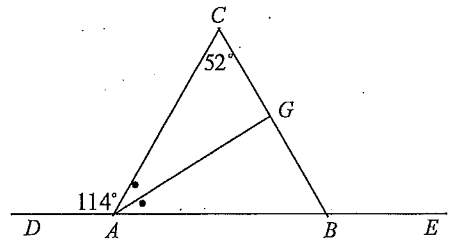
(A) $x = -1$

(B) $x = 3$

(C) $x = 5$

(D) $x = 7$

6 In the diagram, $\angle CAD = 114^\circ$ & $\angle ACB = 52^\circ$. DE is a straight line. AG bisects $\angle CAB$.



What is the value of $\angle AGB$?

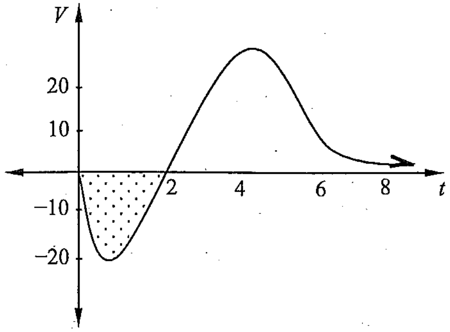
(A) 33°

(B) 52°

(C) 62°

(D) 85°

7



The graph above represents the velocity (V m/s) of a particle after t seconds. The particle is moving in a straight line from rest.

Which of the following best describes the shaded region?

(A) The acceleration of the particle after 2 seconds.

(B) The displacement of the particle after 2 seconds.

(C) The distance travelled by the particle in the first 2 seconds.

(D) The velocity of the particle in the first 2 seconds.

8 Which expression is NOT equal to $\log_2\left(\frac{1}{8}\right)$?

(A) $\dfrac{\log_e 2}{\log_e\left(\frac{1}{8}\right)}$

(B) $\dfrac{\log_e\left(\frac{1}{8}\right)}{\log_e 2}$

(C) $\log_2 1 - \log_2 8$

(D) $\dfrac{\log_e 1 - \log_e 8}{\log_e 2}$

9 The solution of $\sqrt{2}\cos x - 1 = 0$, where $0 \le x \le 2\pi$ is:

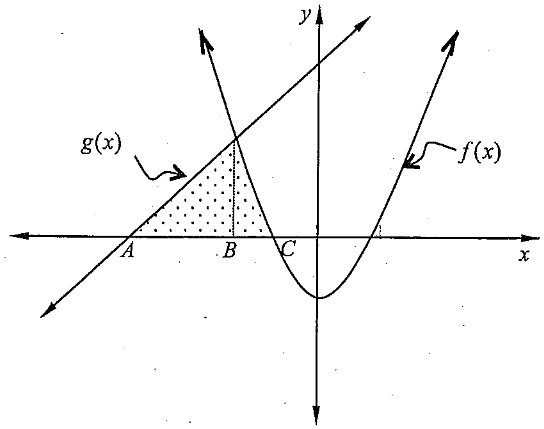
(A) $\frac{\pi}{4}, \frac{5\pi}{4}$

(B) $\frac{\pi}{4}, \frac{7\pi}{4}$

(C) $\frac{\pi}{3}, \frac{2\pi}{3}$

(D) $\frac{\pi}{3}, \frac{5\pi}{3}$

10



The shaded region is best described by:

(A) $\int_A^B g(x)\,dx + \int_B^C f(x)\,dx$

(B) $\int_A^B g(x)\,dx - \int_B^C f(x)\,dx$

(C) $\int_A^C g(x)+f(x)\,dx$

(D) $\int_A^C f(x)-g(x)\,dx$

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 40 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Given that the point $(-3, k)$ lies on the line $x + 5y - 7 = 0$, evaluate k. **2**

(b) Solve for x such that $2x - \frac{x+3}{2} = 6$. **2**

(c) Factorise completely $x^2 - 4y^2 - x + 2y$. **2**

(d) $\frac{5-2\sqrt{3}}{2-\sqrt{3}}$ can be expressed in the form $a + b\sqrt{3}$. **2**

Find the value of a and b.

