2013
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

General Instructions

- Reading Time - 5 minutes.
- Working Time - 3 hours.
- Write using a blue or black pen.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**100**)

- Attempt Questions 1-10. (Multiple Choice - 10 marks)
- Attempt Questions 11-16 (All questions - 15 marks)

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n\neq -1;\ x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\ a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x>0$

Section 1- Multiple choice (10 marks)

1. If $\dfrac{4+\sqrt{3}}{2+\sqrt{3}} = b - 2\sqrt{3}$, then b is equal to

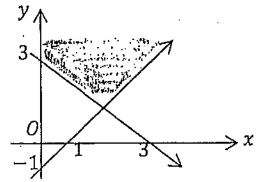
   a. 2 c. -5

   b. -2 d. 5

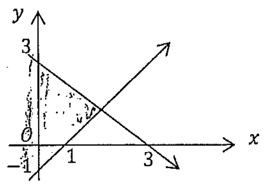
2. In the Cartesian number plane, the region whose points simultaneously satisfies the inequalities:

$$x + y \leqslant 3$$
$$x - y \geqslant 1$$

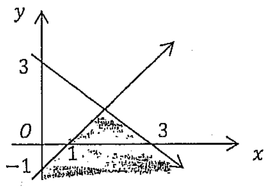
a.



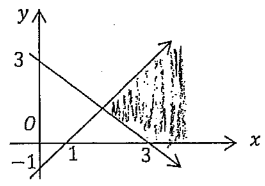
c.



b.



d.



3. $2x^2 \geq x$ for:

   a. $0 \leq x \leq \frac{1}{2}$ c. $x \leq 0$ and $x \geq \frac{-1}{2}$

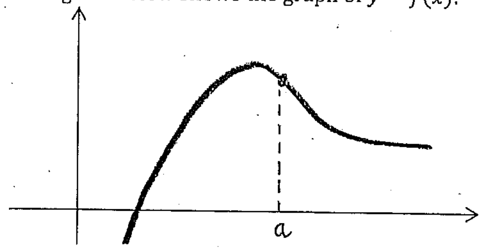
   b. $-\frac{1}{2} \leq x \leq 0$ d. $x \leq 0$ and $x \geq \frac{1}{2}$

4. $\log_2 8$ equals

   a. 2   c. 3

   b. 8   d. Can't be determined

5. The diagram below shows the graph of $y = f(x)$:



Which statement is true?

   a. $f'(a) < 0, f''(a) < 0$ c. $f'(a) < 0, f''(a) > 0$

   b. $f'(a) > 0, f''(a) < 0$ d. $f'(a) > 0, f''(a) > 0$

6. The number of solutions to $\tan^2 3\theta = 1$ are:

   a. 2   c. 4

   b. 12   d. Infinite

7. If the roots of $x^2 - 5x + 2 = 0$ are α and β, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is:

   a. $\frac{5}{2}$ c. $-\frac{5}{2}$

   b. $\frac{2}{5}$ d. $-\frac{2}{5}$

8. If the parabola P has focus $(2,5)$ and directrix $y = -3$, then the equation of P is:

   a. $x^2 = 16(y+1)$ c. $(x-2)^2 = 16(y+1)$

   b. $(x-2)^2 = 16(y-1)$ d. $(x-2)^2 = -16(y+1)$

9. The sum of the first n terms of a series is given by:

$$S_n = \frac{n(3n+1)}{2}$$

the first three terms of the series are:

   a. $2, 7, 15$ c. $2, 5, 8$

   b. $2, 7, 8$ d. $5, 7, 15$

10. The solution(s) to $e^{2x} + 3e^x - 10 = 0$ is/are:

   a. $e^x = 2, e^x = -5$ c. $x = \ln 2$

   b. $e^x = 2$ d. $x = \ln 2, x = \ln(-5)$

End of Section 1

Question 11 (15 marks) Start a new booklet — Marks

a. Differentiate:

i. $\log_e(\sin x)$ — 2

ii. $x \cos x$ — 1

b. Integrate:

