SOUTH SYDNEY HIGH SCHOOL

Number: ....

2013

Year 12

TRIAL HSC

Monday 5th August 2013

(ALSO NSW INDEPENDENT SCHOOLS PAPER)

TRIAL HSC

Mathematics Ext 1

Weighting 40%

Working time: 3 hours

Total marks: 70

Outcomes:

PE1 – E6
HE1 – HE 7

Topics examined:

All topics

Question	Mark
1 -10	/10
11	/15
12	/15
13	/15
14	/15
TOTAL	

General Instructions:

- Write using blue or black pen
- Board-approved calculators and templates may be used
- All necessary working should be shown in every question
- Questions are of equal value
- Full marks may not be awarded for careless or badly arranged work
- Questions are not necessarily arranged in order of difficulty
- Allow 15 minutes for section 1
- Allow 2 hours 45 minutes for section 2

Student name / number ....

Marks

Section 1

10 marks
Attempt Questions 1-10
Allow 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

1 What is the value of $\lim_{x\to 0}\left(\dfrac{\sin\frac{1}{3}x}{2x}\right)$? 1

(A) $\frac{1}{6}$

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) 6

2 Which of the following is an expression for $\dfrac{d}{dx}\left(2^x\right)$? 1

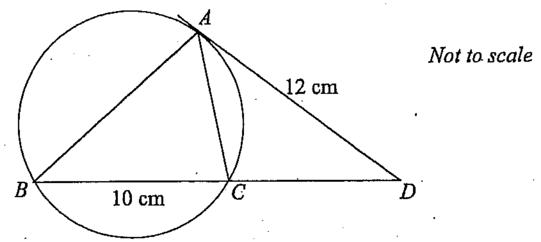
(A) $x2^{x-1}$

(B) 2^{x-1}

(C) 2^x

(D) $2^x\log_e 2$

3



ABC is a triangle inscribed in a circle. The tangent to the circle at A meets BC produced at D where $BC = 10$ cm and $AD = 12$ cm. What is the length of CD ? 1

(A) 6 cm

(B) 7 cm

(C) 8 cm

(D) 9 cm

Marks

4 The equation $2x^3+x^2-13x+6=0$ has roots α, $\frac{1}{\alpha}$ and β. What is the value of β? **1**

(A) 3

(B) 2

(C) −3

(D) −6

5 Which of the following is an expression for $\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)$? **1**

(A) $\frac{-x^2}{1+x^2}$

(B) $\frac{-1}{1+x^2}$

(C) $\frac{1}{1+x^2}$

(D) $\frac{x^2}{1+x^2}$

6 Which of the following lines is a horizontal asymptote of the curve $y=\frac{e^x-2}{e^x+2}$? **1**

(A) $y=-2$

(B) $y=-1$

(C) $y=0$

(D) $y=2$

7 After t years the number N of individuals in a population is given by $N=400+100e^{-0.1t}$. What is the difference between the initial population size and the limiting population size? **1**

(A) 100

(B) 300

(C) 400

(D) 500

Marks

8 A particle is moving in a straight line. At time t seconds it has velocity v ms^{-1} given by $v=2\sin x$ and acceleration a ms^{-2}. Which of the following is an expression for a? **1**

(A) $\cos 2x$

(B) $2\cos x$

(C) $\sin 2x$

(D) $2\sin 2x$

9 What is the coefficient of x^5 in the expansion of $(1-x)(1+x)^9$? **1**

(A) −126

(B) 0

(C) 126

(D) 252

10 Four fair dice are rolled together. What is the probability that exactly three of the dice show the same score? **1**

(A) $\frac{5}{1296}$

(B) $\frac{5}{324}$

(C) $\frac{5}{54}$

(D) $\frac{5}{36}$

Student name / number ....