(e) Evaluate $\int_1^9 t^{\frac{3}{2}}\,dt$. **2**

(f) Find the equation of the normal to $y = x\sin x$ at the point where $x = \frac{\pi}{2}$. **3**

(g) Using the Trapezoidal Rule, find an approximation for $\int_0^2 e^{x^2}\,dx$ with 3 function values. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Differentiate with respect to x:

(i) $\dfrac{\log_e x}{4x}$ **2**

(ii) $e^{\tan 2x}$. **2**

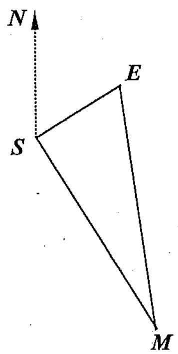
(b) Find $\displaystyle\int \frac{e^{2x}}{4+e^{2x}}\,dx$. **2**

(c) Each hour, a grandfather clock chimes the number of times that corresponds with the time of day. **2**

How many times does the clock chime in one day?

(d) The curve $y = ax^2 + bx + 2$ has a maximum turning point at $(-1, 1)$. Evaluate a and b. **3**

(e) Two cruise ships set sail from Sydney Harbour (**S**). The *Elvis Presley Tribute Cruise* (**E**) sails at 18 km/h on a bearing of 049° while the *Michael Jackson Tribute Cruise* (**M**) sails at 21 km/h along a bearing of 151°.

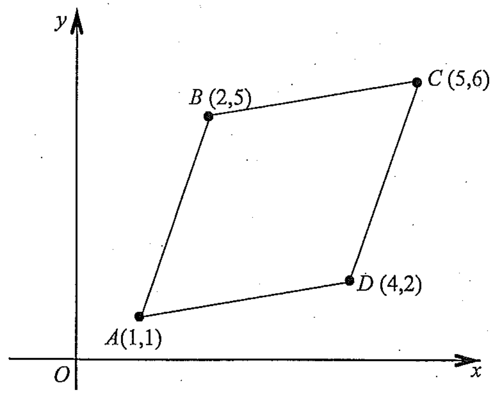


(i) Show that $\angle ESM = 102°$. **1**

(ii) Calculate the distance between the cruise ships to the nearest kilometre after 3 hours. **3**

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a)



$ABCD$ is a parallelogram with coordinates as shown in the diagram above.

(i) Show that the equation of line BC is given by $x - 3y + 13 = 0$. **2**

(ii) Find the length of AD. **1**

(iii) Find the perpendicular distance between the line BC and $A(1,1)$. **1**

(iv) Hence, or otherwise, find the area of $ABCD$. **1**

(b) The area of the sector of a circle with radius 8 cm is $\dfrac{56\pi}{5}$ cm^2. Find the angle that is subtended at the centre of the sector. **2**

(c) Consider the curve $y = 2x^3 + 3x^2 - 36x + 4$ for $-5 \le x \le 5$.

(i) Find the stationary points and determine their nature. **3**

(ii) Find the point of inflexion. **2**

(iii) Sketch the curve for $-5 \le x \le 5$. **2**

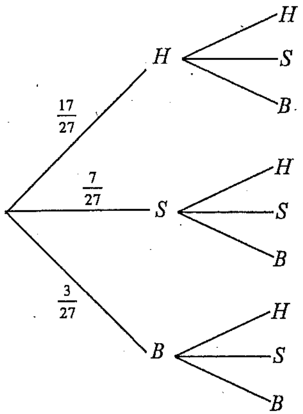
(iv) Find the maximum value in the domain given. **1**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) A jar contains 27 buttons. Twenty of the buttons are heart-shaped and ten of the buttons have silver sparkles painted on them. Each button has least one of these two characteristics.

(i) Explain why the probability that a button chosen at random which is heart-shaped and has silver sparkles is $\frac{1}{9}$. **1**

(ii) Two buttons are selected from the jar at random without replacement. Copy and complete the tree diagram below indicating the probabilities on each branch. **1**

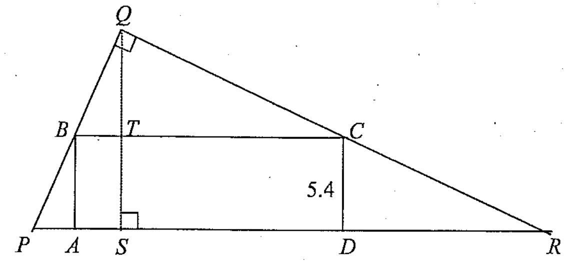


KEY
H = heart-shaped
S = silver sparkles
B = both

(iii) What is the probability that when two buttons are drawn that exactly two heart-shaped buttons are selected? **2**

Question 14 continues on page 13

(b) A rectangle $ABCD$ with height 5.4 cm is inscribed inside a right-angled triangle PQR. $PQ = 12$ cm and $QR = 16$ cm.
QS is drawn perpendicular to PR.

