Year 12 HSC 2U Mathematics

Probability, Geometrical Applications of Differentiation, and Applications of Calculus to the Physical World.

Term 3, 2011 | Week 7 — *Time Allowed*: 50 mins — *Marks*: 38

Show all working to gain maximum marks — Marks will be deducted for poor or illegible work

PART 1 – Probability (12 Marks) CRA

(a) A die is thrown and a coin is tossed. Find the probability of obtaining:

(i) a head and a six **1**

(ii) a tail and a number less than four **1**

(b) Assuming that the probability of a girl being born is 60%, find the probability that a family of 3 children will comprise:

(i) all boys **1**

(ii) two boys and a girl **2**

(iii) at least one boy **1**

(c) A Winter Sport survey of 66 Australians revealed that 44 like Netball, 33 like Wrestling and 6 don't like either. If one individual is selected at random from this group, what is the probability that:

(i) they do not like Netball or Wrestling **1**

(ii) they like Netball but not Wrestling **1**

(d) A faulty machine produces items of which 34% are defective.

(i) What is the probability of the machine producing 3 good items in succession? **1**

(ii) What is the probability of the machine producing n faulty items in succession? **1**

(iii) What is the smallest number of items that are to be produced so that the probability of obtaining at least one good item exceeds 99%? **2**

PART 2 – Geometrical Applications of Differentiation (13 Marks) GHW

(a) If $f''(x) = 18x + 4$, $f'(1) = 13$ and $f(1) = 6$ find $f(x)$ **2**

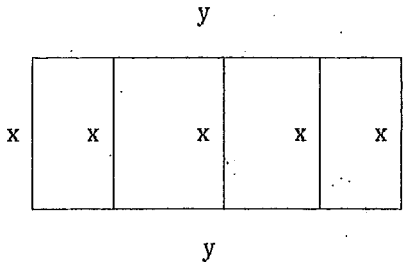
(b) Consider the function $f(x) = x - 3\ln x$ *for* $1 \le x \le 7$

(i) Find the coordinates of the stationary point(s) and determine their nature. **3**

(ii) Sketch the curve for $1 \le x \le 7$. **2**

(iv) What is the maximum value for $1 \le x \le 7$ **1**

(c) Build a rectangular pen with three parallel partitions using 500 feet of fencing.



(i) Show $y = 250 - (5/2)x$ **1**

(ii) Show $A = 250x - (5/2)x^2$ **1**

(ii) What is the maximum area of the pen? **3**

PART 3 – Applications of Calculus to the Physical World (13 Marks) RABS

(a) A weight is tied to a spring. The displacement d (in cm) of the weight is found by the equation $d = 3\sin 3t$ where t is time in seconds.

(i) Following release, when does the weight return to the origin? **1**

(ii) What is the velocity of the weight at this point in time? **2**

(iii) Determine when the weight is at its maximum displacement from the origin. **2**

(b) Following discovery, the population of thylacines was found to be decreasing at a rate proportional to its population.

Two years after discovery, the population of thylacines was 13000.
Five years after discovery, the population of thylacines was 4500.

(i) Find the value of the constant of decay. **2**

(ii) Find the initial population of thylacines. **1**

(iii) After how many years will the number of thylacines be less than 5% of the original population? **2**

(c) The acceleration of a particle is given by $a = 4t$ where a is in ms^{-2} and t is in seconds.

If initially its velocity is $5ms^{-1}$ and its displacement is 6m to the left of the origin, find its displacement after 10 seconds. **3**

PART 1 - PROBABILITY

MARKED BY CRA

(a)

(i) $P(HG) = \frac{1}{2} \times \frac{1}{6}$

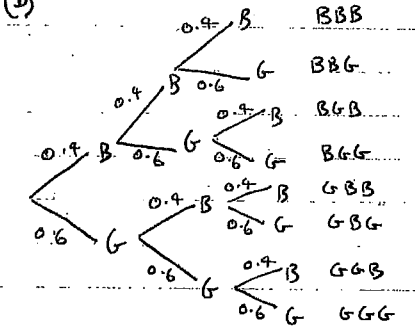
$= \frac{1}{12}$ ✓

· Well done on the whole.

