Sydney Technical High School

Mathematics Department

TRIAL H.S.C. - MATHEMATICS 2 UNIT

AUGUST 2013

General Instructions

- Reading time – 5 minutes
- Working Time – 180 minutes.
- Approved calculators may be used.
- Write using blue or black pen.
- A table of Standard Integrals is provided at the back of this paper.
- In Question 11-16, show relevant mathematical reasoning and/or calculations.
- Begin each question on a new side of the answer booklet.
- Marks shown are a guide and may need to be adjusted.
- Full marks may not be awarded for careless work or illegible writing.

NAME ____________________

TEACHER ____________________

Total Marks – 100

SECTION 1 **Pages 2 – 5**

10 marks

- **Attempt Questions 1 – 10**
- **Allow about 15 minutes.**

SECTION 2 **Pages 6 – 12**

90 marks

- **Attempt Questions 11 – 16**
- **Allow about 2 hours 45 mins.**

TABLE OF STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}; \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

Note: $\ln x = \log_e x, \quad x > 0$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Question 1

For what values of k does the equation $x^2 - 6x - 3k = 0$ have real roots?

A. $k \geq -3$

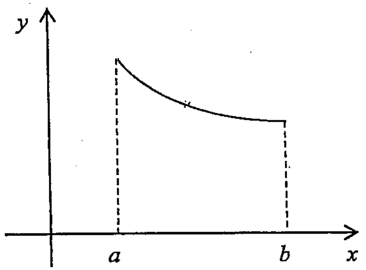
B. $k \leq -3$

C. $k \geq 3$

D. $k \leq 3$

Question 2

For the function $y = f(x)$, $a < x < b$ graphed below:



which of the following is true?

A. $f'(x) > 0$ and $f''(x) > 0$

B. $f'(x) > 0$ and $f''(x) < 0$

C. $f'(x) < 0$ and $f''(x) > 0$

D. $f'(x) < 0$ and $f''(x) < 0$

Question 3

An infinite geometric series has a first term of 8 and a limiting sum of 12.

What is the common ratio?

A. 1/6

B. 5/3

C. 1/2

D. 1/3

Question 4

What are the domain and range of the function $f(x) = \sqrt{4 - x^2}$?

A. Domain: $-2 \leq x \leq 2$, Range: $0 \leq y \leq 2$

B. Domain: $-2 \leq x \leq 2$, Range: $-2 \leq y \leq 2$

C. Domain: $0 \leq x \leq 2$, Range: $-4 \leq y \leq 4$

D. Domain: $0 \leq x \leq 2$, Range: $0 \leq y \leq 4$

Question 5

What is the maximum value of $6 + 2x - x^2$?

A. 6

B. 1

C 7

D. cannot be determined.

Question 6

The sine curve with amplitude 3 units and period 4π units has equation:

A. $y = 4 \sin 3x$

B. $y = 3 \sin 4x$

C. $y = 3 \sin 2x$

D. $y = 3 \sin \frac{x}{2}$

Question 7

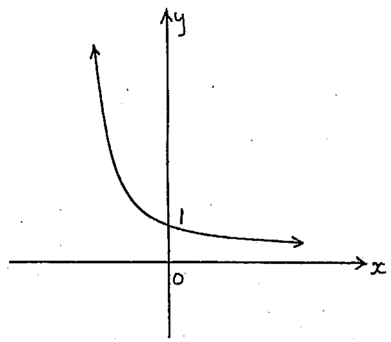
The illustrated graph could be:

A. $y = 2^x$

B. $y = 2^{-x}$

C. $y = (\frac{1}{2})^x$

D. $y = (\frac{1}{2})^{-x}$



Question 8

Janet works out the sum of n terms of an arithmetic series. Her answer, which is correct, could be:

A. $S_n = 2(2^n - 1)$

B. $S_n = 9 - 2n$

C. $S_n = 8n - n^2$

D. $S_n = 7 \times 2^{n-1}$

Question 9

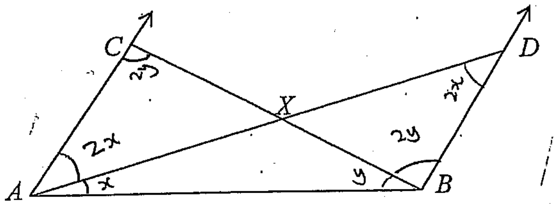


Figure not to scale

In the diagram above: $AC \parallel BD$, $\angle CAX = 2\angle BAX$, $\angle DBX = 2\angle ABX$.