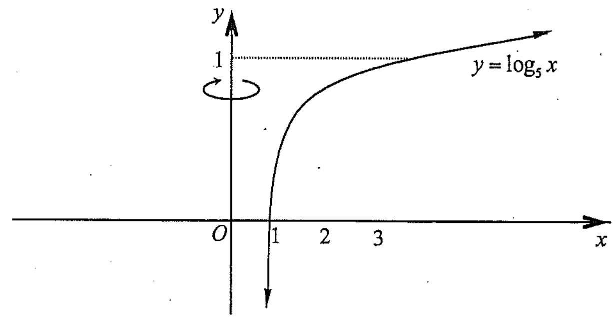


(i) By considering the area of ΔPQR show that $QS = 9.6$ cm. **2**

(ii) Show that $QC = 7$ cm. **2**

(iii) Deduce the area of the rectangle $ABCD$ is 47.25 cm^2. **2**

(c)



The diagram shows the graph of $y = \log_5 x$. The region bounded by $y = \log_5 x$, the line $y = 1$ and the x and y axis is rotated about the y-axis to form a solid.

(i) Show that the volume of the solid is given by **3**

$$V = \pi \int_0^1 e^{y \ln 25}\, dy.$$

(ii) Hence find the volume of the solid. **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) A particle is moving according to the function $x = 9t^3 - \ln t, \quad t > 0$ where x is the displacement in metres after t seconds.

(i) Find an expression for the velocity of the particle. **1**

(ii) Hence, find when the particle comes to rest. **1**

(iii) Explain why the acceleration remains positive for all values of t. **2**

(b) The following equation $T = 16 + 75e^{-0.2t}$ models the temperature in degrees Celsius of Helen's cup of coffee, t minutes after she pours it.

(i) Find the initial temperature of the coffee. **1**

(ii) How long will it take for the coffee to cool to 35°? **2**

(iii) According to the model, what temperature must the coffee eventually cool to? **1**

(iv) Find the rate of change in the temperature of the coffee after 10 minutes. **2**

(c) Reymark borrows \$50 000 at the beginning of 2013 from his local Building Society. The loan is to be repaid in equal monthly repayments of \$900, with interest charged at 7.2% p.a. at the end of each month, just before each repayment.

Let A_n be the amount owing after the nth repayment.

(i) Show that **2**

$$A_n = 50\,000(1.006)^n - 900(1 + 1.006 + 1.006^2 + \dots + 1.006^{n-1}).$$

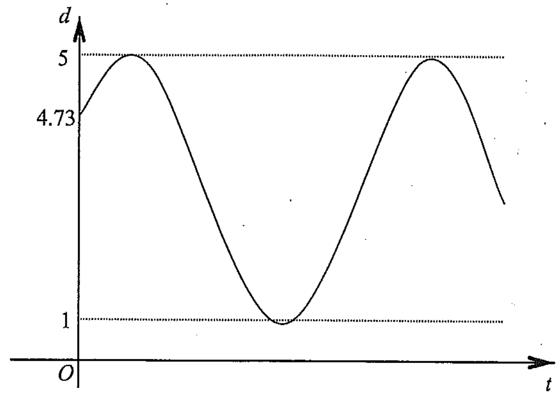
(ii) After how many months will Reymark have halved his loan? **3**

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a) A series is defined as: $1, \frac{y^2}{(1-y)^2}, \frac{y^4}{(1-y)^4}, \dots \quad y \neq 0 \;\&\; y \neq 1$ **2**

Determine the values of y so that the above series does NOT have an infinite sum.

(b)



Samantha is a keen mathematician. She has developed an equation, which is also drawn above, for the depth of water in a river near her home. The depth is modelled by the function

$$d = a\sin\left(nt + \frac{\pi}{3}\right) + c$$

where d is measured in metres and t is the time in hours. The time between successive peaks in her model is exactly 12 hours.