(ii) $P(T,B,2,1) = \frac{1}{2} \times \frac{3}{6}$

$= \frac{1}{4}$ ✓

(b)



(i) $P(BBB) = (0.4)^3$

$= 0.064$ ✓

(ii) $P(BBG) + P(BGB) + P(GBB)$

$= (0.4 \times 0.4 \times 0.6) + (0.4 \times 0.6 \times 0.4) + (0.6 \times 0.4 \times 0.4)$ ✓

$= 0.288$ ✓

· Many did not recognise 3 possible combinations for 2 boys + 1 girl.

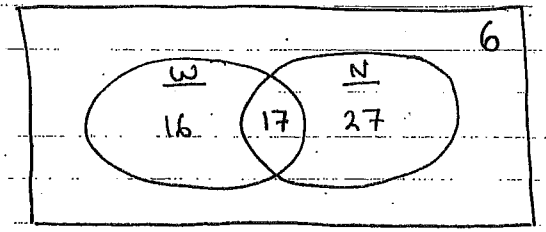
(iii) $P(\quad) = 1 - P(GGG)$

$= 1 - (0.6)^3$

$= 0.784$ ✓

(c) 6 don't like Netball or Wrestling

$\therefore 66 - 6 = 60$ like a sport.



- Venn diagram would be very useful here... many chose to go without!

(i) $P = \frac{6}{66} = \frac{1}{11}$ ✓

(ii) $P = \frac{27}{66}$

$= \frac{9}{22}$ ✓

(d) (i) 34% defective (D)

$\therefore$ 66% good items (G)

$\therefore P(GGG) = (0.66)^3$

$= 0.287496$ ✓

(ii) $P(nD) = (0.34)^n$ ✓

(iii) $1 - (0.34)^n > 0.99$ ✓

$-(0.34)^n > -0.01$

$(0.34)^n < 0.01$

$n \cdot \ln(0.34) < \ln 0.01$

$n > \frac{\ln 0.01}{\ln 0.34}$

$n > 4.268$

$\therefore$ 5 items ✓

- Smallest number of items must be 5. (can't have 4.3 items...)

Part 2.

a) $f''(x) = 18x + 4$

$f'(x) = \int (18x + 4)\,dx$

$= 9x^2 + 4x + C$

$13 = 9(1)^2 + 4(1) + C$

$C = 0.$

$\therefore f'(x) = 9x^2 + 4x$ ✓

$f(x) = \int (9x^2 + 4x)\,dx$

$= 3x^3 + 2x^2 + C$

$6 = 3(1)^3 + 2(1)^2 + C$

$C = 1$

$\therefore f(x) = 3x^3 + 2x^2 + 1$ ✓

Very well done.

b) $f(x) = x - 3\ln x \quad 1 \le x \le 7$

i) $f'(x) = 1 - \frac{3}{x}$

when $f'(x) = 0.$

$0 = 1 - \frac{3}{x}$

$x = 3$ ✓

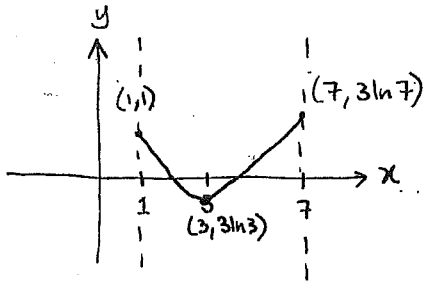
$\therefore$ turning pt @ $(3, 3 - 3\ln 3)$ ✓

x	2	3	4
$f'(x)$	−	0	+

$\therefore (3, 3 - 3\ln 3)$ is minimum ✓

Generally well done, some forgot to check nature

ii)



$f(1) = 1 - 3\ln 1 = 1$

$f(7) = 7 - 3\ln 7$

✓ labelling endpts + t.p.
✓ graph shape.