$\angle AXB = ?$

A. 150°

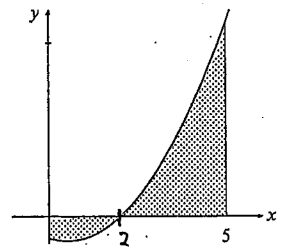
B. 120°

C. 160°

D. 135°

Question 10

Which expression below will give the area of the shaded region bounded by the curve $y = x^2 - x - 2$, the x-axis and the lines $x = 0$ and $x = 5$?



A. $A = \left|\int_0^1 (x^2 - x - 2)dx\right| + \int_1^5 (x^2 - x - 2)dx$

B. $A = \int_0^1 (x^2 - x - 2)dx + \left|\int_1^5 (x^2 - x - 2)dx\right|$

C. $A = \left|\int_0^2 (x^2 - x - 2)dx\right| + \int_2^5 (x^2 - x - 2)dx$

D. $A = \int_0^2 (x^2 - x - 2)dx + \left|\int_2^5 (x^2 - x - 2)dx\right|$

END OF SECTION 1

SECTION 2

90 marks

Attempt Question 11 – 16

Allow about 2 hours 45 minutes for this section.

Answer each question in the writing book provided. Start each question on a new page.
All necessary working should be shown. Full marks cannot be given for illegible writing.

Question 11 (15 marks)

			Marks
a)	Differentiate:		
	(i)	$x \sin 2x$	2
	(ii)	$e^{4x} + \frac{1}{x}$	2
	(iii)	$\frac{x+1}{3+2x}$	2
b)	Find $\int (4x+2)^6\, dx$		2
c)	Solve for x: $3^{1-x} = \frac{1}{\sqrt{27}}$		2
d)	Solve $(\sin x + 1)(2\sin x + 1) = 0$ for $0 \le x \le 2\pi$		3
e)	Evaluate $\sum_{n=1}^{50}(2n+3)$		2

Question 12 (15 marks)

			Marks
a)	Solve $\lvert x+2 \rvert = 3x$		2
b)	Use a change of base to evaluate $\log_2 50$ correct to 2 decimal places.		1
c)	Find the gradient of the curve $y = e^{\sin x}$ at the point where $x = 0$.		2
d)	If α and β are the roots of $x^2 + 4x + 1 = 0$, find without solving:		
	i)	$\alpha + \beta$ and $\alpha\beta$.	1
	ii)	$\frac{1}{\alpha^2} + \frac{1}{\beta^2}$	2
e)	Differentiate:		
	i)	$\ln(x^2+3)$	1
	ii)	$tan^2 4x$	2
f)	Given the parabola $4y = x^2 - 12$, find the:		
	i)	focal length.	1
	ii)	coordinates of the focus.	2
g)	Use Simpson's Rule and the five function values in the table below to estimate $\int_2^4 f(x)dx$.		2

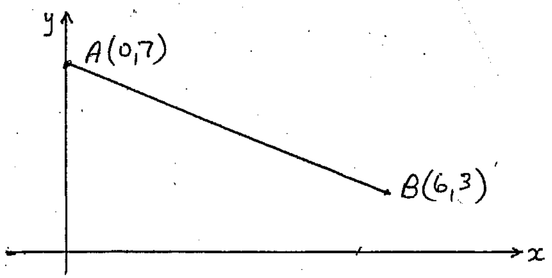
x	2	2.5	3	3.5	4
$f(x)$	4	1	−2	3	8

Question 13 (15 marks) Marks

a) i) Factorise $24 + 2m - m^2$ 1

ii) Hence solve $24 + 2m - m^2 < 0$ 1

b)