(i) Write down the value of the amplitude, a. **1**

(ii) Find the value of c. **1**

(iii) Find the value of n. **1**

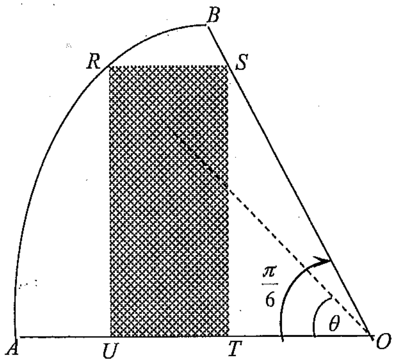
Samantha would like to cross the river when the depth is at its lowest.

(iv) After how many hours will the depth be 1 metre? **2**

Question 16 continues on page 16

Question 16 (continued)

(c) Consider the diagram below.



OAB is a sector of a circle with centre at O and radius r such that $\angle AOB = \frac{\pi}{6}$.

$RSTU$ is a rectangle drawn inside the sector and $\angle ROA = \theta$ as shown in the diagram where $0 < \theta < \frac{\pi}{2}$.

(i) Show that $UT = r\cos\theta - \sqrt{3}\, r\sin\theta$. **3**

(ii) Show that the area of the rectangle can be expressed as: **2**

$$A = r^2\left(\sin\theta\cos\theta - \sqrt{3}\,\sin^2\theta\right)$$

(iii) Hence show that the value of θ which will produce the rectangle of maximum area is $\frac{\pi}{12}$. **3**

End of Paper

Kimon Kousparis (Convenor)	Casimir Catholic College, Marrickville
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CSSA 2013 TRIAL HSC – MATHS SOLUTIONS

Section I
10 Marks

Questions 1-10 (1 mark each)

Question	Answer	Outcomes Assessed	Targeted Performance Bands
1	D	P3	2-3
2	B	P4	2-3
3	A	P4	2-3
4	D	H5	3-4
5	C	P6	4-5
6	D	P4	3-4
7	C	H4	3-4
8	A	H3	4
9	B	H5	4
10	A	H5	4-5

Section II
90 Marks

Question 11 (15 marks)

(a) (2 marks)

Sample answer:
Substitute $(-3, k)$ into the line $x + 5y - 7 = 0$

$-3 + 5(k) - 7 = 0$

$5k = 10$

$k = 2$

(b) (2 marks)

Sample answer:

$2x - \frac{x+3}{2} = 6$

$4x - x - 3 = 12$

$3x = 15$

$x = 5$

(c) (2 marks)

Sample answer:

$x^2 - 4y^2 - x + 2y = (x+2y)(x-2y) - (x-2y)$

$= (x-2y)(x+2y-1)$

(d) (2 marks)

Sample answer:

$\frac{5-2\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{(5-2\sqrt{3})(2+\sqrt{3})}{4-3} = 10 + 5\sqrt{3} - 4\sqrt{3} - 6$

$= 4 + \sqrt{3} \qquad \therefore a = 4, b = 1$

(e) (2 marks)

Sample answer:

$\int_1^9 t^{\frac{3}{2}}\, dt = \left[\frac{2}{5}t^{5/2}\right]_1^9 = \frac{2}{5}(9)^{5/2} - \frac{2}{5} = 96\frac{4}{5}$

(f) (3 marks)

Sample answer:

$y = x\sin x$

$y' = x\cos x + \sin x$

at $x = \frac{\pi}{2}$, $m = 1$

$\therefore$ gradient of normal $= -1$

Equation of tangent: $y - \frac{\pi}{2} = -1\left(x - \frac{\pi}{2}\right)$

$\therefore y = -x + \pi$

(g) (2 marks)

Sample answer:

$\int_0^2 e^{x^2}\,dx \approx \frac{h}{2}\left[f(a) + 2f\left(\frac{a+b}{2}\right) + f(b)\right]$

$\approx \frac{1}{2}\left[1 + 2(e) + e^4\right]$

≈ 30.517

Question 12 (15 marks)

(a) (i) (2 marks)

Sample answer:

Let $y = \frac{\log_e x}{4x}$ using quotient rule $\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$ $\qquad u = \ln x, \quad v = 4x$

$u' = \frac{1}{x}, \quad v' = 4$

$\frac{dy}{dx} = \frac{4x \times \frac{1}{x} - \ln x \times 4}{(4x)^2}$

$= \frac{4 - 4\ln x}{16x^2}$

$= \frac{1 - \ln x}{4x^2}$

(a) (ii) (2 marks)