A mixed bag.
All ~~the~~ points of interest needed to be labelled.

iii) max value = $3\ln 7$ ✓

Some students gave the coordinates without identifying the max value

C i) Perimeter = $5x + 2y$

$P = 500$

$\therefore 5x + 2y = 500$

$2y = 500 - 5x$

$y = 250 - \frac{5}{2}x$

Well done

ii) $A = xy$

$= x(250 - \frac{5}{2}x)$

$= 250x - \frac{5}{2}x^2$

Well done.

iii) $A' = 250 - 5x$

$A' = 0 \quad 0 = 250 - 5x$

$x = 50$ ✓

$A'' = -5$ ✓

as $A'' < 0$, A is max

$\therefore A = 250(50) - \frac{5}{2}(50)^2$

$= 6250 \text{ ft}^2$ ✓

Well done.

PART 3 ... APP OF CALC TO PHYS WORLD

(a)

(i) $0 = 3\sin 3t$

$\div 3$

$0 = \sin 3t$

$0 = \sin\theta$

$\therefore \theta = 0, \pi, 2\pi \ldots$

$\therefore t = \pi/3$ s ✓

MANY STUDENTS USED DEGREES AS A MEASUREMENT OF TIME RATHER THAN RADIANS. t MUST BE IN RADIAN FORM FOR THESE TYPES OF Q'NS!

SOME STUDENTS USED t=0. THE QUESTION STATED "FOLLOWING RELEASE..."

(ii) $V = 9\sin 3t$ ✓

$t = \pi/3 \Rightarrow V = 9\cos\pi$

$= -9 \text{ ms}^{-1}$ ✓

MANY STUDENTS FOUND OUT THE VELOCITY BUT NEGLECTED THE -VE SIGN. VELOCITY HAS DIRECTION!

(iii) x IS MAX WHEN $\frac{dx}{dt}$ (or v) $= 0$.

$0 = 9\cos 3t$

$0 = \cos 3t$

$0 = \cos\theta \quad \therefore \theta = \pi/2, \ 3\pi/2, \ \ldots$

$\therefore t = \pi/6$ s ✓

CHECK IF MAX!

$V' = -27\sin 3t$

$= -27\sin \pi/2$

$= -27$ ⌒ $\therefore$ MAX! ✓

MANY STUDENTS DID NOT CHECK IF INDEED x WAS MAX WHEN $t = \pi/6$ OR NEGLECTED TO MENTION WHY THEY WERE ALLOWING $V = 0$.

(b)

(i) $4500 = 13000\,e^{-3k}$ → 3 yrs between pop. count.

$9/26 = e^{-3k}$ → −VE BECAUSE IT IS DECREASING

$\log_e(9/26) = -3k$ ✓

$k = \dfrac{\log_e(9/26)}{-3}$

$= 0.35362\ldots$ ✓ $= 0.35$ (2dp)

(ii) $13000 = P_0\,e^{-2k}$

$\dfrac{13000}{e^{-2k}} = P_0$

$= 26369.217\ldots$

$= 26\,369$ ✓

THERE WERE SOME ODD ROUNDINGS IN THIS QUESTION. WHY ROUND TO THE NEAREST 10? ie 26 369.21 ⇒ 26370?

CONVERSELY, SOME STUDENTS LEFT THE ANSWER AS A DECIMAL.
CAN YOU HAVE 0.22 OF A THYLACINE? (TASMANIAN TIGER)

(iii) $0.05 > e^{-kt}$

$\ln(0.05) > -kt$

$\dfrac{\ln(0.05)}{-k} < t$ ✓

$8.4715\ldots < t$ ✓

$\therefore t = 9$ years ... or correctly rounded answer.

MOSTLY CORRECT.
SOME STUDENTS FOUND 5% OF 26369, WHICH WAS UNNECESSARY.

(c) $a = 4t$

$v = 2t^2 + c$

$t=0, v=5 \Rightarrow 5 = 0 + c$

$\therefore c = 5$

$v = 2t^2 + 5$ ✓

$d = \frac{2}{3}t^2 + 5t + c$

$t=0, d=-6 \Rightarrow -6 = 0 + c$

$\therefore d = \frac{2}{3}t^2 + 5t - 6$ ✓

$t=10 \Rightarrow d = \frac{2}{3} \times 1000 + 50 - 6$

$d = 710\frac{2}{3}$ m to right ✓

MOSTLY CORRECT. SOME STUDENTS FAILED TO RECOGNISE THE $d = -6$ ASPECT.