TOPIC 1 SETS

FORMULAE AND IMPORTANT NOTES

1. A set is any well-defined list, collection, or class of objects. These objects are called elements or members of the set. A finite set has finite number of members. An infinite set has infinite number of members.

2. Sets will usually be denoted by capital letters A, B, C, etc. There are many ways to define sets. Here are some examples: $A = \{x : 2 < x < 100\}$, $B = \{(x, y): 2x + 3y = 5\}$, $C = \{x : x \text{ is a perfect square}\}$, $D = \{a, b, c, d, e\}$.

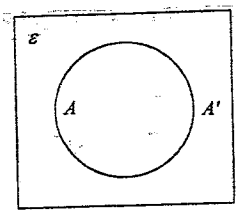
3. **Quick reference**

NOTATIONS	WHAT IT MEANS
$A \cup B$	Union of A and B
$A \cap B$	Intersection of A and B
n(A)	Number of elements in set A
$\in$	"... is an element of ..."
$\notin$	"... is not an element of ..."
A'	Complement of set A
$\varnothing$	The empty set or null set
ε	Universal set
$A \subseteq B$	A is a subset of B
$A \subset B$	A is a proper subset of B
$A \not\subseteq B$	A is not a subset of B
$A \not\subset B$	A is not a proper subset of B
$\{x_1, x_2, ...\}$	The set with elements x_1, x_2, ...
$\{x: ...\}$	The set of all x such that ...

4. (i) The null (empty) set is a set with no number. Example: {a real number whose square is negative} = ϕ. A set with "zero" as the only member is not a null set.
   (ii) A null set is a subset of all sets.
   (iii) If $A \subseteq B$ then $n(A) \leq n(B)$. The converse needs not be true.
   (iv) If $A \subset B$ then $n(A) < n(B)$. The converse needs not be true.
   (v) If $A \subseteq B$ and $B \subseteq A$ then A and B have the same members. We write $A = B$. The members of each of the two sets do not have to be arranged in the same order.

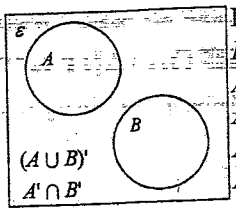
5. Study the following diagrams carefully. Pay particular attention to the notations that are used to define each region.

(i)



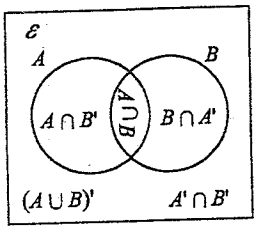
In diagram (i), $A \cup A' = \varepsilon$

(ii)



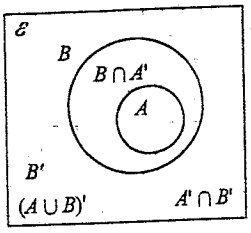
In diagram (ii),
$B \subset A'$
$A \subset B'$
$A \cap B = \varnothing$
$A \cup B' = B'$.
$A' \cup B = A'$
$A \cap B' = A$
$A' \cap B = B$
A and B are mutually exclusive.
They are disjoint sets.

(iii)



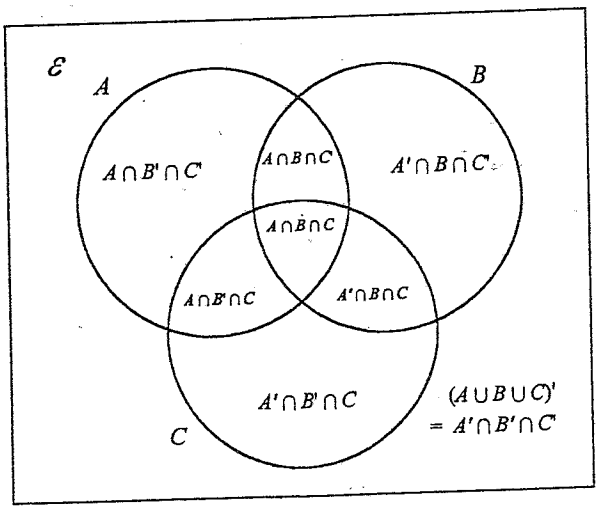
In diagram (iii), $A \cap B \neq \phi$

(v)



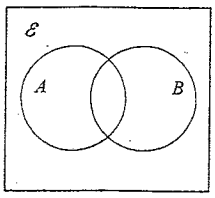
In diagram (iv),
$B \not\subset A$
$B' \subset A'$
$A \cup B = B$
$A \subset B$
$A \cap B = A$
B contains or includes A. A is a proper subset of B.

(v)



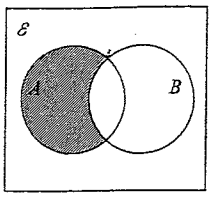
6. Study the name given to each shaded region of the following diagrams.

