Topic 1. Exercises on Complex Numbers I
Level 1

1. Find (a) $z_1 + z_2$; (b) $z_1 - z_2$; (c) $z_1 z_2$; (d) $\frac{z_1}{z_2}$, when $z_1 = 4 + i$, $z_2 = 2 + 3i$.

(a) $6 + 4i$; (b) $2 - 2i$; (c) $5 + 14i$; (d) $\frac{11}{13} - \frac{10}{13}i$.

2. If $z = -3 + 2i$

(a) Evaluate $\bar{z}$. Verify that $z\bar{z}$ is real.

$\boxed{\bar{z} = -3 - 2i}$

(b) Use $\frac{1}{z} = \frac{\bar{z}}{z\bar{z}}$ to find $\frac{1}{z}$ in the form $a + ib$, $a, b \in \mathbf{R}$.

$\boxed{\frac{1}{z} = -\frac{3}{13} - \frac{2}{13}i}$

3. (a) $z \in \mathbf{C}$ such that $\operatorname{Im} z = 2$ and z^2 is real. Find z.

$\boxed{z = 2i}$

(b) Find the square roots of the following complex numbers (i) -25; (ii) i.

(i) $5i\,,\ -5i$ (ii) $\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$.

4. (a) Solve the quadratic equation $x^2 + x + 1 = 0$.

$$\boxed{x = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}}$$

(b) Solve the quadratic equation $4x^2 - 4(1+2i)x - (3-4i) = 0$.

$$\boxed{x = \frac{1}{2} + i.}$$

5. Prove the following results above complex conjugates :

(a) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$;

(b) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$;

(c) $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$;

(d) $\overline{\left(\frac{1}{z}\right)} = \frac{1}{(\bar{z})}$;

(e) $\overline{z_1 \div z_2} = \bar{z}_1 \div \bar{z}_2$;

(f) $\overline{5z} = 5\bar{z}$.

6. (a) $z \in \mathbf{C}$ such that $\operatorname{Re} z = 2\operatorname{Im} z$, and $z^2 - 4i$ is real. Find z .

$2+i; -2-i$

(b) Find the square root of $z = -6i$.

$\boxed{\sqrt{3} - i\sqrt{3}}, \boxed{-\sqrt{3} + i\sqrt{3}}$