SYDNEY TECHNICAL HIGH SCHOOL

2013

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION

Mathematics

Extension 1

General Instructions

- Reading time – 5 minutes.
- Working Time – 2 hours.
- Write using a blue or black pen.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 11 – 14.
- Begin each question on a new page.
- Write your name and your teachers name on the booklet and your multiple choice answer sheet.

Total marks (70)

Section I

10 marks

- Attempt questions 1 – 10.
- Answer on the multiple choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II

60 marks

- Attempt questions 11 – 14
- Answer in the booklet provided and show all necessary working.
- Start a new page for each question and clearly label it.
- Allow about 1 hour 45 minutes for this section.
- Marks are shown beside each question

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Section 1 Total marks - 10

1. The smallest positive value of x for which $\tan(2x) = 1$ is

A. 0

B. $\frac{\pi}{8}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

2. The inverse of the function $f(x) = e^{2x+3}$ is

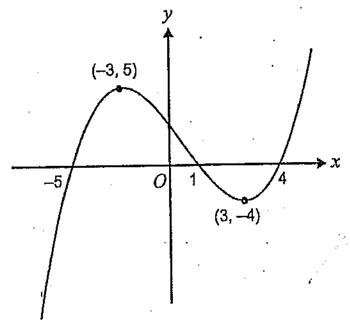
A. $f^{-1}(x) = e^{-2x-3}$

B. $f^{-1}(x) = e^{\frac{x-3}{2}}$

C. $f^{-1}(x) = \log_e(\sqrt{x}) - \frac{3}{2}$

D. $f^{-1}(x) = -\log_e(2x-3)$

3.



For the graph $y = f(x)$ shown above, $f'(x)$ is negative when

A. $-3 < x < 3$

B. $-3 \leq x \leq 3$

C. $x < -3 \text{ or } x > 3$

D. $x \leq -3 \text{ or } x \geq 3$

4. The solutions to the equation $e^{4x} - 5e^{2x} + 4 = 0$ are

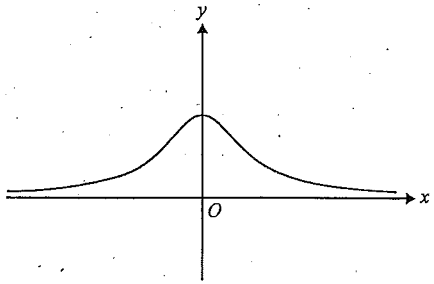
A. $1 \text{ and } 4$

B. $-4 \text{ and } -1$

C. $-\log_e 2, 0, \log_e 2$

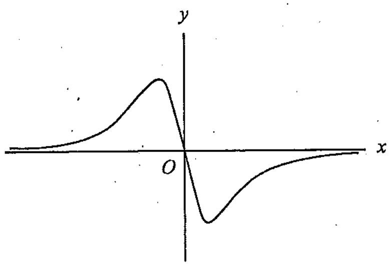
D. $0, \log_e 2$

5. The graph of a function f is shown below

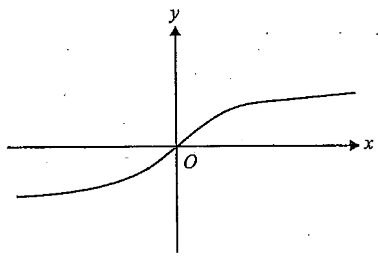


The graph of a primitive function of f could be

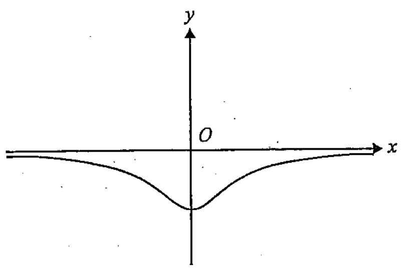
A.



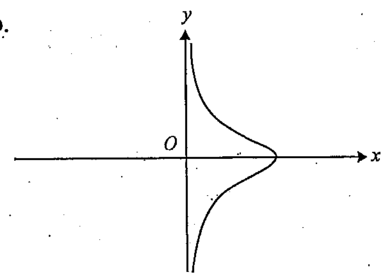
B.



C.



D.