A(0,7) and B(6,3) are points on the number plane and the equation of AB is $2x + 3y - 21 = 0$.

i) Find the length of AB. 1

ii) Find the gradient of AB. 1

iii) Show that the equation of the perpendicular from D(-2,0) to AB is $3x - 2y + 6 = 0$. 2

iv) Find the perpendicular distance from D to AB. 2

v) Find the coordinates of a point C such that ABCD is a parallelogram. 1

c) An amount of money doubles in value over a period of n months. Interest is compounded at the rate of 1% per month. Use the compound interest formula to find the number of months required, correct to the nearest month. 2

d) i) Find $\frac{d}{dx}(\operatorname{cosec} x)$ 2

ii) Hence evaluate $\int_{\pi/3}^{\pi/2} \cot x \operatorname{cosec} x \, dx$. Give your answer in exact form. 2

Question 14 (15 marks) Marks

a) Find the angle that the line $3x + 5y + 2 = 0$ makes with the positive direction of the x-axis. 2

b) Find: i) $\int \sin\frac{2x}{3} \, dx$ 2

ii) $\int \frac{x^2 e^{x^2} + 1}{x} \, dx$ 2

c) Prove that $\frac{\cos\theta}{1+\sin\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$ 2

d) Solve for m: $\log_m 8 + 3\log_m 4 = 6$. Leave your answer in exact form. 3

e)

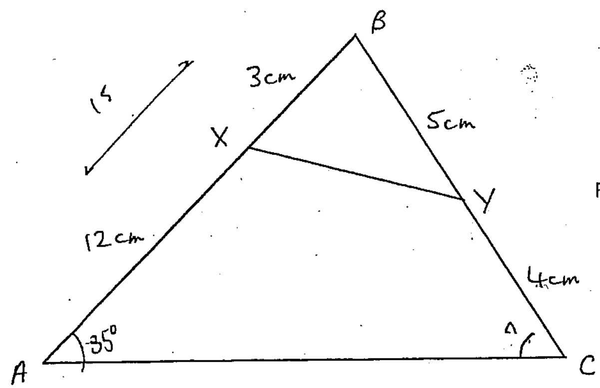


Figure not to scale

i) Prove that ΔBXY is similar to ΔABC. 2

ii) If angle A is 35°, use the Sine Rule to find the size of angle C, correct to the nearest degree. 2

Question 15 (15 marks) Marks

a)

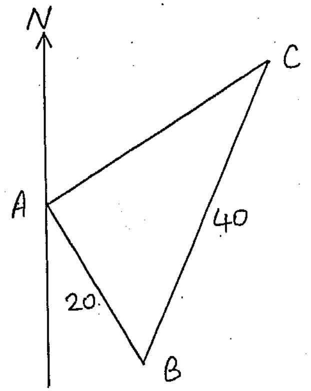


Figure not to scale

Two geologists on a large level area of land drive 20 km from point A on a bearing of 150°T to a point B. They then drive 40 km on a bearing of 020°T to point C.

i) Copy the above diagram into your answer booklet, and find the size of $\angle ABC$. 1

ii) Use the Cosine Rule to find the distance AC to the nearest kilometre. 2

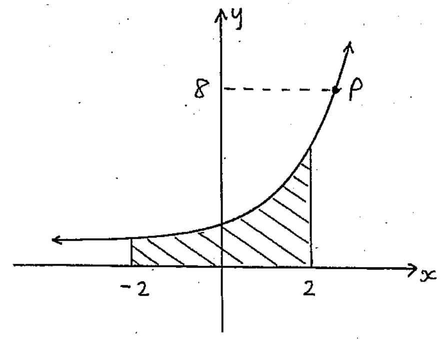
b) Consider the curve defined by $y = 4 - \cos 2x$.

i) State the amplitude <u>and</u> period of this curve. 2

ii) Sketch the curve for $0 \le x \le \pi$. Show clear, relevant information on the axes. 2

iii) Find the area between the curve and the line $y = 2$ for $0 \le x \le \pi$. 3

c) The diagram shows the curve $y = e^x$, a shaded area from $x = -2$ to $x = 2$, and a point P on the curve.