Marks

Section 2

60 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

Attempt each question in a SEPARATE writing booklet. Extra writing booklets are available.
In Questions 11-14 your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) The letters of the word NUMBER are arranged at random in a row. Find the probability that consonants occupy both end positions. **2**

(b) $A(-1,4)$ and $B(7,-2)$ are two points. Find the coordinates of the point P that divides the interval AB internally in the ratio 3 : 2. **2**

(c) Find correct to the nearest degree the acute angle θ between the lines $3x-2y=0$ and $x+3y=0$. **2**

(d) Find the exact value of $\int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx$. **3**

(e) Use the substitution $t=\tan\frac{x}{2}$ to show that $\frac{1+\cos x+\sin x}{1-\cos x+\sin x}=\cot\frac{x}{2}$. **3**

(f) $P(2at, at^2)$ is a point on the parabola $x^2=4ay$ with focus $F(0,a)$.

(i) Use differentiation to show that the tangent to the parabola at P has gradient t and equation $tx-y-at^2=0$. **2**

(ii) Show that the shortest distance between the focus and this tangent is $a\sqrt{1+t^2}$. **1**

Student name / number ....

Marks

Question 12 (15 marks) Use a SEPARATE writing booklet.

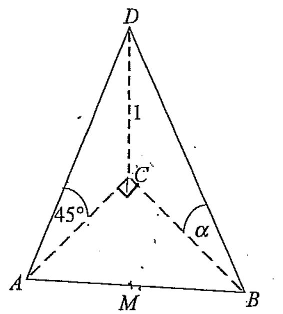
(a) Show that the equation $\cos x-x=0$ has a root between $x=0$ and $x=1$. **2**

(b) Solve the inequality $\frac{2x-1}{x+2}>1$. **3**

(c) Use the substitution $u=6-x$ to find the exact value of $\int_1^6 x\sqrt{6-x}\,dx$. **3**

(d) Use Mathematical Induction to show that $n!>e^n$ for all positive integers $n\geq 6$. **3**

(e)



CD is a vertical pole of height 1 metre that stands with its base C on horizontal ground.
A is a point due South of C such that the angle of elevation of D from A is 45°.
B is a point due East of C such that the angle of elevation of D from B is α.
M is the midpoint of AB.

(i) Show that $AB=\operatorname{cosec}\alpha$. **2**

(ii) Show that $CM=\frac{1}{2}\operatorname{cosec}\alpha$. **2**

Student name / number ....

Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) P is a fixed point on a circle with centre O and radius 10 cm. Q is a variable point which moves on the circle such that $\angle POQ = \theta$ radians. The length L cm of the chord PQ is given by $L = 20\sin\frac{\theta}{2}$ and the area A cm^2 of the minor segment cut off by PQ is given by $A = 50(\theta - \sin\theta)$. The length of the chord PQ is increasing at a constant rate of $0 \cdot 01$ cms^{-1}. Find the rate at which the area of the minor segment is changing when $\theta = \frac{2\pi}{3}$. **3**

(b) Consider the function $f(x) = \sin^{-1}(x-1)$.

(i) Find the domain of the function. **1**

(ii) Sketch the graph of the curve $y = f(x)$ showing the endpoints and the x intercept. **2**

(iii) The region in the first quadrant bounded by the curve $y = f(x)$ and the y axis between the lines $y = 0$ and $y = \frac{\pi}{2}$ is rotated through one complete revolution about the y axis. Find in simplest exact form the volume of the solid of revolution. **3**

(c) A particle is performing Simple Harmonic Motion in a straight line. At time t seconds its displacement from a fixed point O on the line is x metres, given by $x = 4\sqrt{2}\cos\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$, its velocity is v ms^{-1} and its acceleration is $\ddot{x}$ ms^{-2}.

(i) Find the amplitude and period of the motion. **2**

(ii) Find the initial position of the particle and determine if it is initially moving towards or away from O. **2**

(iii) Find the distance travelled by the particle in the first 3 seconds of its motion. **2**

Student name / number ....

Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the function $f(x) = 2 - \log_e x$.

