Student's Name: ___________

St Catherine's School

Waverley

Year 11 Mathematics

Preliminary Task 3

9th June 2015

Time allowed: **Working time 55 minutes + reading time 3 minutes**

Total marks: **42 marks**

Weighting: **20%**

INSTRUCTIONS

- There are 5 questions each of different value.
- Complete Questions 1, 2 and 3 in one booklet and Questions 4 and 5 in the second booklet.
- Marks for each question are indicated.
- All necessary working must be shown.
- Diagrams should be drawn using pencil and ruler.
- Approved scientific calculators may be used.
- Marks may be deducted for careless or badly arranged work.

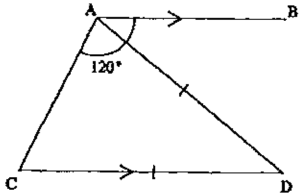
Question 1	3
Question 2	/10
Question 3	/10
Question 4	/7
Question 5	/12
TOTAL	/42

QUESTION 1 **START A NEW BOOKLET** **3 marks**

Answer in your writing booklet.

Choose the correct answer.

(a)



In the diagram above, $AB \parallel CD$. $AD = CD$. $\angle CAB = 120^0$.

Which of the following statements is *False?*

(A) $\angle CAD = 60^0$ (B) ΔADC is equilateral

(C) $\angle CAB = \angle ACD$ (D) $\angle BAD = \angle ADC$

(b) A line L makes an angle of 45^0 with the positive x – axis and passes through the point $(-1,5)$. What is the equation of line L?

(A) $y = x + 4$ (B) $y = -x - 4$

(C) $y = -x + 4$ (D) $y = x + 6$

(c) The equation of the line $\frac{x}{3} + \frac{y}{5} = 4$ written in gradient-intercept form is?

(A) $y = -\frac{5x}{3} + 20$ (B) $y = -\frac{3x}{5} + 20$

(C) $y = -\frac{5x}{3} + 60$ (D) $y = \frac{5x}{3} + 20$

End of Question 1

QUESTION 2 10 marks

(a)

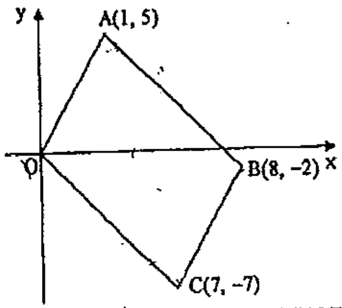


DIAGRAM NOT DRAWN TO SCALE

In the diagram, $O(0,0)$, $A\,(1,5)$, $B(8,-2)$ and $C(7,-7)$ are the vertices of a quadrilateral $OABC$.

(i) Find the midpoint of the interval joining AC. **1**

(ii) Find the gradient of OC. **1**

(iii) Show that the equation of OC is $x + y = 0$. **1**

(iv) Find the exact length of OC. **2**

(v) Show that AB is parallel to OC. **1**

(vi) Find the exact perpendicular distance from B to OC. **2**

(b) The interior angle of a regular polygon is 150^0. How many sides does the polygon have? **2**

End of Question 2

QUESTION 3 10 marks

(a) (i) Find the point of intersection of the lines $2x + 5y = 8$ and $3x - 2y = -7$. **2**

(ii) Hence find the equation of the line passing through the point of intersection of the lines $2x + 5y = 8$ and $3x - 2y = -7$ and perpendicular to the line $4x - 3y - 1 = 0$. Give your answer in general form. **3**

(b)

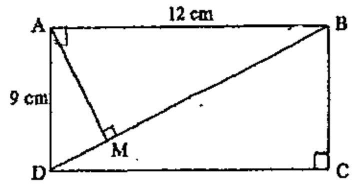


DIAGRAM NOT DRAWN TO SCALE

$ABCD$ is a rectangle with $AB = 12cm$, $AD = 9cm$ and AM is perpendicular to BD.

(i) Copy or trace the diagram onto your answer sheet.