i. $\int \left(\sin 2x - e^{-\frac{x}{5}}\right) dx$ — 2

ii. $\int \frac{1+x}{x^2} dx$ — 2

c. Evaluate: — 2

$$\int_2^5 \frac{dx}{x}$$

d. The gradient function of a curve is given by:

$$\frac{dy}{dx} = 3x^2 - 12$$

i. For what values of x does the curve increase with downward concavity? — 1

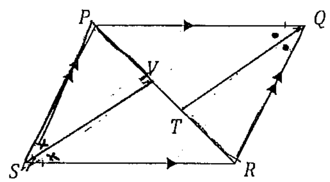
ii. If this curve passes through the point $(-3, 2)$, find the equation of the curve. — 2

e. For what values of m does the line $y = m(x+1)$ have no intersection with the parabola $y = 2x^2$? — 3

End of Question 11

Question 12 (15 marks) Start a new booklet — Marks

a.



$PQRS$ is a parallelogram. TQ bisects $\angle PQR$ and VS bisects $\angle PSR$.

i. Copy this diagram into your answer booklet.

ii. State why $\angle PQR = \angle PSR$ — 1

iii. Prove that $\Delta PVS \equiv \Delta RTQ$ — 3

iv. Hence find the length of TV if $PR = 20$cm and $TR = 8$cm. — 1

b. The point $Q(-2, 1)$ lies on the line k whose equation is $9x - 2y + 20 = 0$.
The point $R(4, -2)$ lies on the line l whose equation is $3x + y - 10 = 0$.

i. Show that k and l intersect at a point P on the y axis. — 1

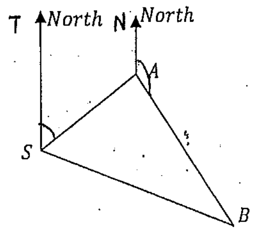
ii. Find the equation of the line m which joins Q and R. — 2

iii. Find the area of the triangle PQR. — 3

c. Solve $\tan 2\theta = \sqrt{3}$ for $0° \leq \theta \leq 360°$ — 2

d. A ship sails from Sydney for 200km on a bearing of 040°, then sails on a bearing of 157° for 345km. — 2

How far from Sydney is the ship, to the nearest kilometre?



End of Question 12

Question 13 (15 marks) **Start a new booklet** **Marks**

a. Consider the parabola $y = x^2 - 8x + 4$.

Find:

i. The coordinates of the vertex. **2**

ii. The coordinates of the focus. **2**

b. Find the value of k in the quadratic equation $x^2 - 5x + k - 1 = 0$ if:

i. One root is equal to 2. **2**

ii. One root is the reciprocal of the other. **2**

c. At what point on the curve $y = x^2 + x$ is the line $5x - y - 4 = 0$ a tangent. **2**

d. The acceleration a ms^{-2} of a moving object is given at time t seconds ($t \geq 0$) by

$a = 4\pi^2 \cos \pi t$.

At time $t = 0$, the object is at the point $x = 0$ and travelling with velocity

$v = 2\pi$ ms^{-1}.

i. Find the velocity v and displacement x as a function of t. **2**

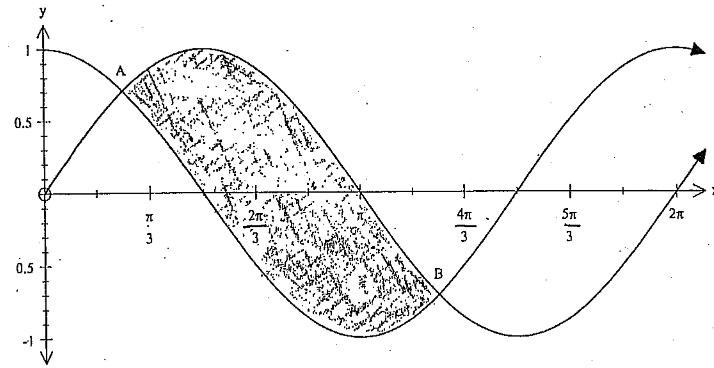
ii. Find the times in the first 4 seconds when the particle is stationary. **3**

End of Question 13

Question 14 (15 marks) **Start a new booklet**

a. The diagram shows the graphs $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq 2\pi$.