(i) Find the equation of the inverse function $f^{-1}(x)$. **1**

(ii) Explain why the x coordinate X of the point of intersection P of the graphs $y = f(x)$ and $y = f^{-1}(x)$ satisfies the equation $e^{2-X} - X = 0$. **2**

(iii) Use two applications of Newton's Method with an initial value of $X = 1 \cdot 5$ to find the value of X correct to two decimal places. **3**

(b) A vertical building of height 60 metres stands on horizontal ground. A particle is projected from a point O at the top of the building with speed $V = 20\sqrt{2}$ ms^{-1} at an angle α above the horizontal. It moves in a vertical plane under gravity where the acceleration due to gravity is $g = 10$ ms^{-2} and hits the ground at a distance 120 metres from the foot of the building. At time t seconds its horizontal and vertical displacements from O are x metres and y metres respectively, given by $x = 20\sqrt{2}\,t\cos\alpha$ and $y = 20\sqrt{2}\,t\sin\alpha - 5t^2$. (Do NOT prove these results.)

(i) Show that $\alpha = \frac{\pi}{4}$ *or* $\alpha = \tan^{-1}\frac{1}{3}$. **2**

(ii) If $\alpha = \tan^{-1}\frac{1}{3}$, find the exact time taken for the particle to hit the ground. **2**

(iii) If $\alpha = \frac{\pi}{4}$, find the exact speed of the particle after 6 seconds. **2**

(c) Use the Binomial expansion of $(1+x)^n$ and integration to show that **3**

$$^nC_0 + \tfrac{1}{3}\,^nC_2 + \tfrac{1}{5}\,^nC_4 + \ldots = \frac{2^n}{n+1} \quad \text{for } n = 1, 2, 3, \ldots .$$

Independent Trial HSC 2013 Mathematics Extension 1 Marking Guidelines

Section I

Questions 1-10 (1 mark each)

Question	Answer	Solution	Outcomes
1.	A	$\lim_{x\to 0}\left(\frac{\sin\frac{1}{3}x}{2x}\right)=\frac{1}{6}\lim_{x\to 0}\left(\frac{\sin\frac{1}{3}x}{\frac{1}{3}x}\right)=\frac{1}{6}\times 1=\frac{1}{6}$	H5
2.	D	$\frac{d}{dx}\left(2^x\right)=\frac{d}{dx}\left(e^{\log_e 2^x}\right)=\frac{d}{dx}\left(e^{x\log_e 2}\right)=e^{x\log_e 2}\log_e 2=2^x\log_e 2$	H5
3.	C	Let $BD=x$ cm. Then $(10+x)x=12^2$ and $x>0$ $\therefore (x+18)(x-8)=0$ $\therefore x=8$ $CD=8$ cm	PE3
4.	C	$\alpha\times\frac{1}{\alpha}\times\beta=-\frac{6}{2}$ $\therefore \beta=-3$	PE3
5.	B	$\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)=\frac{1}{1+\left(\frac{1}{x}\right)^2}\left(-\frac{1}{x^2}\right)=\frac{-1}{1+x^2}$	HE4
6.	B	$\lim_{x\to-\infty}\left(\frac{e^x-2}{e^x+2}\right)=\frac{0-2}{0+2}=-1$. Hence $y=-1$ is an asymptote as $x\to-\infty$.	H5
7.	A	$N=400+100e^{-0.1t}$ $\therefore t=0\Rightarrow N=500$ and $\lim_{t\to\infty}N=400$. Hence *initial population* − *limiting population* $=500-400=100$	HE3
8.	D	$a=v\frac{dv}{dx}=2\sin x\,.\,2\cos x=2\sin 2x$	HE5
9.	B	Term in x^5 in expansion of $(1-x)(1+x)^9$ is $1\,.\,{}^9C_5\,x^5-x\,.\,{}^9C_4\,x^4$. Hence coefficient of x^5 is ${}^9C_5-{}^9C_4=0$	HE3
10.	C	$6\times{}^4C_3\left(\frac{1}{6}\right)^3\frac{5}{6}=\frac{5}{54}$	HE3

Question 11

a. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
• counts arrangements with consonants at both ends	1
• divides by total number of arrangements and simplifies	1

Answer

$$P(\text{consonants at both ends})=\frac{4\times 3\times 4!}{6!}=\frac{2}{5}$$

b. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• finds x	1
• finds y	1

Answer

$A(-1,4)$ $B(7,-2)$

3 : 2

$$P\left(\frac{21+(-2)}{5},\ \frac{(-6)+8}{5}\right) \qquad \therefore P\left(\frac{19}{5},\frac{2}{5}\right)$$

c. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• finds value of $\tan\theta$	1
• calculates θ	1