(ii) Find the length of BD. **1**

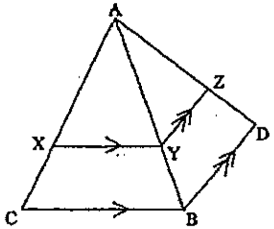
(iii) Prove that ΔABM is similar to ΔDBA. **2**

(iv) Hence find the length of BM. **2**

End of Question 3

QUESTION 4 START A NEW BOOKLET 7 marks

(a)



NOT TO SCALE

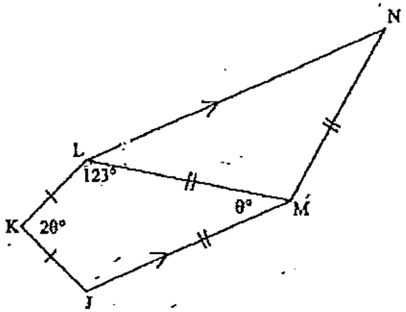
ABC and ABD are two triangles. X, Y and Z are points such that $XY \parallel CB$ and $YZ \parallel BD$.

$AX = 5cm$, $XC = 3cm$ and $AZ = 7cm$.

(i) Copy the diagram into your booklet and mark on it the given information. 1

(ii) Find the value of ZD, giving reasons. 2

(b)



NOT TO SCALE

In the diagram above $JKLM$ is a kite and LMN is a triangle.

$JM \parallel LN$, $JK = KL$, $JM = ML = MN$, $\angle JKL = 2\theta$ and $\angle JML = \theta$.

(i) Copy or trace the diagram into your booklet.

(ii) Show that $\angle JML = 38^0$, giving reasons. 2

(iii) Determine the size of $\angle LNM$ giving reasons. 2

End of Question 4

QUESTION 5 12 marks

(a) (i) Find the perpendicular distance from the point (0,0) to the line $5x + 6y = 30$. 2

(ii) Hence determine how many times the line $5x + 6y = 30$ intersects the circle $x^2 + y^2 = 4$. Give reasoning with your answer. 2

(b) In the diagram CT bisects $\angle ACB$, AE is perpendicular to CT and M is the midpoint of AB. AE produced meets BC at the point P.

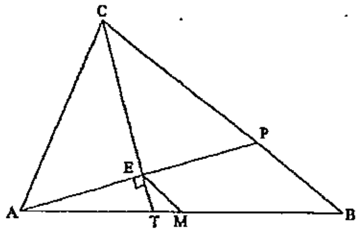


FIGURE NOT TO SCALE

(i) Copy this diagram onto your answer booklet and mark in all the given information. 1

(ii) Prove that ΔACE is congruent to ΔPCE. 3

(iii) Explain why $AE = EP$. 1

(iv) Hence prove that EM is parallel to PB. 3

End of Question 5

END OF TASK

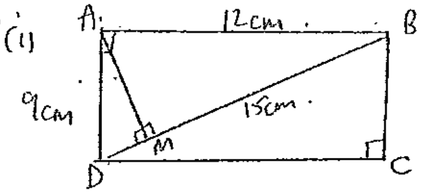
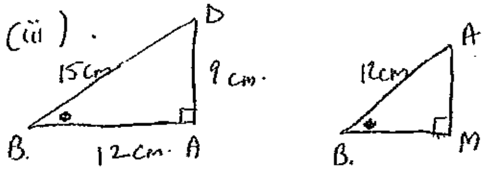
11 MATHEMATICS TASK 3 SOLUTIONS 2015

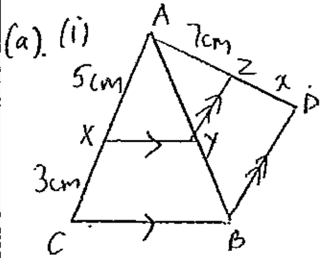
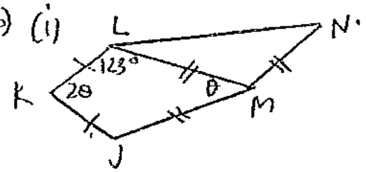
Solutions	Marks	Comments: Criteria
Question 1		
(a) C	1	
(b) $\tan 45 = m$ $\therefore$ gradient of line $= 1$. $\therefore y - 5 = 1(x+1)$ $y - 5 = x + 1$ $y = x + 6$ D	1	• For multiple choice please use capital letters to answer. • Do not just leave the answer, with no letter.
(c) $\frac{x}{3} + \frac{y}{5} = 4$ $\frac{5x+3y}{15} = 4$ $5x + 3y = 60$ $3y = -5x + 60$ $\therefore y = \frac{-5}{3}x + 20$. A	1	
Solutions a. C b. D c. A		