6. The derivative of $log_e\ (2f(x))$ with respect to x is

A. $\frac{f'(x)}{f\ (x)}$

B. $2\ \frac{f'(x)}{f\ (x)}$

C. $\frac{f'(x)}{2f(x)}$

D. $log_e\ (2f'(x))$

7. The normal to the curve with equation $y = x^{\frac{3}{2}} + x$ at the point (4,12) is parallel to the straight line with equation

A. $4x = y$

B. $4y + x = 7$

C. $y = \frac{x}{4} + 1$

D. $x - 4y = -5$

8. The function with rule $f\ (x) = -3\sin\ (\frac{\pi x}{5})$ has period

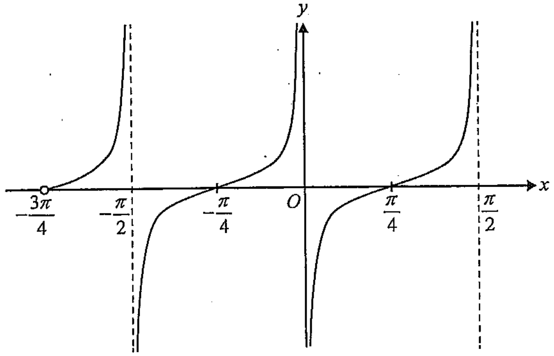
A. 3

B. 5

C. 10

D. $\frac{\pi}{5}$

9. A section of the graph of f is shown below:



The equation of f could be

A. $f(x) = \tan x$

B. $f(x) = \tan\left(x - \frac{\pi}{4}\right)$

C. $f(x) = \tan\left[2\left(x - \frac{\pi}{4}\right)\right]$

D. $f(x) = \tan\left[2\left(x - \frac{\pi}{2}\right)\right]$

10. The equation of the chord of contact of the tangents to the parabola $x^2 = 8y$ from the point (3,-2) is;

A. $3x - 4y + 8 = 0$

B. $3x - 8y + 16 = 0$

C. $3x - 8y - 8 = 0$

D. $3x - 4y + 16 = 0$

Section 2 — Total marks - 60

Answer all questions starting each question on a new side of paper with your name and question number at the top of the page

Question 11 (15 marks)

A. Find the coordinates of the point P which divides the interval from A(-3,6) to B(12, -4) in the ratio of -2:3 — 2

B. Find the value $\cos 105°$ in simplest exact form with a rational denominator. — 3

C. Solve the inequality $\frac{2x+1}{x-1} \geq 3$ and graph your solution on a number line — 3

D. Find $\lim_{x \to 0} \frac{\sin 6x}{7x}$ — 1

E. Use the substitution u = t + 1 or otherwise to evaluate $\int_0^1 \frac{t}{\sqrt{t+1}}\,dt$ — 3
(Leave your answer in exact form)

F. Find the acute angle, to the nearest degree, between the lines $y = 3x + 1$ and $y = -x + 6$ — 3

Question 12 (15 marks) (Start a new page)

A. (i) Show that the equation of the tangent at $T\ (-2t,\ t^2\ )$ on the parabola $y = \frac{1}{4}x^2$ is given by $y + tx + t^2 = 0$ 2

(ii) The point $M\ (x,y)$ is the midpoint of the interval TA where A is the x intercept of the equation of the tangent at T.
Find the equation of the locus of M as T moves on the parabola.

B. Find $\int \frac{dx}{4+x^2}$

C. Given $f\ (x) = sin^{-1}\,2x$

(i) Write down the domain and range of $f(x)$ 2

(ii) Sketch the curve 1

D. A spherical balloon is expanding so that its volume $V\ mm^3$ increases at a constant rate of $72mm^3$/second. What is the rate of increase of its surface area $A\ mm^2$ when the radius is 12mm. 3

E. Use mathematical induction to prove that 3
$n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9 for all positive integers n

Question 13 (15 marks) (Start a new page)

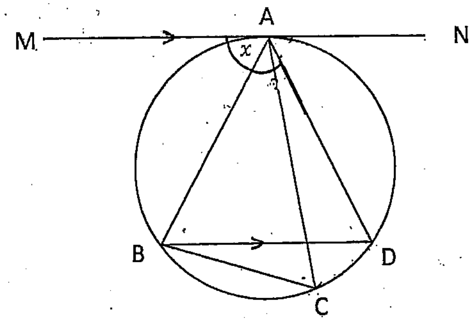
A. A particle moves in a straight line and at time t seconds, its distance xcm from a fixed point is given by $x = 1 + \frac{1}{2}\ cos\ 2t$

(i) Show that the motion of the particle is simple harmonic by expressing $\ddot{x} = -n^2\,(x - A)$ 1

(ii) State the period of its motion 1

(iii) Find the displacement of the particle from the origin when it is at rest, and determine its amplitude. 2

B.