Answer

Lines have gradients $\frac{3}{2}$ and $-\frac{1}{3}$. $\therefore \tan\theta=\left|\frac{\frac{3}{2}-\left(-\frac{1}{3}\right)}{1+\frac{3}{2}\times\left(-\frac{1}{3}\right)}\right|=\frac{11}{3}$ $\therefore \theta\approx 75°$ (to the nearest degree)

d. Outcomes assessed : HE4

Marking Guidelines

Criteria	Marks
• finds primitive function	1
• substitutes limits and evaluates one inverse sine expression in terms of π	1
• completes evaluation in terms of π	1

Answer

$$\int_{\sqrt{2}}^{\sqrt{3}}\frac{1}{\sqrt{4-x^2}}\,dx=\left[\sin^{-1}\frac{x}{2}\right]_{\sqrt{2}}^{\sqrt{3}}=\sin^{-1}\frac{\sqrt{3}}{2}-\sin^{-1}\frac{1}{\sqrt{2}}=\frac{\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$$

Q11 (cont)

e. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• writes expressions for $\cos x$ and $\sin x$ in terms of t	1
• simplifies expressions $1 \pm \cos x + \sin x$	1
• simplifies quotient expressed in terms of t to complete proof	1

Answer

$t = \tan\frac{x}{2} \Rightarrow$

$$1+\cos x+\sin x = 1+\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2} = \frac{(1+t^2)+(1-t^2)+2t}{1+t^2} = \frac{2(1+t)}{1+t^2}$$

$$1-\cos x+\sin x = 1-\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2} = \frac{(1+t^2)-(1-t^2)+2t}{1+t^2} = \frac{2t(1+t)}{1+t^2}$$

Hence $\dfrac{1+\cos x+\sin x}{1-\cos x+\sin x} = \dfrac{1}{t} = \cot\frac{x}{2}$

f. Outcomes assessed : PE4, H5

Marking Guidelines

Criteria	Marks
i • uses differentiation to show tangent has gradient t	1
• finds equation of tangent	1
ii • writes expression for perpendicular distance from F to the tangent and simplifies	1

Answer

i.

$$y = at^2 \Rightarrow \frac{dy}{dt} = 2at$$

$$x = 2at \Rightarrow \frac{dx}{dt} = 2a$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = t$$

Hence tangent at $P(2at, at^2)$ has gradient t

and equation $y - at^2 = t(x-2at)$

$tx - y - at^2 = 0$.

ii. $\perp$ distance from $F(0,a)$ to line $tx - y - at^2 = 0$ is $d = \dfrac{|0-a-at^2|}{\sqrt{t^2+(-1)^2}} = \dfrac{a(1+t^2)}{\sqrt{1+t^2}} = a\sqrt{1+t^2}$

Question 12

a. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
• notes that $f(x) = \cos x - x$ is continuous	1
• shows that $f(0)$ and $f(1)$ have opposite signs	1

Answer

$f(x) = \cos x - x$ is a continuous function such that $f(0) = 1 - 0 = 1 > 0$ and $f(1) = \cos 1 - 1 < 0$.
Hence $\cos x - x = 0$ for some x such that $0 < x < 1$.

b. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
• multiplies by $(x+2)^2$, or multiplies by $(x+2)$ and considers cases $x+2>0$, $x+2<0$	1
• finds one of the two inequalities describing possible values of x	1
• finds the second inequality and combines the two inequalities correctly	1

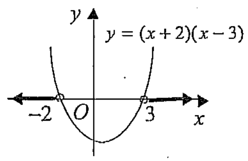
Answer

$$\frac{2x-1}{x+2} > 1$$

$$(2x-1)(x+2) > (x+2)^2 \quad \text{and} \quad x \neq -2$$

$$(x+2)\{(2x-1)-(x+2)\} > 0$$

$$(x+2)(x-3) > 0$$



$\therefore x < -2$ or $x > 3$

c. Outcomes assessed : HE6

Marking Guidelines

Criteria	Marks
• applies substitution to convert definite integral	1
• finds primitive in terms of u	1
• evaluates in surd form	1