$\frac{d}{dx} e^{\tan 2x} = 2\sec^2 2x \times e^{\tan 2x}$

$= 2e^{\tan 2x}\sec^2 2x$

(b) (2 marks)

$\int \frac{e^{2x}}{4 + e^{2x}}\, dx = \frac{1}{2}\ln\left(4 + e^{2x}\right) + C$

(c) (2 marks)

Arithmetic progression = 2(1 + 2 + 3 +... + 12), where $n = 12$, $a = 1$, $l = 12$

$$S_n = \frac{n}{2}(a+l)$$
$$S_{12} = 2\times\left[\frac{12}{2}(1+12)\right] = 156$$

(d) (3 marks)

$y = ax^2 + bx + 2$

$y' = 2ax + b$

substituting (-1, 1), $1 = a - b + 2$

$-1 = a - b$ .... ①

Turning point occurs at (-1, 1), substituting $0 = 2a(-1) + b$, $0 = -2a + b$ .... ②

Equating equation ① and ②, $0 = -2a + b$, $-1 = a - b$ $\therefore a = 1, b = 2$

(e) (i) (1 mark)

$\angle ESM = 151° - 49° = 102°$

(e) (ii) (3 marks)

Distance $SE = 18 \times 3 = 54$ km

Distance $SM = 21 \times 3 = 63$ km

Using cosine rule,

$$EM^2 = 54^2 + 63^2 - 2 \times 54 \times 63\cos 102°$$
$$= 8299.631144$$
$$EM = 91 \text{ km}$$

Question 13 (15 marks)

(a) (i) (2 marks)

$m_{BC} = \frac{6-5}{5-2} = \frac{1}{3}$

equation of line: $y - 5 = \frac{1}{3}(x-2)$

$3y - 15 = x - 2$

$x - 3y + 13 = 0$, as required.

(a) (ii) (1 mark)

Sample answer:

$AD = \sqrt{(4-1)^2 + (2-1)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$ units

(a) (iii) (1 mark)

Perpendicular distance $= \frac{|1-3(1)+13|}{\sqrt{1+9}}$

$= \frac{11}{\sqrt{10}}$ units

(a) (iv) (1 mark)

Area $= \frac{11}{\sqrt{10}} \times \sqrt{10}$

$= 11$ units^2

(b) (2 marks)

$\frac{56\pi}{5} = \frac{1}{2} \times 8^2 \times \theta$

$\frac{56 \times 2 \times \pi}{5 \times 64} = \theta$

$\frac{112\pi}{320} = \theta$

$\frac{7\pi}{20} = \theta$ or 63°

(c) (i) (3 marks)

$y = 2x^3 + 3x^2 - 36x + 4$

$\frac{dy}{dx} = 6x^2 + 6x - 36$

stationary when $\frac{dy}{dx} = 0$

$6x^2 + 6x - 36 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3$, $x = 2$

$\frac{d^2y}{dx^2} = 12x + 6$

When $x = -3$, $y = 85$

and when $x = 2$, $y = -40$

x	-3	2
$\frac{d^2y}{dx^2}$	-30	30
concavity	down	up

So (-3, 85) is a MAXIMUM turning point and (2, -40) is a MINIMUM turning point.

(c) (ii) (2 marks)

Possible point of inflexion at $\frac{d^2y}{dx^2} = 0$

$\frac{d^2y}{dx^2} = 12x + 6$

$12x + 6 = 0$

$x = -\frac{1}{2}$

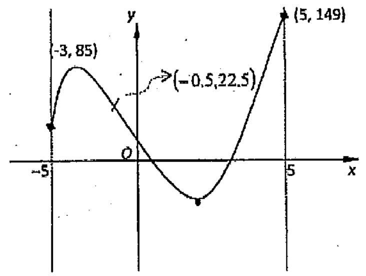
when $x = -\frac{1}{2}$, $y = 22.5$

Test for change in concavity:

x	-1	-0.5	0
$\frac{d^2y}{dx^2}$	-6	0	6
concavity	down		up

A change in concavity exists, so $\left(-\frac{1}{2}, 22.5\right)$ is a point of inflexion.

