Question 3 — **Start a new Booklet** — Marked by CRA

Consider the function $f(x)=\frac{x^3}{3}-x^2-3x$

a) Find the coordinates of any stationary points and determine their nature. **3**

b) Find the coordinates of any points of inflexion. **2**

c) Sketch the curve showing all critical points. **2**

d) For what values of x is the curve concave up? **1**

TASK CONTINUES ON PAGE 4

Year 12 Mathematics: HSC Task 2:

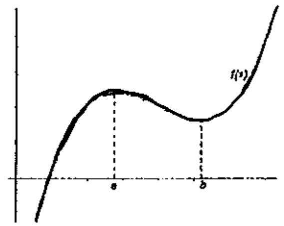
Geometrical Applications of Calculus, Integration — Time: 50 minutes

Term 1, 2015 — MARKS: 28

Teacher: HRK, JJA, GHW, CRA, CJC

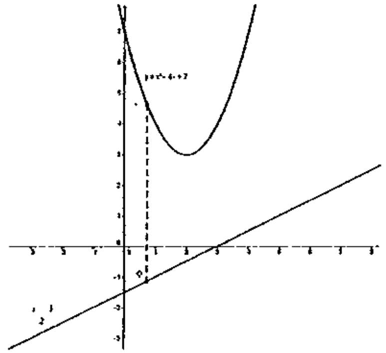
Question 4 — **Start a new Booklet** — Marked by CRA

a) Consider the curve below of $f(x)$, with turning points at $x=a$ and $x=b$



For what values of x is the curve decreasing? **1**

b) In the diagram below, P is a point on the curve $y=x^2-4x+7$, and Q is a point on the line $y=\frac{x-3}{2}$. P and Q have the same x coordinate.



i) Show that $PQ=x^2-\frac{9x}{2}+\frac{17}{2}$ **1**

ii) Hence, find the minimum distance of PQ. **3**

END OF TASK

$\checkmark$ = 1 MARK

1(i) $\int 3x^2 - 4x + 1 \, dx = x^3 - 2x^2 + x + c$

(ii) $\int 3x\sqrt{x} \, dx$

$= \int 3x^{\frac{3}{2}} \, dx$

1ST Make user friendly ie x^n

$= \dfrac{3x^{\frac{5}{2}}}{\frac{5}{2}} + c$

$= \dfrac{2(3x^{\frac{5}{2}})}{5} + c$

$= \dfrac{6x^{\frac{5}{2}}}{5} + c$ OR $\begin{cases} \dfrac{6x^2\sqrt{x}}{5} + c \\ \text{OR} \\ \dfrac{6\sqrt{x^5}}{5} + c \end{cases}$

(iii) $\int \dfrac{2}{3(\sqrt{x+1})^3} \, dx = \dfrac{2}{3}\int (x+1)^{-\frac{3}{2}} \, dx$

again here FIRST Rearrange

$= \dfrac{2}{3} \dfrac{(x+1)^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

$= -\dfrac{4}{3}(x+1)^{-\frac{1}{2}} + c$

OR $= \dfrac{-4}{3\sqrt{x+1}} + c$

1b $A = \left| \int_{-1}^{0} 2x^3 \, dx \right| + \int_{0}^{2} 2x^3 \, dx$

$= \left| \left[\dfrac{x^4}{2} \right]_{-1}^{0} \right| + \left[\dfrac{x^4}{2} \right]_{0}^{2}$

$= \left| (0) - \left(\tfrac{1}{2}\right) \right| + \left[(8) - (0) \right]$

$= \frac{1}{2} + 8$

$= 8\frac{1}{2}$

$\therefore$ Shaded Area $= 8\frac{1}{2}$ units^2

Start here. QUESTION 2 SOLUTIONS

(a)

x	1	2	3
$f(x)$	-1	$\frac{1}{5}$	$\frac{1}{15}$

$h = 1$ ① correct values

$$\int_a^b f(x)\,dx \doteq \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + \dots) + 2(y_2 + y_4 + \dots)\right]$$

$$\doteq \frac{1}{3}\left[\left(-1 + \frac{1}{15}\right) + 4\left(\frac{1}{5}\right)\right]$$ ① correct application of simpsons rule

$$\doteq -0.04 \ (2.d.p)$$ ① correct calculation

(b)(i) $y = x^2$ $\quad y = 2x$

$x^2 = 2x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \quad x = 2$ ①

(ii) $y = 2x \quad y = x^2$

$y^2 = 4x^2 \quad y = x^4$

$$V = \pi\int_0^2 4x^2 - x^4\,dx$$ ① π, squared functions

① subtraction in correct order.

$$= \pi\left[\frac{4x^3}{3} - \frac{x^5}{5}\right]_0^2$$

$$= \pi\left[\frac{32}{3} - \frac{32}{5}\right]$$

$$= \frac{64\pi}{15} \text{ units}^3$$ ① correct exact answer.