Not to scale

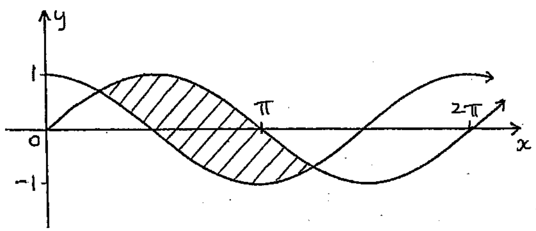
i) The point P has a y coordinate of 8. Find its x coordinate. 1

ii) The shaded area is rotated about the x-axis. Find the volume of the generated solid, giving your answer correct to <u>3 significant figures</u>. 3

d) Factorise $x^2 + 2xy + y^2 - 1$ 1

Question 16 (15 marks)

a)



The diagram shows the curves $y = \sin x$ and $y = \cos x$.

Write an appropriate integral expression to represent the shaded area above. 2

DO NOT EVALUATE THIS INTEGRAL.

b) Given the curve $y = x\log x - x$, for $x > 0$.

i) Find where the curve crosses the x-axis. 2

ii) Find any stationary points and determine their nature. 2

iii) Write a statement for the concavity of this curve. 1

iv) Find y when $x = e^2$, and sketch the curve for $0 < x \leq e^2$ 2

c) A man has 1 million (10^6) dollars in a bank account. The account earns a steady $\frac{1}{2}$% interest per month, compounded monthly.

At the same time, however, a bank employee is stealing a constant amount \$M per month from this account, immediately after the month's interest is added to the man's account.

Let A_n be the amount remaining in the man's account at the end of n months.

i) Write an expression for A_1, and show that 2

$A_2 = 10^6(1.005)^2 - M(1.005 + 1)$

ii) Write a simplified expression for A_n 2

iii) Determine the value of \$M that is stolen each month, such that the man will have only \$20 remaining in his account after 10 years. 2

END OF PAPER

S.T.H.S – 2U TRIAL 2013 – SOLNS

Section I **Multiple Choice** **10 Marks**

Attempt Question 1 – 10 (1 mark each)
Allow approximately 15 minutes for this section.

Use the multiple choice answer sheet below to record your answers to Question 1 – 10.

Select the alternative: A, B, C or D that best answers the question.

Colour in the response oval completely.

Sample:

2 + 4 = ? (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, draw a cross through the incorrect answer and colour in the new answer

ie A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word "*correct*" and draw an arrow as follows:

correct

A ● B ⊗ C ○ D ○

Start here →

1. A ● B ○ C ○ D ○ ✓
2. A ○ B ○ C ● D ○ ✓
3. A ○ B ○ C ○ D ● ✓
4. A ● B ○ C ○ D ○ ✓
5. A ○ B ○ C ● D ○ ✓
6. A ○ B ○ C ○ D ● ✓
7. A ○ B ● C ○ D ○ ✓
8. A ○ B ● C ○ D ○ × C
9. A ○ B ● C ○ D ○ ✓
10. A ○ B ○ C ● D ○ ✓

9

Question 11

a) i) $\frac{d}{dx}(x\sin 2x) = \sin 2x + 2x\cos 2x$ ✓ 2

$u = x$ $v = \sin 2x$

$u' = 1$ $v' = 2\cos 2x$

$\frac{15}{15}$

ii) $\frac{d}{dx}\left(e^{4x} + \frac{1}{x}\right) = \frac{d}{dx}\left(e^{4x} + x^{-1}\right)$

$= 4e^{4x} - x^{-2}$

$= 4e^{4x} - \frac{1}{x^2}$ ✓ 2

iii) $\frac{d}{dx}\left(\frac{x+1}{3+2x}\right) = \frac{3+2x-(x+1)(2)}{(3+2x)^2}$