(c) (iii) (2 marks)



(c) (iv) (1 mark)

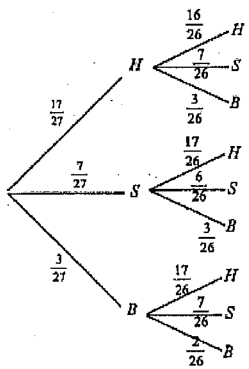
The endpoints of the curve across this domain are (-5, 9) and (5, 149). So the maximum value of the curve is 149.

Question 14 (15 marks)

(a) (i) (1 mark)

P(heart-shaped & silver sparkles) $= \frac{3}{27} = \frac{1}{9}$.

(a) (ii) (1 mark)



(a) (iii) (2 marks)

$P(2\text{ hearts}) = P(heart/heart) + P(heart/both) + P(both/heart) + P(both/both)$

$= \frac{17}{27}\times\frac{16}{26} + \frac{17}{27}\times\frac{3}{26} + \frac{3}{27}\times\frac{17}{26} + \frac{3}{27}\times\frac{3}{26}$

$= \frac{190}{351}$

(b) (i) (2 marks)

Area of $\Delta PQR = \frac{1}{2}\times 12\times 16 = 96\,\text{cm}^2$

By Pythagoras' Theorem, $PR = 20\,\text{cm}$.

Therefore, $96 = \frac{1}{2}\times 20\times QS$

$QS = 9.6$ cm.

(b) (ii) (2 marks)

ΔQTC and ΔQSR are similar (equiangular).

$\frac{QT}{QS} = \frac{QC}{QR}$ (matching sides in similar triangles are in proportion)

$\frac{4.2}{9.6} = \frac{QC}{16}$

$QC = \frac{4.2\times 16}{9.6}$

$= 7$, as required.

(b) (iii) (2 marks)

ΔQTB and ΔQSP are similar (equiangular).

$\frac{QB}{QT} = \frac{QP}{QS}$ (matching sides in similar triangles are in proportion)

$\frac{QB}{12} = \frac{4.2}{9.6}$

$QB = \frac{4.2\times 12}{9.6}$

$= 5.25$

By Pythagoras' Theorem $BC = 8.75$ units

So the area $= 8.75\times 5.4 = 47.25$ units^2

(c) (i) (3 marks)

$y = \log_5 x$

$= \frac{\log_e x}{\log_e 5}$

$x = (e^{y\ln 5})$

$x^2 = (e^{y\ln 5})^2$

$x^2 = e^{y\ln 25}$

So volume $= \pi\int_0^1 e^{y\ln 25}\,dy$, as required.

(c) (ii) (2 marks)

$$V=\pi\int_0^1 e^{y\ln 25}dy=\pi\left[\frac{e^{y\ln 25}}{\ln 25}\right]_0^1$$
$$=\pi\left(\frac{e^{\ln 25}-1}{\ln 25}\right)$$
$$\approx 23.42(2d.p.)$$

Question 15 (15 marks)

(a) (i) (1 mark)

$$v=27t^2-\frac{1}{t}$$

(a) (ii) (1 mark)

$$v=27t^2-\frac{1}{t}=0$$

$27t^3=1 \quad \therefore t=\frac{1}{3}$ sec

(a) (iii) (2 marks)

$$a=54t+\frac{1}{t^2}$$

As $t>0$, $54t>0$ & $\frac{1}{t^2}>0$ $\therefore a>0$

Acceleration will remain positive for all values of t.

(b) (i) (1 mark)

$T=16+75e^{-0.2t}$ $\therefore$ When $t=0$, $T=16+75e^0=16+75=91°$

(b) (ii) (2 marks)

When $T=35$ $\therefore 35=16+75e^{-0.2t}$ $\quad\therefore \frac{19}{75}=e^{-0.2t}$ $\quad\therefore \ln\frac{19}{75}=-0.2t$

$$\therefore t=\frac{\ln\frac{19}{75}}{-0.2}=6.86\approx 7 \text{ minutes}$$

(b) (iii) (1 mark)

As $t\to\infty$, $T\to 16$

The coffee will cool to the surrounding temperature, 16°

(b) (iv) (2 marks)

$T=16+75e^{-0.2t}$ $\quad\therefore \frac{dT}{dt}=-15e^{-0.2t}$ and when $t=10$ minutes

$\therefore \frac{dT}{dt}=-15e^{-2}=-2.03$ degrees per minute.