ABC is a triangle inscribed in a circle. MAN is a tangent to the circle at A. BD is a chord of the circle such that BD||MN. Let ∠MAB= x 3
Copy diagram onto your answer sheet.
Show that CA bisects ∠BCD.

C. Newton's law of cooling states that the rate of change of the temperature T of a body at any time t is proportional to the difference in temperature T of the body and the temperature m of the surrounding medium ie: $\frac{dT}{dt} = k(T - m)$ where k is a constant.

(i) Show that $T = m + Ae^{kt}$ where A is a constant, satisfies this equation 1

(ii) If the temperature of the surrounding air is 40°C and the temperature of the body drops from 170°C to 105°C in 45 minutes, find the temperature of the body in another 90 minutes (nearest whole degree) 3
[Find k correct to 3 decimal places]

(iii) Find the time taken for the temperature of the body to drop to 80°C (to the nearest minute) 2

D. Find $\int cos^2 2x\, dx$ 2

Question 14 (15 marks) (Start a new page)

A. (i) Prove $\frac{d^2x}{dt^2} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ where v denotes velocity 2

(ii) The acceleration of a particle moving in a straight line is given by $\ddot{x} = -2e^{-x}$ where x is the displacement from the origin. Initially the particle is at the origin with velocity 2m/s.

α. Prove that $v^2 = 4e^{-x}$ 2

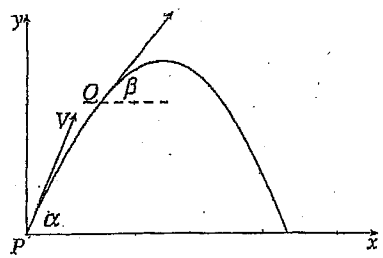
β. Describe the subsequent motion of the particle with reference to its speed and direction 2

B. P(x) is a monic polynomial of degree 3. P(x) has a quadratic factor of $x^2 - 1$ and when P(x) is divided by $x - 2$, the remainder is -9. Form an equation for P(x) and hence solve P(x) =0

C. A particle is projected from a point P on horizontal ground, with initial Speed $V\ m/s$ at an angle of elevation α to the horizontal. Its equations of motion are $\ddot{x} = 0\ and\ \ddot{y} = -g$. The horizontal and vertical components of velocity and displacement of the particle at any time t are given by

$$\frac{dx}{dt} = V\cos\alpha\ \ and\ \ \frac{dy}{dt} = V\sin\alpha - gt$$

$$x = Vt\cos\alpha\ and\ \ y = Vt\sin\alpha - \frac{1}{2}gt^2\ (do\ not\ prove\ these)$$



(i) Show that the time of the flight of the of the particle is given by $t = \frac{2Vsin\alpha}{g}$ 2

(ii) The particle reaches a point Q, as shown, where the direction of the flight makes an angle β with the horizontal. Show that 1

$$tan\beta = \frac{Vsin\alpha - gt}{Vcos\alpha}$$

(iii) Hence show that the time taken to travel from P to Q is 2

$$\frac{Vsin(\alpha-\beta)}{gcos\,\beta} \text{ seconds}$$

(iv) Consider the case where $\beta = \frac{\alpha}{2}$. If the time taken to travel 2

from P to Q is one third of the total time of the flight, find the value of α.

Multiple Choice Answer Sheet

Name ________________________

Completely fill the response oval representing the most correct answer.