$u = x+1$ $v = 3+2x$

$u' = 1$ $v' = 2$

$= \frac{3+2x-2x-2}{(3+2x)^2}$

$= \frac{1}{(3+2x)^2}$ ✓ 2

b) $\int (4x+2)^6 = \frac{(4x+2)^7}{7\times 4} + c$

$= \frac{(4x+2)^7}{28} + c$ ✓ 2

c) $3^{1-x} = \frac{1}{\sqrt{27}}$

$3^{1-x} = 27^{-\frac{1}{2}}$

~~$3^{1-x} = 3^{\frac{3}{2}}$~~

$3^{1-x} = 3^{-\frac{3}{2}}$

$1 - x = -\frac{3}{2}$

$\therefore x = 1 + \frac{3}{2}$ ✓ 2

$\therefore x = \frac{5}{2}$

d) $(\sin x+1)(2\sin x+1)=0$

$\sin x+1=0$

$\sin x=-1$

$x=\frac{3\pi}{2}$

$2\sin x+1=0$

$\sin x=-\frac{1}{2}$

$x=\frac{7\pi}{6}, \frac{11\pi}{6}$

3

$\therefore x=\frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ for $0\le x\le 2\pi$

e) $\sum_{n=1}^{50}(2n+3)$

$T_1=2(1)+3$
$=5$

$T_2=2(2)+3$
$=7$

$T_3=2(3)+3$
$=9$

$S_{50}=\frac{50}{2}\left(2(5)+(50-1)2\right)$

$=25(10+98)$

$S_{50}=2700$

2

Question 12 $\frac{11}{15}$

a) $|x+2|=3x$

$x+2=3x$
$2x=2$
$x=1$
Check:
$|1+2|=3$
$3=3$ ✓

$x+2=-3x$
$-4x=2$
$x=-\frac{1}{2}$
Check:
$|-\frac{1}{2}+2|=3(-\frac{1}{2})$
$\frac{3}{2}\ne-\frac{3}{2}$ X

$\therefore x=1$ only ✓ 2

b) $\log_2 50=\frac{\log 50}{\log 2}$

$=5.64$ ✓ 1

c) $y=\cancel{esi}\ e^{\sin x}$

$y'=\cos x\, e^{\sin x}$ ✓ sub. $x=0$

$y'=\cos(0)e^{\sin(0)}$

$=1\times e^0$

$=1$

$\therefore$ Gradient $=1$ ✓ 2

d) i) $\alpha+\beta=-\frac{b}{a}$

$=\frac{-4}{1}$ ✓

$=-4$

$\alpha\beta=\frac{c}{a}$ 1

$=\frac{1}{1}$ ✓

$=1$

ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\beta^2+\alpha^2}{\alpha^2\beta^2}$

$(\alpha+\beta)^2 = \alpha^2+2\alpha\beta+\beta^2$

$\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$

$= (-4)^2-2(1)$

$= 14$

$\alpha^2\beta^2 = (\alpha\beta)^2$

$= 1^2$

2

$\therefore \frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{14}{1}$

$= 14$

e) i) $\frac{d}{dx}\left(\ln(x^2+3)\right) = \frac{2x}{x^2+3}$

ii) $\frac{d}{dx}(\tan^2 4x) = \cancel{\frac{d}{dx}(\sec^2 4x - 1)} = \cancel{\frac{d}{dx}(\tan 16x^2)}$

$= 2\tan 4x \times \sec^2 4x \times 4$ $\cancel{= 4\tan 4x}$ $\cancel{= 16\sec^2 16x^2 + C}$

$= 8\tan 4x \sec^2 4x$

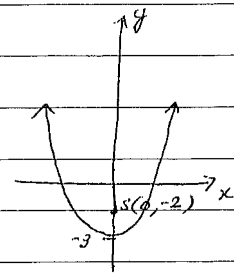
Mathematics 2 Unit Trial HSC 2013

f) i) $4y = x^2-12$

$x^2 = 4y+12$

$x^2 = 4(y+3) \Rightarrow 4a = 4,\ a = 1$

$\therefore$ Focal length is $\cancel{2}$ 1 units.



ii) Focus $(0,-a)$

$4a = 4$

$a = 1$ Focal length!