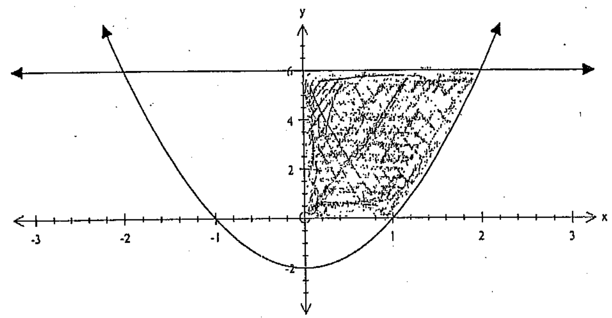
The graphs intersect at A and B.



i. Show that A has coordinates $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ and find the coordinates of B. **3**

ii. Find the area enclosed by the two graphs. **3**

b. The diagram shows the region bounded by the curve $y = 2x^2 - 2$, the line $y = 6$ and the x and y axes. **3**



Find the volume of the solid of revolution formed when the region is rotated about the y-axis.

c. Find an approximation to $\int_0^1 2^x dx$ by using Simpson's Rule with 5 function values. **4**

d. Find the equation of the straight line passing through $(0,0)$ and (h, r) **2**

End of Question 14

Question 15 (15 marks) Start a new booklet

a. Consider the arithmetic series

$$3 + 8 + 13 + \cdots + 488$$

i. How many terms are in this series? 1

ii. Find the sum of all the terms in this series. 1

b. Consider the geometric series:

$$1 + \frac{4}{3}\sin^2 x + \frac{16}{9}\sin^4 x + \frac{64}{27}\sin^6 x + \cdots$$

i. When the limiting sum exists, find the value in simplest form. 2

ii. For what values of x in the interval $0 \le x \le \frac{\pi}{2}$ does the limiting sum of this series exist? 2

c. If α and β are the roots of the equation $2x^2 - 7x - 5 = 0$ find the values of:

i. $\alpha + \beta$ 1

ii. $\alpha\beta$ 1

iii. $(\alpha + 1)(\beta + 1)$ 2

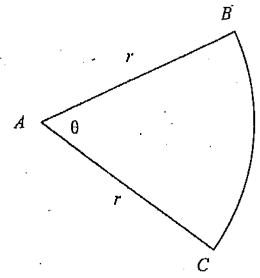
iv. $(\alpha + 1)^{-1} + (\beta + 1)^{-1}$ 2

d. Show that the quadratic equation in x, $(a^2 - b^2)x^2 + 2b(a - c)x + (b^2 - c^2) = 0$ has real and rational roots for all values of x, if a, b and c are rational. 3

End of Question 15

Question 16 (15 marks) Start a new booklet

a.



In the figure, AB and AC are radii of length r metres of a circle whose centre is A. The arc BC of the circle subtends an angle θ radians at A.

i. Write down the formulae for:

α. The length of arc BC 1

β. The area of the sector ABC 1

ii. The perimeter of the figure above is 12 metres. Show that the area y square metres of the sector ABC is given by: 2

$$y = \frac{72\theta}{(\theta + 2)^2}$$

iii. Hence show that the maximum area of the sector is 9m^2. 3

b. A block of ice originally of mass 84kg is melting at a rate equal to 3% of its mass. Assuming that at any time t hours, its mass M is given by