Answer

$u = 6 - x$

$du = -dx$

$x = 1 \Rightarrow u = 5$

$x = 6 \Rightarrow u = 0$

$$\int_1^6 x\sqrt{6-x}\,dx = -\int_5^0 (6-u)\sqrt{u}\,du$$

$$= \int_0^5 \left(6u^{\frac{1}{2}} - u^{\frac{3}{2}}\right) du$$

$$= \left[4u^{\frac{3}{2}} - \tfrac{2}{5}u^{\frac{5}{2}}\right]_0^5$$

$$= 4\times 5\sqrt{5} - \tfrac{2}{5}\times 5^2\sqrt{5}$$

$$= 10\sqrt{5}$$

Q12 (cont)

d. Outcomes assessed : HE2

Marking Guidelines

Criteria	Marks
• defines an appropriate sequence of statements and verifies that the first is true	1
• shows that if $S(k)$ is true, then $(k+1)!>(k+1)e^k$	1
• deduces that since $k\geq 6$, $S(k)$ true implies $S(k+1)$ true, and completes the induction process	1

Answer

Let $S(n)$, $n=6,7,8,\ldots$ be the sequence of statements defined by $S(n):\ n!>e^n$

Consider $S(6)$: $6!=720>e^6\approx 403\cdot 4$ Hence $S(6)$ is true.

If $S(k)$ is true: $k!>e^k$ **

Consider $S(k+1)$: $(k+1)!=(k+1)\,k!$

$>(k+1)e^k$ *if $S(k)$ is true using ***

$>e\,.\,e^k$ *for $k\geq 6$*

$=e^{k+1}$

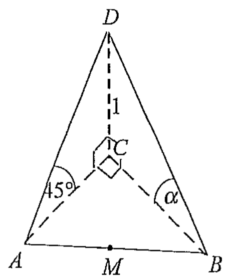
Hence if $S(k)$ is true, then $S(k+1)$ is true. But $S(6)$ is true, hence $S(7)$ is true and then $S(8)$ is true and so on. Therefore by Mathematical Induction, $n!>e^n$ for all integers $n\geq 6$.

e. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
i • writes down the length of AC and writes an expression for BC in terms of α	1
• uses Pythagoras' theorem in ΔABC to find AB in terms of α	1
ii • deduces that A, B, C lie on a circle with centre M	1
• uses equal radii to deduce result	1

Answer



i. In ΔABC : $AC=1$, $BC=\cot\alpha$ and $\angle ACB=90^\circ$

$\therefore AB^2=1+\cot^2\alpha=\operatorname{cosec}^2\alpha$

$AB=\operatorname{cosec}\alpha$

ii. A unique circle can be drawn through A, B and C. Since $\angle ACB=90^\circ$, AB is a diameter of this circle and hence M is its centre and CM, BM are radii.

$\therefore CM=BM=\frac{1}{2}\operatorname{cosec}\alpha$

Question 13

a. Outcomes assessed : HE5

Marking Guidelines

Criteria	Marks
• finds $\frac{d\theta}{dt}$ in terms of θ using value of $\frac{dL}{dt}$	1
• finds $\frac{dA}{dt}$ in terms of θ	1
• evaluates rate of change of A when $\theta=\frac{2\pi}{3}$ and notes that A is increasing	1

Answer

$$L=20\sin\tfrac{\theta}{2}$$

$$\frac{dL}{dt}=10\cos\tfrac{\theta}{2}\,\frac{d\theta}{dt}$$

$$\therefore \frac{d\theta}{dt}=\frac{1}{10\cos\frac{\theta}{2}}\times 0\cdot 01$$

$$A=50(\theta-\sin\theta)$$

$$\frac{dA}{dt}=50(1-\cos\theta)\frac{d\theta}{dt}$$

$$=50(1-\cos\theta)\times\frac{0\cdot 01}{10\cos\frac{\theta}{2}}$$

$\therefore$ when $\theta=\frac{2\pi}{3}$, $\frac{dA}{dt}=50\times 1\cdot 5\times\frac{0\cdot 01}{5}=0\cdot 15$ $\therefore A$ is increasing at a rate $0\cdot 15$ cm^{2}s^{-1}

b. Outcomes assessed : HE4, H8

Marking Guidelines

Criteria	Marks
i • states domain	1
ii • sketches curve with correct shape and x-intercept	1
• shows endpoints of curve	1
iii • writes definite integral for V	1
• expands and finds primitive function	1
• evaluates by substitution of limits	1