1. A ○ B ● C ○ D ○ ✓
2. A ○ B ○ C ● D ○ ✓
3. A ○ B ● C ○ D ○ × B
4. A ○ B ○ C ○ D ● ✓
5. A ○ B ● C ○ D ○ ✓
6. A ○ B ○ C ● D ○ × C
7. A ○ B ○ C ○ D ● × B
8. A ○ B ○ C ● D ○ ✓
9. A ○ B ● C ○ D ○ × C
10. A ● B ○ C ○ D ○ ✓

6

Section 2

Question 11

A) $A(-3,6)$ $B(12,-4)$

$-2 : 3$

$$P\left(\frac{(-3\times3)+(12\times-2)}{-2+3}, \frac{(6\times3)+(-4\times-2)}{-2+3}\right)$$

$$= P\left(\frac{-33}{1}, \frac{26}{1}\right)$$

$\therefore P(-33, 26)$ ✓ 2

B) $\cos 105 = \cos(45°+60°)$

$= \cos 45 \cos 60 - \sin 45 \sin 60$

$= \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2} - \sqrt{6}}{4}$ ✓ 3

C) $\frac{2x+1}{x-1} \geq 3$ $\quad x \neq 1$

~~$2x+1 \geq 3x-3$~~

~~$-x \geq -4$~~

~~$x \leq 4$~~

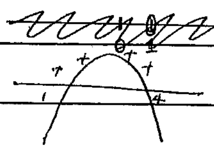
$\frac{2x+1}{x-1}(x-1)^2 \geq 3(x-1)^2$

$2x+1(x-1) \geq 3(x-1)^2 \Rightarrow (x-1)[2x+1-3(x-1)] \geq 0$

~~$(x-1)[2x+1-3] \geq 0$~~

~~$(x-1)(2x-2) \geq 0$~~ $\therefore (x-1)(4-x) \geq 0$

$x \geq 1$ but $x \neq 1$ $\therefore x > 1$ ✗



$1 < x \leq 4$

D) $\lim_{x\to 0} \frac{\sin 6x}{7x}$

$= \frac{\sin 6x}{x} \div \frac{7x}{x}$

$= 6 \times \frac{1}{7}$

$= \frac{6}{7}$ // ✓ 1

E) $\int_0^1 \frac{t}{\sqrt{t+1}}\,dt$ Let $u=t+1 \rightarrow t=u-1$

Change limits $\frac{du}{dt}=1$

$u=1+1$ $dt=du$

$=2$

$u=0+1$ 1

$=1$

$\int_1^2 \frac{u-1}{\sqrt{u}}\,du = \int_1^2 u-1 \div \sqrt{u}$ X

$= \int_1^2 u^{1/2} - u^{-1/2}\,du = \int (u^2-u) \div u^{\frac{1}{2}}$

$= \left[\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right]_1^2 = \left[(u^2-u) \times \frac{1}{2\sqrt{u}}\right]_1^2$

$= \left(\frac{2}{3}\,2^{3/2} - 2\sqrt{2}\right) - = \left[(2^2-2)\times\frac{1}{2\sqrt{2}}\right] - \left(1^2-1\right)\times\frac{1}{2\sqrt{0}}$

$\left(\frac{2}{3}-2\right) = 1\frac{1}{3} - \frac{2\sqrt{2}}{3}$ // $= \frac{1}{\sqrt{2}}$

$\cancel{\frac{1}{\sqrt{2}} = t+1}$

$t = \cancel{\frac{\sqrt{2}-2}{2}}$ //

X

F) $y=3x+1$

$y=-x+6$

$m_1=3$

$m_2=-1$

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\frac{3+1}{1+-3}\right|$

$= |-2|$

$=2$ ✓

$\theta = 63°$ to the nearest degree // 3

10

Question 12 (10)

A)(i) $T(-2t, \cancel{4}t^2)$

$y = \frac{1}{4}x^2$

$\cancel{4y = x^2}$ $\frac{dy}{dx} = 2 \times \frac{1}{4}x$

$= \frac{x}{2}$ sub $x = -2t$

$\frac{-\cancel{2}t}{\cancel{2}} = -t$

$y - \cancel{4}t^2 = -t(x + 2t)$

$y - \cancel{4}t^2 = -tx - 2t^2$

$y - t^2 + tx + 2t^2 = 0$

$\therefore y + tx + t^2 = 0$ as req'd //

(ii) A is where $y = 0$

$0 + tx + t^2 = 0$

$\cancel{t(x+t)=0}$ $tx = -t^2$

$x = -t$

$\cancel{A(0, -t)}$ $A(-t, 0)$ $M_{TA} = \left(\frac{-2t-t}{2}, \frac{t^2}{2}\right)$

$M_{TA} = \left(\frac{-2t+0}{2}, \frac{t^2-t}{2}\right)$ $= \left(-\frac{3t}{2}, \frac{t^2}{2}\right)$