$\therefore$ Focus at $(0,-2)$

g) $A_{\text{simpson}} = \frac{0.5}{3}\left(4+8+4(1+3)+2(-2)\right)$

$= \frac{1}{6}(12+16-4)$

$= 4 \text{ units}^2$ 2

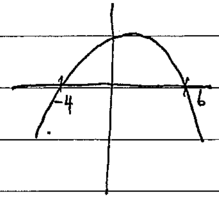
Question 13

a) i) $24 + 2m - m^2$

$(6 - m)(4 + m)$ ✓ 1

$6 \; -m$
$4 \; m$

ii) $m = 6, -4$



$\therefore m < -4, \; m > 6$ // ✓ 1

b) i) $D_{AB} = \sqrt{(7-3)^2 + (0-6)^2}$

$= \sqrt{16 + 36}$

$= 2\sqrt{13}$ units ✓ 1

ii) $M_{AB} = \dfrac{7-3}{0-6}$

$= \dfrac{4}{-6}$

$= -\dfrac{2}{3}$ // ✓ 1

iii) $M_D \times -\frac{2}{3} = -1$ ✓

$M_D = \frac{3}{2}$

$y - 0 = \frac{3}{2}(x+2)$ ✓

$2y = 3x + 6$

$\therefore 3x - 2y + 6 = 0$ // 2

Mathematics 2 Unit Trial HSC 2013

iv) Perp. dist. $= \left| \dfrac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$ $\quad 2x + 3y - 21 = 0$ $\quad (-2, 0)$

$= \left| \dfrac{(2\times -2) + (3\times 0) - 21}{\sqrt{2^2+3^2}} \right|$

$= \left| \dfrac{-25}{\sqrt{13}} \right|$

$= \dfrac{25\sqrt{13}}{13}$ units // ✓ 2

v) $\cancel{C}$ $C(4, -4)$ // ✓ 1

11

c) $2P = P(1.01)^n$

$2 = 1.01^n$

$\log 2 = n \log 1.01$

$n = \dfrac{\log 2}{\log 1.01}$

$= 69.66$ months ✓ 2

$\therefore$ 70 months, correct to the nearest month

d) i) $\frac{d}{dx}(\operatorname{cosec}x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right)$ $= \frac{d}{dx}(\sin x)^{-1}$

$\cancel{= \frac{1}{\cos x} \times \ln(\sin x)}$ $= -1(\sin x)^{-2} \times \cos x$

$=$ ✗ $\cancel{0}$ $= \frac{-\cos x}{\sin x \cdot \sin x}$

$= -\cot x \operatorname{cosec} x$

ii) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot x \operatorname{cosec} x \, dx =$ $\cancel{0}$

$= -\left[\operatorname{cosec} x\right]_{\pi/3}^{\pi/2}$

$= -\left[\frac{1}{\sin \pi/2} - \frac{1}{\sin \pi/3}\right]$

$= -\left[1 - \frac{2\sqrt{3}}{3}\right]$

$= \frac{2\sqrt{3}}{3} - 1$

Question 14

$\frac{12}{15}$

a) $3x + 5y + 2 = 0$

$5y = -3x - 2$

$y = -\frac{3}{5}x - \frac{2}{5}$

Gradient $= -\frac{3}{5}$ ✓ 2

~~Acute angle = tan θ = 3/5~~

Obtuse angle $= 180° - \tan^{-1}\left(\frac{3}{5}\right)$

$= 149°2'$ // ✓

b) i) $\int \sin\left(\frac{2x}{3}\right) = -\frac{1}{\left(\frac{2}{3}\right)}\cos\left(\frac{2x}{3}\right) + c$

$= \frac{-3}{2}\cos\left(\frac{2x}{3}\right) + c$ // ✓ 2

ii) $\int \frac{x^2e^{x^2}+1}{x}\,dx = \int \frac{x^2e^{x^2}}{x}\,dx + \int \frac{1}{x}\,dx$