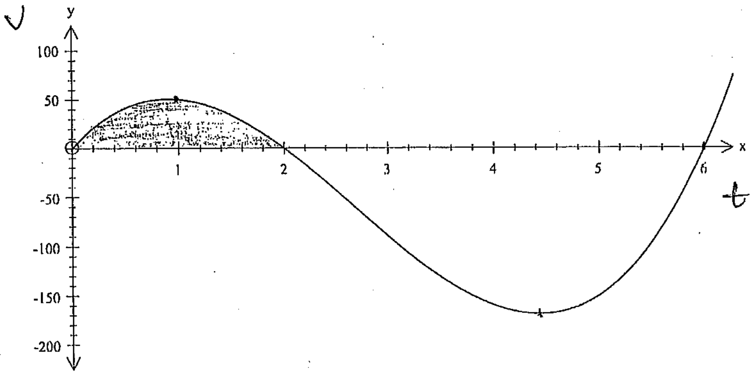
$$M = M_0 e^{-kt}$$

i. What will its mass be after 12 hours? (answer correct to 3 significant figures) 1

ii. What time will elapse before 80% of its mass has melted? (answer correct to 3 significant figures) 2

Question 16 continues on next page

c. The graph represents the velocity v m/s of a particle after t seconds travelling in a straight line. The particle starts from rest.



i. What is the velocity of the particle after 1 second? **1**

ii. When does the particle change direction? **1**

iii. When is the acceleration of the particle zero? **1**

iv. What happens to the particle after 6 seconds? **1**

v. Explain what is represented by the shaded region in the diagram. **1**

End of Exam

SOLUTIONS TO 2U MATHS TRIAL HSC 2013

Section I

1. D
2. B
3. D
4. C
5. A
6. D
7. A
8. B
9. C
10. C

Question 11.

(a)(i) $\log_e(\sin x)$

$\frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x$

(ii) $u = x \quad v = \cos x$

$u' = 1 \quad v' = -\sin x$

$\frac{dy}{dx} = \cos x - x\sin x$

(b)(i) $\int (\sin 2x - e^{-x/5})\,dx$

$= -\frac{1}{2}\cos 2x + 5e^{-x/5} + C$

(ii) $\int \frac{1+x}{x^2} = \int\left(\frac{1}{x^2} + \frac{x}{x^2}\right)dx$

$= \int\left(x^{-2} + \frac{1}{x}\right)dx$

$= -x^{-1} + \log_e x + C$

$= -\frac{1}{x} + \log_e x + C$

(c) $\int_2^5 \frac{dx}{x} = \int_2^5 \frac{1}{x}\,dx$

$= \left[\log_e x\right]_2^5$

$= \log_e 5 - \log_e 2$

$= \log_e\left(\frac{5}{2}\right)$ or

$= 0.916$

(d)(i) $\frac{dy}{dx} = 3x^2 - 12$

Function increases when $\frac{dy}{dx} > 0$

ie $3x^2 - 12 > 0$

$x^2 - 4 > 0$

$x < -2, \quad x > 2$

(ii) $\frac{dy}{dx} = 3x^2 - 12$

$y = \frac{3x^3}{3} - 12x + C$

when $x = -3 \quad y = 2$

$2 = (-3)^3 - 12(-3) + C$

$C = -7$

$\therefore y = x^3 - 12x - 7$

(e) $m(x+1) = 2x^2$

$mx + m = 2x^2$

$2x^2 - mx - m = 0$

No intersection if $2x^2 - mx - m = 0$ has no real roots. ie $\Delta < 0$

$(-m)^2 - 4(2)(-m) < 0$

$m^2 + 8m < 0$

$m(m+8) < 0$

$\therefore \quad -8 < m < 0$

Question 12

(b) Solving Simultaneously

$9x - 2y + 20 = 0$

$3x + y - 10 = 0$

$9x - 2y + 20 = 0$

$+ \quad 6x + 2y - 20 = 0$

$15x \quad = 0$

$x = 0$

$\therefore y = 10$

$P \equiv (0, 10)$ which lies on y-axis

ii) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - -2}{-2 - 4}$

$= -\frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y + 2 = -\frac{1}{2}(x - 4)$

$y + 2 = -\frac{1}{2}x + 2$

$y = -\frac{1}{2}x$

OR $\quad x + 2y = 0$

iii) Need to find perpend. dist. from P to QR

$d = \left|\frac{1(0) + 2(10) + 0}{\sqrt{1^2 + 2^2}}\right|$

$= \frac{20}{\sqrt5} \times \frac{\sqrt5}{\sqrt5} = 4\sqrt5$

Need to find QR

$QR = \sqrt{(4 - -2)^2 + (-2 - 1)^2}$

$QR = \sqrt{45}$

$QR = 3\sqrt5$

$\therefore A = \frac{1}{2} \times 3\sqrt5 \times 4\sqrt5$

$= 30 \text{ units}^2$

c) $\tan 2\theta = \sqrt3$

Base angle $\theta = 60°$

1st & 3rd Quad.