Qu	Solutions	Marks	Comments: Criteria
	Question 2		
(a) (i)	$M_{AC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ $= \left(\frac{1+7}{2}, \frac{5-7}{2}\right)$ $= (4, -1)$	1	
(ii)	$m_{OC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-7-0}{7-0}$ $= -1$	1	
(iii)	$y - 0 = -1(x - 0)$ $y = -x$ $\therefore x + y = 0$, as required	1	
(iv)	$d_{OC} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ $= \sqrt{(0-7)^2 + (0+7)^2}$ $= \sqrt{49+49}$ $= \sqrt{98}$ $= 7\sqrt{2}$ units	2	• If did not simplify $\sqrt{98}$ to $7\sqrt{2}$, gave only $1\frac{1}{2}/2$.
(v)	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-2-5}{8-1}$ $= \frac{-7}{7}$ $= -1$ $m_{OC} = -1$ (part ii). $\therefore$ Since $m_{AB} = m_{OC} = -1$, then the lines AB and OC are parallel.	1	• Needed to have written conclusion "$\therefore$ since $m_{AB} = m_{OC} = -1$, then lines AB and OC are $\parallel$" to get full marks.

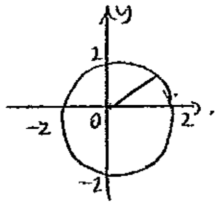
Qn	Solutions	Marks	Comments: Criteria
	Question 2 continued		
	(vi) $B(8,-2)$, $x+y=0$. $\perp d = \frac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ $= \frac{\lvert 1(8)+1(-2)+0\rvert}{\sqrt{1^2+1^2}}$ $= \frac{\lvert 8-2\rvert}{\sqrt{2}}$ $= \frac{6}{\sqrt{2}}$ units.	2.	
	(b) If the interior angle $=150°$, then the exterior angle is $30°$ (angle sum of straight line). $\therefore 360° \div 30° = 12$ $\therefore$ The polygon has 12 sides. OR $150 = \frac{(n-2)180}{n}$, where n = number of sides of polygon. $150n = 180n - 360$ $30n = 360$ $\therefore n = 12$	2	• Some students used trial and error. Working out should be shown in future.

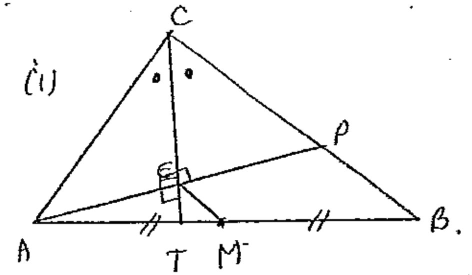
Qn	Solutions	Marks	Comments: Criteria
	Question 3		
	(a) (i) $2x+5y=8$ ① $\times 3$ $6x+15y=24$ ③ $3x-2y=-7$ ② $\times 2$ $6x-4y=-14$ ④ ③ − ④: $6x+15y=24$ $6x-4y=-14$ $19y=38$ $\therefore y=2$ Sub $y=2$ into ①: $2x+10=8$ $2x=-2$ $x=-1$ $\therefore$ Solution is $(-1,2)$	2	• Use of substitution method inevitably caused many algebraic errors. Elimination method much better for this type of question
	(ii) Pt $(-1,2)$, from (i) $4x-3y-1=0$ $3y=4x-1$ $y=\frac{4}{3}x-\frac{1}{3}$ $\therefore m=\frac{4}{3}$ $\perp m = -\frac{3}{4}$. To find equation of line: $y-2=-\frac{3}{4}(x+1)$ $4y-8=-3x-3$ $3x+4y-5=0$.	3	

Qn	Solutions	Marks	Comments: Criteria
	b) (i) 		
	(ii). $BD^2 = 12^2 + 9^2$ (Pythagoras' theorem) $= 144 + 81$ $\therefore BD = 15$ cm	1	
	(iii).  (Placing Δs in same orientation) In Δs BDA, BAM; $\angle DBA = \angle ABM$ (common angle) $\angle DAB = \angle AMB$. (property of rectangle and given $AM \perp BD$) ∴ 3rd angle in each triangle equal (angle sum of Δ) $\therefore \Delta DBA \ \vert\vert\vert \ \Delta ABM$ (equiangular)	2	• 2 angles required for 2 marks; with reason. (No reason, loss of ½ mark)
	(iv). $\frac{BD}{AB} = \frac{AD}{AM} = \frac{AB}{BM}$ (ratio of corresponding sides in similar triangles are equal) $\frac{15}{12} = \frac{9}{AM} = \frac{12}{BM}$. $\therefore 15BM = 12^2$ $BM = \frac{144}{15}$ $\therefore BM = 9.6$ cm.	2	• Correct ratios 1 • Correct answer 1