$= \left(-t, \frac{t(t-1)}{2}\right)$ $\therefore x = -\frac{3t}{2} \Rightarrow t = -\frac{2x}{3}$

$x = -t$ $y = \frac{t(t-1)}{2}$ $\therefore y = \left(-\frac{2x}{3}\right)^2 \div 2$

$t = -x \longrightarrow y = \frac{-x(-x-1)}{2}$ $= +\frac{4x^2}{9} \times \frac{1}{2}$

$\therefore y = \frac{(x^2+x)}{2}$ // $y = \frac{2x^2}{9} \Rightarrow 2x^2 = 9y.$

B) $\int \frac{dx}{4+x^2} = \int \frac{dx}{(2)^2 + x^2}$

$= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$ //

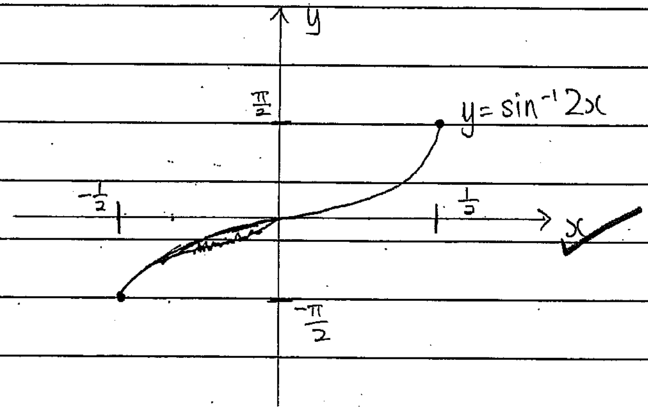
c)(i) $y = \sin^{-1}2x$

D: $-1 \le 2x \le 1$

$\therefore -\frac{1}{2} \le x \le \frac{1}{2}$

R: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(ii)



D. $V = \frac{4}{3}\pi r^3$ given $\frac{dV}{dt} = 72$ mm³/s

$A = 4\pi r^2$

$\frac{dA}{dt} = ?$ $\therefore \frac{dV}{dr} = 4\pi r^2$ $\frac{dA}{dr} = 8\pi r$

$\therefore \frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$

$= \cancel{8\pi r}^{2} \times \frac{1}{\cancel{4\pi r^2}_{r}} \times 72$

$= \frac{144}{12}$ mm²/s when $r = 12$

$= 12$ mm²/s

E) Step ① Show true for $n=1$

$$\frac{1^3+(2)^3+(3)^3}{9} = \frac{36}{9}$$

$$=4$$ ✓

$\therefore$ true for $n=1$

Step ② Assume true for $n=k$, some positive integer

i.e $k^3+(k+1)^3+(k+2)^3 = 9M$ where M is some integer

Step ③ Prove true for $n=k+1$

$(k+1)^3+(k+2)^3+(k+3)^3 = 9M - k^3 + (k+3)^3$??

$= 9M \cancel{-k^3} \cancel{+k^3} + 27 + 9k^2 + 27k$

$= 9M - 27 + 9k^2 + 27k$

$= \cancel{9(M-3)}\ 9(M-3+k^2+3k)$

i.e divisible by 9 $= 9N$ where N is another integer.

Step ④ Since we have shown true for $n=1$, assumed true for $n=k$, some positive integer and proved true for $n=k+1$, this result is true for all positive integers n.

Question 13

A) (i) $x = 1+\frac{1}{2}\cos 2t \Rightarrow \dot{x} = -\sin 2t$

$\ddot{x} = -2\cos 2t = -2\left(2(x-1)\right)$

$\ddot{x} = -4(x-1)$ which is in the form

$= -n^2(x-A)$ $\ddot{X} = -n^2X$

(ii) $T = \frac{2\pi}{n} = \frac{2\pi}{2} = \pi$ s.