$2\theta = 60°, 180° + 60°, 360° + 60°, 360° + 240°$

$2\theta = 60°, 240°, 420°, 600°$

$\theta = 30°, 120°, 210°, 300°$

(d) $\angle SAN = 180° - 40$ (co-int. ∠s)

$= 140°$

$\therefore \angle SAB = 360 - (140 + 157)$ ∠ of revol.

$= 63°$

$a^2 = b^2 + c^2 - 2bc\cos A$

$= 200^2 + 345^2 - 2(200)(345)\cos 63$

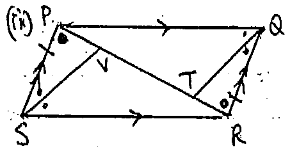
$= 96374.3$

$a = \sqrt{96374.3}$

$\therefore x = 310$

(a)(ii) opposite angles of parallelograms are equal.

(iii)



$PS = QR$ (opp. sides of //grams =)

$\angle PSV = \angle RQT$ ($\angle PQR = \angle PSR$ from (ii) (and VS bisects $\angle PSR$ & TQ bisects $\angle PQR$)

$\angle SPV = \angle QRT$ (Alternate angles)

$\therefore \Delta PVS \equiv \Delta RTQ$ (AAS)

(iv) $PR = 20$

$TR = 8$

$\therefore PR - 2(TR)$ since $TR = PV$

$= 20 - 16$

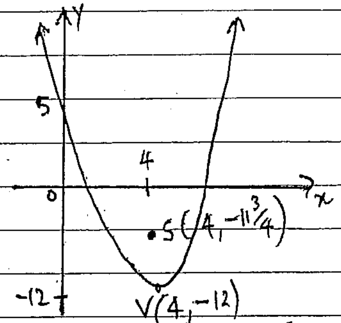
$= 4.$

Question 13

(a) $y = x^2 - 8x + 4$

$\therefore y - 4 + 16 = x^2 - 8x + 16$

$y + 12 = (x-4)^2$



$\therefore S(4, -11\tfrac{3}{4})$ $V(4, -12)$

$[a = 1/4]$

(b)(i) $\alpha = 2$ using sum of roots

$\alpha + \beta = \frac{-b}{a}$

$2 + \beta = 5/1$

$\therefore \beta = 3$

Using product of roots

$\alpha\beta = c/a$

$2 \times 3 = k - 1$

$k = 7$

or since $x = 2$ is a root then must satisfy

$\therefore 2^2 - 5(2) + k - 1 = 0$

$4 - 10 + k - 1 = 0$

$k = 7$

(ii) $\alpha = \frac{1}{\beta}$ Using product of roots

$\alpha\beta = c/a$

$1 = k - 1 \quad \therefore k = 2$

(c) $y = x^2 + x \Rightarrow \frac{dy}{dx} = 2x + 1$

Now $5x - y - 4 = 0$

$y = 5x - 4, \ m = 5$

$\therefore 2x + 1 = 5 \Rightarrow x = 2.$

If $x = 2$ $y = 2^2 + 2$

$\therefore (2, 6).$

(d)(i) $a = 4\pi^2 \cos \pi t$

$V = \int 4\pi^2 \cos \pi t \, dt$

$= \frac{4\pi^2}{\pi} \sin \pi t + C$

$= 4\pi \sin \pi t + C$

when $t = 0$ $V = 2\pi$

$2\pi = 4\pi \sin \pi(0) + C \quad \therefore C = 2\pi$

$\therefore V = 4\pi \sin \pi t + 2\pi$

$x = \int (4\pi \sin \pi t + 2\pi) \, dt$

$x = -\frac{4\pi}{\pi} \cos \pi t + 2\pi t + K$

when $t = 0$ $x = 0$

$0 = -4\cos 0 + 0 + K \quad \therefore K = 4$

$\therefore x = -4\cos \pi t + 2\pi t + 4$

(ii) Object stationary $v = 0$

$4\pi \sin \pi t + 2\pi = 0$

$\sin \pi t = -2\pi/4\pi$

$\sin \pi t = -1/2$

$\therefore \pi t = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, 3\pi + \frac{\pi}{6}, 4\pi - \frac{\pi}{6}$