$= \int xe^{x^2} + \ln x + c$ 1

$= \cancel{\frac{x^2 e^{2x}}{2}} + \ln x + c$ //

$\frac{e^{x^2}}{2} + \ln x + c$

c) $\frac{\cos\theta}{1+\sin\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$

LHS $= \frac{\cos\theta}{1+\sin\theta} + \frac{\cos\theta}{1-\sin\theta}$

$= \frac{\cos\theta(1-\sin\theta) + \cos\theta(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$$= \frac{\cos\theta - \cos\theta\sin\theta + \cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta}$$

$$= \frac{2\cos\theta}{1-\sin^2\theta}$$

$$= \frac{2\cancel{\cos\theta}}{\cancel{\cos^2\theta}_{\cos\theta}}$$

$$= \frac{2}{\cos\theta}$$

$= 2\sec\theta$ ✓ 2

$= RHS$

$\therefore LHS = RHS$

$$\frac{\cos\theta}{1+\sin\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta \;//$$

d) $\log_m 8 + 3\log_m 4 = 6$

$\log_m 8 + \log_m 4^3 = 6$

$\log_m (8\times 64) = 6$

$\log_m 512 = 6$ ✓ 2

$m^6 = 512$

$m = 512^{\frac{1}{6}}$ or $\sqrt[6]{512}$ //

e) i) In ΔBXY and ΔABC,

$\angle ABC = \angle XBY$ (common)

~~No other similarities~~ ✗ $\frac{BY}{BA} = \frac{5}{15} = \frac{1}{3}$; $\frac{BX}{BC} = \frac{3}{9} = \frac{1}{3}$

$\therefore$ ~~They are not similar~~ $\therefore \Delta BXY \parallel\!\mid \Delta ABC$ (2 sides in the same ratio + included angle equal)

ii) $\frac{\sin 35}{9} = \frac{\sin C}{15}$

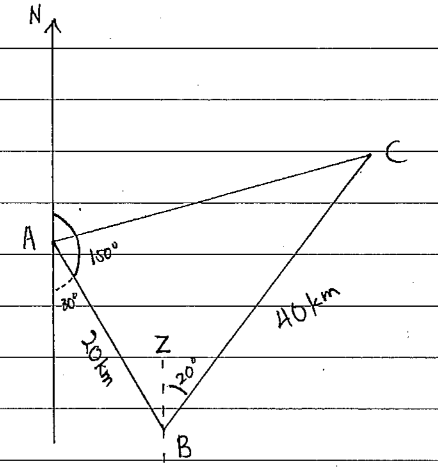
$\sin C = \frac{15\sin 35}{9}$

$C = 73°$ to the nearest degree //

✓ 2

Question 15

a) i)



$\angle ABZ = 30°$ (alternate angles are equal)

$\angle ABC = 30° + 20°$

$= 50°$ //

ii) $AC^2 = 20^2 + 40^2 - 2.20.40.\cos 50$

$AC^2 = 2000 - 1600\cos 50$

$AC^2 = 971.54$

$AC = 31$ km to the nearest km //

b) i) Amplitude $= 1$

Period $= \frac{2\pi}{2}$

$= \pi$

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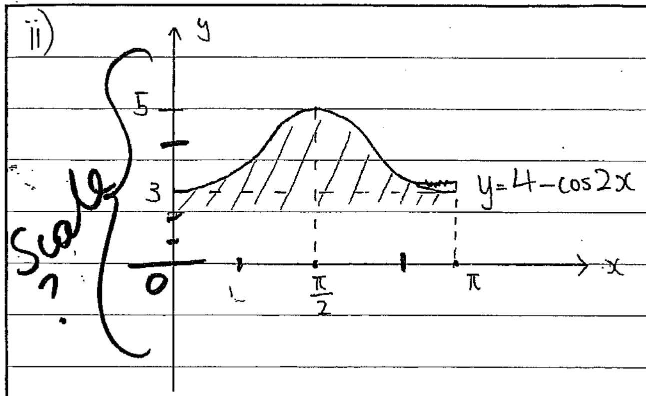
ii)



iii) $\int_0^{\pi} 4 - \cos 2x \, = \left[4x - \frac{1}{2}\sin 2x\right]_0^{\pi}$