Qn	Solutions	Marks	Comments: Crit
	Question 4 (a). (i) 	1	
	(ii) $\frac{5}{3} = \frac{AY}{YB}$ (by ratio of intercepts theorem, $XY \parallel BC$) But $\frac{AY}{YB} = \frac{7}{x}$ (by ratio of intercepts theorem, $YZ \parallel BD$) $\therefore \frac{5}{3} = \frac{7}{x}$ $5x = 21$ $x = \frac{21}{5}$ $x = 4.2$ cm.	2	
	(b) (i) 		• Marks for reason and working
	(ii). JKLM is a kite (given) $\therefore \angle KJM = \angle KLM$ ($\Delta KJM \equiv \Delta KLM$). $= 123°$ $\therefore 3\theta + 123° + 123° = 360°$ (angle sum of kite) $3\theta = 114$ $\therefore \theta = 38°$, as required.	2	

Qn	Solutions	Marks	Comments: Criteria
	Question 4 continued ...		
	(iii) $\angle LMJ = 38^\circ$ $= \angle MLN$ (alternate $\angle$s in $\parallel$ lines, $LN \parallel JM$). $\therefore \angle LNM = 38^\circ$ (equal base angle of isosceles ΔLMN).	2	

Qn	Solutions	Marks	Comments: Criteria
	<u>Question 5</u>		
	(a) (i) $(0,0)$ to $5x+6y=30$ $5x+6y-30=0$ $\perp d = \dfrac{\lvert ax_1+by_1+c \rvert}{\sqrt{a^2+b^2}}$ $= \dfrac{\lvert 5(0)+6(0)-30 \rvert}{\sqrt{5^2+6^2}}$ $= \dfrac{\lvert -30 \rvert}{\sqrt{61}}$ $= \dfrac{30}{\sqrt{61}}$ units $(= 3.841\ldots$ units$)$	2	
	(ii)  Radius of circle $x^2+y^2=4$ is <u>2 units</u>, centre $(0,0)$. $\perp d$ of $5x+6y=30$ from $(0,0)$ is <u>3.841...</u> units. $\therefore$ Since the radius of the circle is <u>less</u> than the perpendicular distance found in (a), then this would mean that the circle and line <u>do not intersect</u> at all.		• Needed to make comparison between circle radius and $\perp$ distance values <u>explicit</u>. Sketches accepted but needed again very clear explanation for full marks.

Qn	Solutions	Marks	Comments: Criteria
(b)	(i) 	1	• Many students just rewrote the question and did not work in the bisected angles ($\frac{1}{2}$) and that M was the midpoint ($\frac{1}{2}$).
	(ii). In Δs ACE, PCE; $\angle ACE = \angle PCE$ (given, CT bisects $\angle ACB$) $\angle CEA = \angle CEP$ (given, $AE \perp CT$). CE is common. $\therefore \Delta ACE \equiv \Delta PCE$ (AAS)	3.	• $\frac{1}{2}$ for correct angle/side, $\frac{1}{2}$ for reasons.
	(iii). $AE = EP$ (corresponding sides of congruent triangles, ΔACE and PCE)	1.	
	(iv). In Δs AEM, APB; $\angle A$ is common $\frac{AM}{AB} = \frac{AE}{AP} = \frac{1}{2}$ (M is midpoint of AB, given, and $AE = EP$) $\therefore$ 2 sides of one triangle are in same ratio as 2 sides of the other and included angle is the same. $\therefore \Delta AEM \parallel\!\mid \Delta APB$. $\therefore \angle AEM = \angle APB$ (corresponding $\angle$s of similar Δs). but since $\angle AEM = \angle APB$, then EM must be parallel to PB, as these angles form corresponding angles in parallel lines if $EM \parallel PB$.	3.	• Many students did not make use of the similar Δs AEM, APB. • Connection between corresponding angles of similar Δs and corresponding $\angle$s of parallel lines not made clear. • Use of ratio of intercepts theorem also accepted.