$\pi t = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$

$t = \frac{7}{6}, \frac{11}{6}, \frac{19}{6}, \frac{23}{6}$

$0 \le t \le 4$

Question 14

(a)(i) $\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

ie $x = \pi/4, \ 5\pi/4$

when $x = \pi/4$ $y = 1/\sqrt{2}$

when $x = 5\pi/4$ $y = -1/\sqrt{2}$

$\therefore A(\pi/4, 1/\sqrt{2})$ $B(\frac{5\pi}{4}, -1/\sqrt{2})$

(ii) $\int_{\pi/4}^{5\pi/4} \sin x \, dx - \int_{\pi/4}^{5\pi/4} \cos x \, dx$

$= \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) \, dx$

$= -\cos x - \sin x \Big]_{\pi/4}^{5\pi/4}$

$= -(\cos\frac{5\pi}{4} - \cos\frac{\pi}{4}) - (\sin\frac{5\pi}{4} - \sin\frac{\pi}{4})$

$= -(-1/\sqrt{2} - 1/\sqrt{2}) - (-1/\sqrt{2} - 1/\sqrt{2})$

$= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = 2\sqrt{2}$

(b) $y = 2x^2 - 2 \Rightarrow y + 2 = 2x^2$

and $x^2 = \frac{y+2}{2}$

$V = \pi \int x^2 \, dy$

$V = \pi \int_0^6 \frac{y+2}{2} \, dy$

$V = \frac{\pi}{2}\left(\frac{1}{2}y^2 + 2y\right)\Big]_0^6$

$V = \frac{\pi}{2}\left(\frac{1}{2}(36) + 2(6)\right)$

$V = 15\pi$ units^3

(c)

x	0	1/4	1/2	3/4	1
2^x	1	$2^{1/4}$	$2^{1/2}$	$2^{3/4}$	2

$A = \frac{h}{3}\left((y_0 + y_n) + 4(\text{odds}) + 2(\text{even})\right)$

$= \frac{1/4}{3}\left(3 + 4(2^{1/4} + 2^{3/4}) + 2(2^{1/2})\right)$

$= 1.4427$

(d) $m = \frac{r-0}{h-0} \quad \therefore m = \frac{r}{h}$

$y - 0 = \frac{r}{h}(x - 0)$

$y = \frac{rx}{h}$ or $xr - hy = 0$

Question 15

(a) $T_n = a + (n-1)d$

$488 = 3 + (n-1)5$

$488 = 5n - 2$

$n = 98$

(b) $S_n = \frac{n}{2}(a + l)$

$= \frac{98}{2}(3 + 488)$

$= 24059$

(b)(i) $a = 1 \quad r = \frac{4}{3}\sin^2 x$

$S = \frac{a}{1-r}$

$S = \frac{1}{1 - 4/3 \sin^2 x} \times \frac{3}{3}$

$S = \frac{3}{3 - 4\sin^2 x}$

(ii) $0 < |\frac{4}{3}\sin^2 x| < 1$

$0 < x < \pi/3$

(c) (i) $\alpha + \beta = -b/a = 7/2$

(ii) $\alpha\beta = c/a = -5/2$

(iv) $(\alpha+1)^{-1} + (\beta+1)^{-1}$

$= \frac{1}{\alpha+1} + \frac{1}{\beta+1}$

$= \frac{\beta+1+\alpha+1}{(\alpha+1)(\beta+1)}$

$= \frac{\alpha+\beta+2}{(\alpha+1)(\beta+1)}$

$= \frac{7/2 + 1}{2} = \frac{11}{4}$

(iii) $(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1$

$= 7/2 - 5/2 + 1$

$= 2$

(d) For rational roots and real roots $\Delta \geq 0$ and Δ needs to be perfect square.