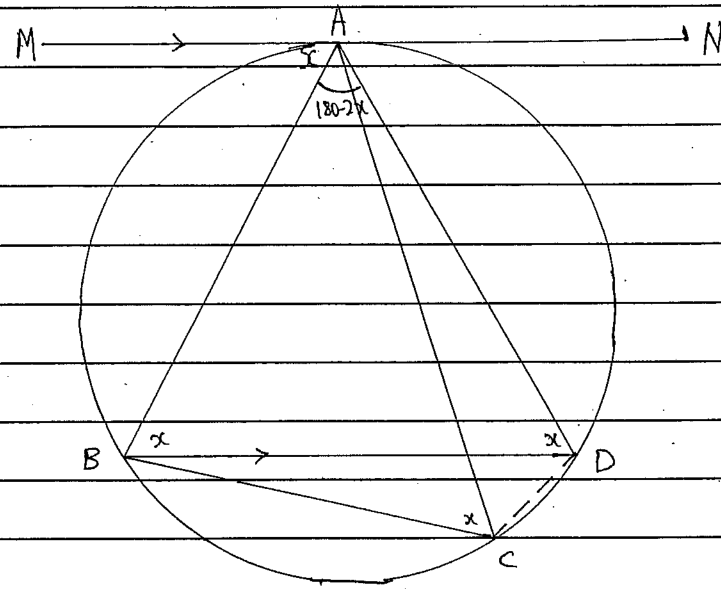
(iii) When $v=0$ $\sin 2t = 0 \Rightarrow 2t = 0, \pi, 2\pi, \ldots$

$t = 0, \frac{\pi}{2}, \pi, \ldots$

$\therefore x = \frac{3}{2}, \frac{1}{2}$

$\therefore$ Amplitude $= \frac{1}{2}$.

B)



Let $\angle MAB = x$

$\angle ABD = x$ (alternate angles $MN \parallel BD$)

$\angle ADB = x$ (alternate segment theorem)

$\angle ACB = x$ (angles in same segment)

~~BACD is a~~ ABCD is a cyclic quadrilateral

$\angle BAD = 180 - 2x$ (angle in triangle BAD)

So opposite angles are supplementary.

$180 - 2x + x + \angle ACD = 180$

$-x + \angle ACD = 0$

$x = \angle ACD$ ✓ 3

$\therefore \angle ACB = x = \angle ACD$

$\therefore$ CA bisects $\angle BCD$

~~Question~~

c) (i) $T = m + Ae^{kt}$

$Ae^{kt} = T - M$

$\frac{dT}{dt} = k \cdot Ae^{kt}$

$= k(T - M)$ as reqd. ✓ 1

(ii) $170 = 40 + Ae^{0k}$

$A = 130$ ✓

$105 = 40 + 130e^{45k}$

$65 = 130e^{45k}$

$e^{45k} = \frac{1}{2}$

$\ln e^{45k} = \ln(\frac{1}{2})$

$45k = \ln(\frac{1}{2})$

$k = \frac{\ln(\frac{1}{2})}{45}$

$= -0.015$ ✓

$T = 40 + 130e^{135k}$

$= 40 + (16°15')$

$= 56°$ $57°$ ✓ 1

(iii) $80 = 40 + 130e^{kt}$

$\frac{4}{13} = e^{kt}$

$t = \frac{\ln(\frac{4}{13})}{k}$

$= 77$ minutes ✓✓ 2

D) $\int \cos^2 2x \, dx = \int \frac{1}{2}(1 + \cos 4x)\, dx$

$= \frac{1}{2}[x + \frac{1}{4}\sin 4x] + c$

$= \frac{x}{2} + \frac{\sin 4x}{8} + c$ ✓✓ 2

Question 14

(A)(i) $\frac{d^2x}{dt^2} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$RHS = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dv}\left(\frac{1}{2}v^2\right) \times \cancel{\frac{dx}{dt}} \frac{dv}{dx}$

$= v \times \cancel{\frac{dx}{dt}} \frac{dv}{dx}$

$= \frac{dx}{dt} \times \frac{dv}{dx}$

$= \frac{dv}{dt}$

$= \ddot{x}$

$= \frac{d^2v}{dt^2}$

(ii) $-2e^{-x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

a) $\int -2e^{-x} = \int \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$\frac{-2e^{-x}}{-1} + c = \frac{1}{2}v^2$

$2e^{-x} + c = \frac{1}{2}v^2$ when $x=0$, $v=2$

$2e^{0} + c = \frac{1}{2}(4)$

$\therefore c = 0$

$\therefore 2e^{-x} = \frac{1}{2}v^2$

$\therefore v^2 = 4e^{-x}$ //

B)