QUESTION 3

$$f(x) = \frac{x^3}{3} - x^2 - 3x$$

(a) $f'(x) = \frac{3x^2}{3} - 2x - 3$

$\therefore 0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$\therefore x = -1, \ 3.$ ✓

when $x = -1$, $f(-1) = \frac{(-1)^3}{3} - (-1)^2 - 3(-1) = \frac{5}{3}$

when $x = 3$, $f(3) = \frac{(3)^3}{3} - (3)^2 - 3(3) = -9.$

$\therefore$ Stat points @ $(-1, \frac{5}{3})$, $(3, -9)$ ✓

$f''(x) = 2x - 2.$

$f''(-1) = 2(-1) - 2 = -4. < 0$

$\therefore$ concave down

$\therefore$ Max @ $(-1, \frac{5}{3})$

$f''(3) = 2(3) - 2 = 4 > 0$

$\therefore$ concave up.

$\therefore$ min @ $(3, -9)$ ✓

(b) $f''(x) = 2x - 2.$

$0 = 2x - 2.$

$2x = 2$

$x = 1$

$f(1) = \frac{(1)^3}{3} - (1)^2 - 3(1) = \frac{-11}{3}$

$\therefore$ possible P.O.I @ $(1, -\frac{11}{3})$ ✓

Test:

x	0.9	1	1.1
$f''(x)$	-0.2	0	0.2
	$\cap$	$-$	$\cup$

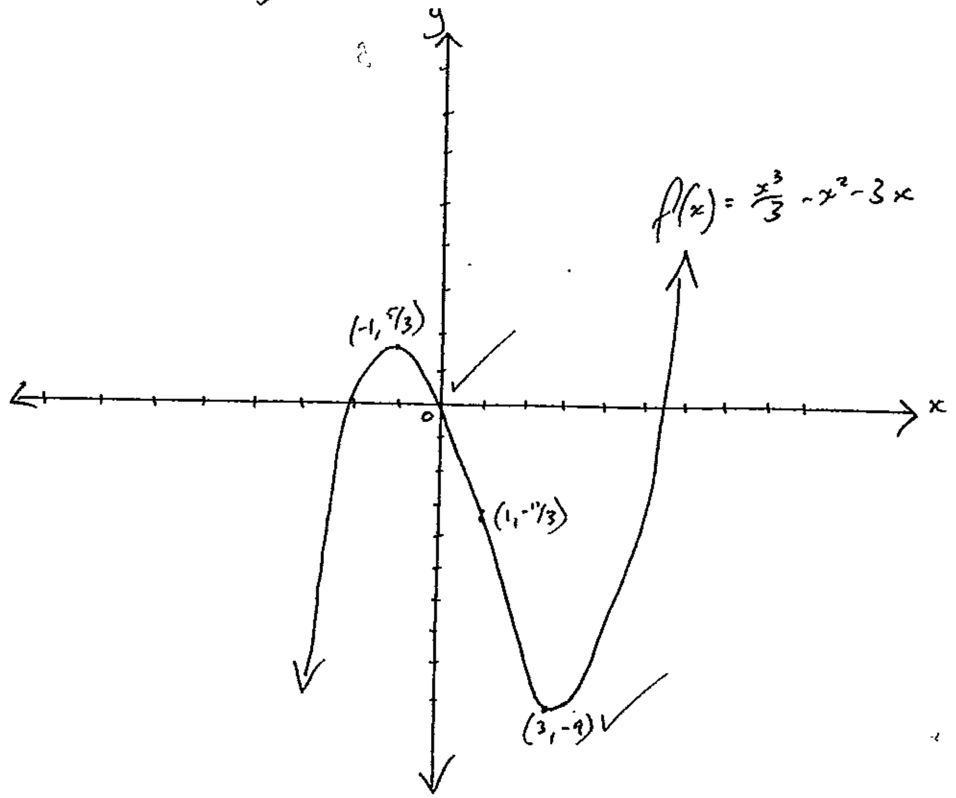
$\therefore$ change in concavity ✓

$\therefore$ P.O.I @ $(1, \frac{-11}{3})$

(c) Max @ $(-1, \frac{5}{3})$

min @ $(3, -9)$

P.O.I @ $(1, -\frac{11}{3})$



(d) concave up $x > 1$

QUESTION 4

(a) Curve decreasing $a < x < b$

(b) (i) $PQ = (x^2 - 4x + 7) - (\frac{x}{2} - \frac{3}{2})$

$= x^2 - 4x + 7 - \frac{x}{2} + \frac{3}{2}$

$= x^2 - x(4 + \frac{1}{2}) + \frac{17}{2}$

$\therefore PQ = x^2 - \frac{9}{2}x + \frac{17}{2}$ as required.

(ii) $PQ' = 2x - \frac{9}{2}$

$0 = 2x - \frac{9}{2}$

$0 = 4x - 9$.

$\therefore 4x = 9$

$x = \frac{9}{4}$.

$PQ'' = 2 > 0$

$\therefore$ concave down.

$\therefore$ min distance occurs when $x = \frac{9}{4}$.

$\therefore PQ = (\frac{9}{4})^2 - \frac{9}{2}(\frac{9}{4}) + \frac{17}{2}$

$= \frac{55}{16}$ units or (3.4375 units)