(c) (i) (2 marks)

$$A_1=50000\left(1+\frac{0.6}{100}\right)-900=50000(1.006)-900$$

$$A_2=50000(1.006)^2-900(1+1.006)$$

$$A_3=50000(1.006)^3-900(1+1.006+1.006^2)\ldots$$

$$\therefore A_n=50000(1.006)^n-900(1+1.006+1.006^2+\ldots+1.006^{n-1})$$

(c) (ii) (3 marks)

Let $A_n=25000$

$$\therefore 25000=50000(1.006)^n-900(1+1.006+1.006^2+\ldots+1.006^{n-1})$$

$$\therefore 25000=50000(1.006)^n-900\left(\frac{1.006^n-1}{0.006}\right)$$

$$\therefore 25000=50000(1.006)^n-150000(1.006^n-1) \qquad (\div 25000)$$

$$\therefore 1=2(1.006)^n-6(1.006^n-1) \Rightarrow 1=2(1.006)^n-6(1.006)^n+6$$

$$\therefore 4(1.006)^n=5 \Rightarrow n=\frac{\ln\frac{5}{4}}{\ln 1.006}=37.3$$

After 37.3 months ReyMarks will have halved his loan.

Question 16 (15 marks)

(a) (2 marks)

The limiting sum does not exist if $|r|>1$

$\therefore \left|\frac{y^2}{(1-y)^2}\right|>1$ $\quad$ i.e. $\frac{y^2}{(1-y)^2}>1$

So $y^2>(1-y)^2$ $\quad\therefore y^2-(1-y)^2>0$

Expanding gives $y^2-1+2y-y^2>0$ $\qquad \therefore y>\frac{1}{2}$

(b) (i) (1 mark)

$5-1=4$ $\therefore a=2$

(b) (ii) (1 mark)

By symmetry, $c=3$

(b)(iii) (1 mark)

$\frac{2\pi}{n}=12$ $\therefore n=\frac{\pi}{6}$

(b) (iv) (2 marks)

$1 = 2\sin\left(\frac{\pi}{6}t+\frac{\pi}{3}\right)+3$

$\sin\left(\frac{\pi}{6}t+\frac{\pi}{3}\right)=-1$

$\therefore \frac{\pi}{6}t+\frac{\pi}{3}=\frac{3\pi}{2}$ $\quad \therefore t = 7$ *hours*

(c) (i) (3 marks)

In ΔORU, $\quad \cos\theta=\frac{OU}{r}$ and $\sin\theta=\frac{RU}{r}$

$\therefore\ OU = r\cos\theta \ldots ①$ and $RU = r\sin\theta$

Since $ST = RU$ $\quad \therefore ST = r\sin\theta$

In ΔOST $\quad \tan\frac{\pi}{6}=\frac{ST}{OT}$ $\quad \therefore \frac{1}{\sqrt{3}}=\frac{r\sin\theta}{OT}$

$\therefore\ OT = \sqrt{3}\, r\sin\theta \ldots ②$

Now using ① & ② $\rightarrow UT = OU - OT$

$\therefore UT = r\cos\theta - \sqrt{3}\, r\sin\theta$.

(c) (ii) (2 marks)

Area of rectangle $RSTU = ST \times TU$

$\therefore A = r\sin\theta\,(r\cos\theta - \sqrt{3}\, r\sin\theta) = r^2(\sin\theta\cos\theta - \sqrt{3}\sin^2\theta)$

(c) (iii) (3 marks)

$\frac{dA}{d\theta}=r^2\left[-\sin^2\theta+\cos^2\theta-2\sqrt{3}\sin\theta\cos\theta\right]$

and for maximum area $\frac{dA}{d\theta}=0$

$\therefore\left[-\sin^2\theta+\cos^2\theta-2\sqrt{3}\sin\theta\cos\theta\right]=0$

$\therefore \sin^2\theta+2\sqrt{3}\sin\theta\cos\theta-\cos^2\theta=0$ $\qquad [\div \text{ by } \cos^2\theta]$

$\therefore \tan^2\theta+2\sqrt{3}\tan\theta-1=0$

$\therefore \tan\theta=\frac{-2\sqrt{3}\pm\sqrt{12+4}}{2}=\frac{-2\sqrt{3}\pm 4}{2}=-\sqrt{3}+2$ $\qquad (0<\theta<\frac{\pi}{2})$

$\therefore \theta=15^\circ=\frac{\pi}{12}$