(B) Let $P(x) = (x^2-1)(x+k)$

$\therefore P(2) = (4-1)(2+k) = -9$

$\therefore k+2 = -3 \Rightarrow k = -5$

$\therefore P(x) = (x^2-1)(x-5)$

C) $0 = vt\sin\alpha - \frac{1}{2}gt^2$

$0 = 2vt\sin\alpha - gt^2$

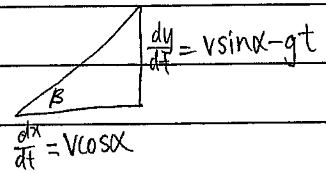
~~$0 = t(2v\sin\alpha - gt)$~~

$gt^2 = 2vt\sin\alpha$

$t^2 = \frac{2vt\sin\alpha}{g}$

$t = \frac{2v\sin\alpha}{g}$ //

(ii)



$\therefore \tan\beta = \frac{v\sin\alpha - gt}{v\cos\alpha}$ //

(B) Let $P(x) = (x^2-1)(x+k)$

$\therefore P(2) = (4-1)(2+k) = -9$

$\therefore k+2 = -3 \Rightarrow k=-5$

$\therefore P(x) = (x^2-1)(x-5)$

(D) $\int \cos^2 2x \, dx$

$= \int \frac{1}{2}(\cancel{\#}\cos 4x + 1)\, dx$

$= \frac{1}{2}\left(\frac{\sin 4x}{4} + x\right) + c$ #

c) $0 = vt\sin\alpha - \frac{1}{2}gt^2$

$0 = 2vt\sin\alpha - gt^2$

~~$0 = t(2v\sin\alpha - gt)$~~

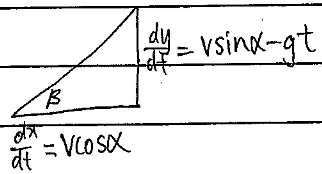
$gt^2 = 2vt\sin\alpha$

$t^2 = \frac{2vt\sin\alpha}{g}$

$t = \frac{2v\sin\alpha}{g}$ //

2

(ii)



$\therefore \tan\beta = \frac{V\sin\alpha - gt}{V\cos\alpha}$ //

1

Q14 (c) $\dot{x} = V\cos\alpha \quad \dot{y} = V\sin\alpha - gt$

$x = Vt\cos\alpha \quad y = Vt\sin\alpha - \frac{gt^2}{2}$

(i) Let $y = 0$

$t\left(V\sin\alpha - \frac{gt}{2}\right) = 0$

$t = 0$ or $\frac{gt}{2} = V\sin\alpha$

$t = \frac{2V\sin\alpha}{g}$ = time of flight.

(ii) At Q

$\tan\beta = \left|\frac{\dot{y}}{\dot{x}}\right| = \left|\frac{V\sin\alpha - gt}{V\cos\alpha}\right|$ as req^d.

(iii) From (ii)

$\frac{\sin\beta}{\cos\beta} = \frac{V\sin\alpha - gt}{V\cos\alpha}$

$\therefore V\sin\beta\cos\alpha = V\sin\alpha\cos\beta - g\cos\beta\, t$

$\therefore t = \frac{V\sin\alpha\cos\beta - V\sin\beta\cos\alpha}{\cancel{\cos\beta}\, g\cos\beta}$

$= \frac{V(\sin(\alpha-\beta))}{g\cos\beta}$ seconds as req^d.

(iv) Let $\beta = \frac{\alpha}{2}$

$\therefore t_{PQ} = \frac{V\sin(\alpha - \frac{\alpha}{2})}{g\cos\frac{\alpha}{2}}$

$= \frac{V}{g}\tan\frac{\alpha}{2}$

Given that $t_{PQ} = \frac{1}{3}\cdot\frac{2V\sin\alpha}{g}$

$\frac{\cancel{V}}{\cancel{g}}\frac{\cancel{\sin\frac{\alpha}{2}}}{\cos\frac{\alpha}{2}} = \frac{1}{3}\cdot\frac{2\cancel{V}}{\cancel{g}}\cdot 2\cancel{\sin\frac{\alpha}{2}}\cos\frac{\alpha}{2}$

$\therefore \cos^2\frac{\alpha}{2} = \frac{3}{4}$

$\cos\frac{\alpha}{2} = \pm\frac{\sqrt{3}}{2}$ But α is acute

$\frac{\alpha}{2} = 30^\circ \quad \therefore \alpha = 60^\circ$