$= [(4\pi - 0) - 0]$

$= 4\pi$

$4\pi - 2\pi = 2\pi$ (Area minus rectangle)

$\therefore$ Area is 2π units^2 //

c) i) $8 = e^x$

$x = \log_e 8$

$= \ln 8$ //

ii) $y = e^x$

$\log_e y = x$

$\cancel{V_x = \pi \int_{-2}^{2} (\ln y)^2 \, dy}$

$\cancel{= \pi \left[2\ln y\right]_{-2}^{2}}$

$\cancel{\approx 4.355}$ units^3 //

$V_x = \pi \int_{-2}^{2} (e^x)^2 \, dx$

$= \pi \left[\frac{e^{2x}}{2}\right]_{-2}^{2}$

$= \frac{\pi}{2}\left[e^4 - e^{-4}\right]$

$= 85.7$ units^3 (to 3 s.f)

d) $x^2 + 2xy + y^2 - 1$

$= (x+y)^2 - 1$

$= [(x+y) - 1][(x+y) + 1]$

$= (x+y-1)(x+y+1)$ //

Question 16

a) $\sin x = \cos x$

$\sin x - \cos x = 0$

$\frac{\sin x}{\cos x} - \frac{\cos x}{\cos x} = 0$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

S | A ✓
T ✓ | C

They intersect at $x = \frac{\pi}{4}$ and $\frac{5\pi}{4}$

$\therefore \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin x - \cos x \, dx$

2

b) i) $y = x \log x - x$

$y = x \log_e x - x$

$x(\log_e x - 1) = 0$

$x = 0$ | $\log_e x = 1$, $x = e^1$

$\therefore x = 0, e$

$\therefore$ ~~Crosses at x = 0 and e~~ Crosses at $x = e$ because $x > 0$

2

ii) $y = x \ln x - x$

$\cancel{y' = \frac{1}{x} - 1}$

$y'' = -\frac{1}{x^2}$

Stationary points at $y' = 0$

$\frac{1}{x} - 1 = 0$

$1 - x = 0$

$y' = x \cdot \frac{1}{x} + \ln x - 1$ $\quad D: x > 0$

$= \ln x = 0 \quad \therefore x = e^0 = 1$

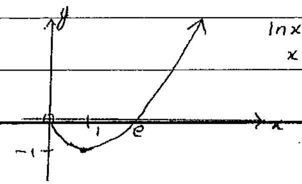
$y = -1$

$y'' = \frac{1}{x}$ for $x = 1$ $y'' = 1 > 0$ ∪

$\therefore$ Min at $(1, -1)$; x-intercept at $y = 0$

$\ln x = 1$

$x = e^1$.



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c) i) $A_1 = 10^6\left(1 + \frac{5}{1000}\right)^1 - M$

$A_2 = \left[10^6\left(1 + \frac{5}{1000}\right)^1 - M\right]1.005^1 - M$

$A_2 = 10^6\left(1 + \frac{5}{1000}\right)^2 - M(1.005^1 + 1)$

$A_2 = 10^6(1.005)^2 - M(1.005^1 + 1)$

2

ii) $A_n = 10^6(1.005)^n - M\underbrace{(1 + 1.005 + 1.005^2 + \cdots + 1.005^{n-1})}_{G.P \quad a=1 \quad r=1.005}$

$A_n = 10^6(1.005)^n - M\left(\frac{1(1.005^n - 1)}{1.005 - 1}\right)$

2

iii) $20 = 10^6(1.005)^{120} - M\left(\frac{1.005^{120} - 1}{0.005}\right)$

$M\left(\frac{1.005^{120} - 1}{0.005}\right) = 1819376.73$

$M = \frac{1819376.73}{163.88}$

$M = 11101.93$

$\therefore$ \$11101.93 is being stolen every month.

2

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