$\Delta = b^2 - 4ac$

$= [2b(a-c)]^2 - 4(a^2-b^2)(b^2-c^2)$

$= 4b^2(a-c)^2 - 4(a^2-b^2)(b^2-c^2)$

$= 4b^2(a-c)^2 - 4(a^2b^2 - a^2c^2 - b^4 + b^2c^2)$

$= 4[b^2(a^2-2ac+c^2) - a^2b^2 + a^2c^2 + b^4 - b^2c^2]$

$= 4[a^2b^2 - 2ab^2c + b^2c^2 - a^2b^2 + a^2c^2 + b^4 - b^2c^2]$

$= 4[b^4 - 2ab^2c + a^2c^2]$

$= 4(b^2 - ac)^2$

$= [2(b^2 - ac)]^2$

$\therefore$ since Δ is a perfect square, roots are real and rational.

Q16.

(a) (i) (α) $l = r\theta$

(β) $A = \frac{1}{2}r^2\theta$

(ii) $Y = \frac{1}{2}r^2\theta$

$P = AB + \text{arc } BC + CA$

$12 = r + r\theta + r$

$12 = r\theta + 2r$

$12 = r(\theta + 2)$

$r = \frac{12}{\theta+2}$

$\therefore Y = \frac{1}{2}\left(\frac{12}{\theta+2}\right)^2\theta$

$Y = \frac{1}{2}\left(\frac{144}{(\theta+2)^2}\right)\theta$

$Y = \frac{72\theta}{(\theta+2)^2}$

(iii) $\frac{dY}{d\theta} = \frac{(\theta+2)^2 \cdot \frac{d}{d\theta}(72\theta) - 72\theta \cdot \frac{d}{d\theta}(\theta+2)^2}{((\theta+2)^2)^2}$

$= \frac{(\theta+2)^2(72) - 72\theta \times 2(\theta+2)^1 \cdot 1}{(\theta+2)^4}$

$= \frac{72(\theta^2+4\theta+4) - 144\theta(\theta+2)}{(\theta+2)^4}$

$= \frac{72\theta^2 + 288\theta + 288 - 144\theta^2 - 288\theta}{(\theta+2)^4}$

$= \frac{288 - 72\theta^2}{(\theta+2)^4}$

$$= \frac{72(4-\theta^2)}{(\theta+2)^2}$$

$$= \frac{72(2-\theta)(2+\theta)}{(\theta+2)^4}$$

$$= \frac{72(2-\theta)}{(\theta+2)^3}$$

Stationary points occur when $\frac{dY}{d\theta} = 0$

ie. $\frac{72(2-\theta)}{(\theta+2)^3} = 0$

$72(2-\theta) = 0$

$\theta = 2$

when $\theta < 2$ $\frac{dY}{d\theta} > 0$

when $\theta > 2$ $\frac{dY}{d\theta} < 0$

$\therefore$ max at $\theta = 2$

when $\theta = 2$ $Y = \frac{72(2)}{(2+2)^2} = \frac{144}{16} = 9\,m^2$

(b) (i) $\frac{dM}{dt} = -km$ where $k = 0.03$

$\therefore M = M_0 e^{-kt}$ ie $M = 84e^{-0.03t}$

when $t = 12$: $M = 84e^{-0.03 \times 12}$

$M = 58.6$ kg

(ii) $0.2 \times 84 = 84e^{-0.03 \times t}$

$0.2 = e^{-0.03t}$

$\log_e 0.2 = -0.03t$

$t = 53.6$ hours

(c) (i) $V = 50$

(ii) $t = 2$ (not $t = 6$)

(iii) $t = 1$ $t = 4$

(iv) Particle stops

(v) Distance travelled (pos. direction)