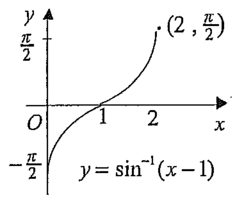
Answer

$f(x)=\sin^{-1}(x-1)$

i. Domain : $-1\leq x-1\leq 1$

$\{x: 0\leq x\leq 2\}$

ii.



iii. $V=\pi\int_0^{\frac{\pi}{2}}(1+\sin y)^2\ dy$

$=\pi\int_0^{\frac{\pi}{2}}\left\{1+2\sin y+\frac{1}{2}(1-\cos 2y)\right\}\ dy$

$=\pi\left[\frac{3}{2}y-2\cos y-\frac{1}{4}\sin 2y\right]_0^{\frac{\pi}{2}}$

$=\pi(\frac{3\pi}{4}+2)$

Volume is $\pi(\frac{3\pi}{4}+2)$ cubic units.

Q13 (cont)

c. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • writes down the amplitude	1
• finds the period	1
ii • finds the initial position	1
• determines the initial direction of travel	1
iii • finds the position of the particle when $t=3$	1
• determines the distance travelled in the first three seconds.	1

Answer

$x=4\sqrt{2}\cos\left(\frac{\pi}{4}t-\frac{\pi}{4}\right),\quad \dot{x}=-\pi\sqrt{2}\sin\left(\frac{\pi}{4}t-\frac{\pi}{4}\right)$

i. Amplitude $4\sqrt{2}$ m, period $2\pi\div\frac{\pi}{4}=8$ s

ii. $t=0\Rightarrow x=4$, $v=\pi>0$. Particle is initially 4m to the right of O and moving away from O.

iii. Particle first reaches its positive extreme at $x=4\sqrt{2}$ when $t=1$. In the next 2 seconds ($\frac{1}{4}$ period) the particle travels from this extreme back to O. Hence the distance travelled in the first three seconds is $\left(4\sqrt{2}-4\right)+4\sqrt{2}=8\sqrt{2}-4$ metres.

Question 14

a. Outcomes assessed : HE4, PE3

Marking Guidelines

Criteria	Marks
i • finds the inverse function	1
ii • explains that any intersection point of the graphs of inverse functions must lie on $y=x$	1
• deduces the required equation for X	1
iii • writes an expression for the next approximation using Newton's Method	1
• calculates the next approximation to 2 decimal places.	1
• uses this value in a second application of Newton's method	1

Answer

i.
$$f(x)=2-\log_e x$$
$$y=2-\log_e x$$
$$\log_e x=2-y$$
$$x=e^{2-y}$$
$$f^{-1}(x)=e^{2-x}$$

ii. The graphs of $f(x)$ and $f^{-1}(x)$ are reflections in the line $y=x$, hence any intersection points of these graphs must lie on the line $y=x$. If the graphs $y=f(x)$ and $y=f^{-1}(x)$ intersect at P where $x=X$, then $f(X)=f^{-1}(X)=X$. $\therefore\ e^{2-X}=X$ giving $e^{2-X}-X=0$.

iii. Let $g(x)=e^{2-x}-x$. Then $g'(x)=-e^{2-x}-1$ and $x-\dfrac{g(x)}{g'(x)}=\dfrac{x(e^{2-x}+1)+e^{2-x}-x}{e^{2-x}+1}=\dfrac{x+1}{1+e^{x-2}}$

$$X=1\cdot5\ \Rightarrow\ X-\frac{g(X)}{g'(X)}=\frac{2\cdot5}{1+e^{-0\cdot5}}\approx1\cdot56$$
$$X=1\cdot56\Rightarrow X-\frac{g(X)}{g'(X)}=\frac{2\cdot56}{1+e^{-0\cdot44}}\approx1\cdot56$$

Hence $X\approx1\cdot56$ correct to 2 dec. places.

Q14 (cont)

b. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • writes equation for $\tan\alpha$	1
• solves for $\tan\alpha$ to deduce required values of angle of projection	1
ii • finds exact value of $\cos\alpha$ given $\tan\alpha$	1
• uses this value to find exact time to hit the ground	1
iii • finds expressions for horizontal and vertical components of velocity	1
• uses Pythagoras' theorem to find the speed	1

Answer

i. $x=20\sqrt{2}\,t\cos\alpha$, $\quad y=20\sqrt{2}\,t\sin\alpha-5t^2$

Ground is 60m below O.

When $x=120$, $y=-60$.

$$20\sqrt{2}\sin\alpha\left(\frac{120}{20\sqrt{2}\cos\alpha}\right)-5\left(\frac{120}{20\sqrt{2}\cos\alpha}\right)^2=-60$$
$$120\tan\alpha-5\times6\times3\sec^2\alpha=-60$$
$$4\tan\alpha-3(1+\tan^2\alpha)=-2$$
$$3\tan^2\alpha-4\tan\alpha+1=0$$
$$(\tan\alpha-1)(3\tan\alpha-1)=0$$
$$\therefore\ \tan\alpha=1\quad\text{or}\quad\tan\alpha=\tfrac{1}{3}$$
$$\alpha=\tfrac{\pi}{4}\qquad\qquad\alpha=\tan^{-1}\tfrac{1}{3}$$

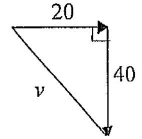
ii. $\tan\alpha=\frac{1}{3}\Rightarrow\sec^2\alpha=\frac{10}{9}$, $\cos\alpha=\frac{3}{\sqrt{10}}$

Particle hits ground when

$$t=\frac{120}{20\sqrt{2}\cos\alpha}=\frac{6}{\sqrt{2}}\cdot\frac{\sqrt{10}}{3}=2\sqrt{5}\ \text{s}$$

iii. $\alpha=\frac{\pi}{4}\ \Rightarrow\ \dot{x}=20$ and $\dot{y}=20-10t$

Then $t=6\Rightarrow\dot{x}=20$ and $\dot{y}=-40$



$$v=\sqrt{20^2+40^2}=20\sqrt{5}\ \text{ms}^{-1}$$

c. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
• writes the binomial expansion and finds the primitive function of both sides	1
• substitutes the limits of an appropriate definite integral or evaluates the constant of integration	1
• simplifies value of definite integral or substitutes appropriate x values into primitive	1

Answer

$$(1+x)^n\equiv{}^nC_0+{}^nC_1x+{}^nC_2x^2+\ldots+{}^nC_nx^n$$
$$\int_{-1}^{1}(1+x)^n\,dx=\int_{-1}^{1}\left\{{}^nC_0+{}^nC_1x+{}^nC_2x^2+\ldots+{}^nC_nx^n\right\}dx$$
$$\frac{1}{n+1}\Big[(1+x)^{n+1}\Big]_{-1}^{1}=\Big[{}^nC_0x+\tfrac{1}{2}{}^nC_1x^2+\tfrac{1}{3}{}^nC_2x^3+\ldots+\tfrac{1}{n+1}{}^nC_nx^{n+1}\Big]_{-1}^{1}$$
$$\frac{2^{n+1}-0}{n+1}={}^nC_0\{1-(-1)\}+\tfrac{1}{2}{}^nC_1\{1^2-(-1)^2\}+\tfrac{1}{3}{}^nC_2\{1^3-(-1)^3\}+\ldots+\tfrac{1}{n+1}{}^nC_n\{1^{n+1}-(-1)^{n+1}\}$$
$$\frac{2^{n+1}}{n+1}={}^nC_0.2+\tfrac{1}{3}{}^nC_2.2+\tfrac{1}{5}{}^nC_4.2+\ldots+\tfrac{1}{n+1}{}^nC_n\{1^{n+1}-(-1)^{n+1}\}$$
$$\therefore\ {}^nC_0+\tfrac{1}{3}{}^nC_2+\tfrac{1}{5}{}^nC_4+\ldots=\frac{2^n}{n+1}$$