(i)



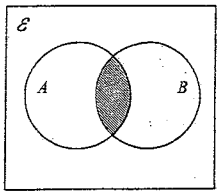
$\varnothing = \varepsilon'$

(ii)



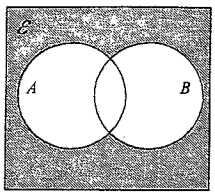
$A \cap B' = (A' \cup B)'$

(iii)



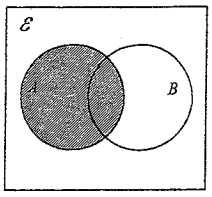
$A \cap B$

(iv)



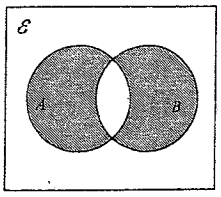
$(A \cup B)' = A' \cap B'$

(v)



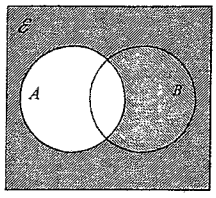
A

(vi)



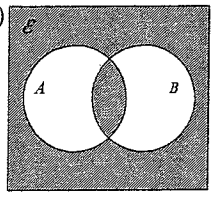
$(A \cap B') \cup (B \cap A')$

(vii)



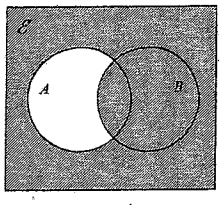
A'

(viii)



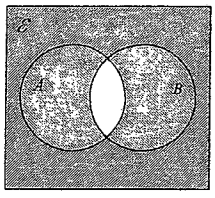
$(A \cup B)' \cup (A \cap B)$
$= (A' \cap B') \cup (A \cap B)$
$= (A' \cup B) \cap (B' \cup A)$

(ix)



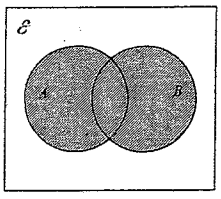
$A' \cup B = (A \cap B')'$

(x)



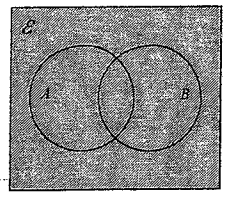
$(A \cap B)' = A' \cup B'$

(xi)



$A \cup B$

(x)

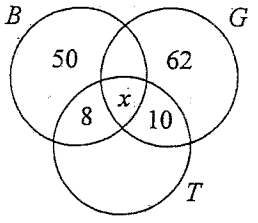


ε

7. (i) For any two sets: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
   (ii) For any three sets:
   $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

WORKED EXAMPLES

1. Each member of a sports club plays at least one of the games: badminton, golf, tennis. Denoting the sets of members who play badminton, golf, tennis by B, G, T respectively, display the following information on a Venne diagram:
$n(B' \cap G \cap T') = 62$, $n(B \cap G' \cap T') = 50$, $n(B \cap G' \cap T) = 8$, $n(B' \cap G \cap T) = 10$.
Denoting $n(B \cap G \cap T)$ by x and given that $n(G) = 112$ and $n(T) = 60$, express $n(B \cap G \cap T')$ and $n(B' \cap G' \cap T)$ in terms of x.
Given that the total membership of the club is 200, evaluate x and find (i) $n((B \cup G) \cap T')$, (ii)$n(B' \cup G' \cup T')$. (N84/P2/2a)



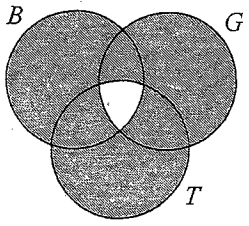
$$n(B \cap G' \cap T') = n(G) - n(B' \cap G \cap T') - n(B' \cap G \cap T) - (B \cap G \cap T)$$
$$= 112 - 62 - 10 - x$$
$$= 40 - x$$

$$n(B' \cap G' \cap T') = n(T) - n(B' \cap G \cap T') - n(B \cap G \cap T) - n(B \cap G' \cap T)$$
$$= 60 - 10 - x - 8$$
$$= 42 - x$$

$$n(\varepsilon) = n(G) + n(B \cap G' \cap T') + n(B \cap G' \cap T) - n(B' \cap G' \cap T)$$
$$= 112 + 50 + 8 + 42 - x$$
$$= 212 - x$$
$$200 = 212 - x$$
$$x = 12$$

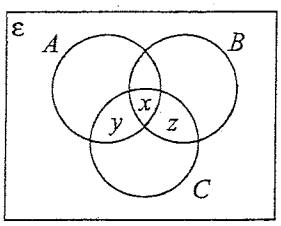
(i) $$n((B \cup G) \cap T') = n(B \cap G' \cap T') + n(B \cap G \cap T') + n(B' \cap G \cap T')$$
$$= 50 + (40 - x) + 62$$
$$= 50 + 40 - 12 + 62$$
$$= 140$$

(ii)



$B' \cup G' \cup T' = (B \cap G \cap T)'$
$\therefore n(B' \cup G' \cup T')$
$= n(B \cap G \cap T)'$
$= n(\varepsilon) - n(B \cap G \cap T)$
$= 200 - x$
$= 200 - 12$
$= 188$

2. (a) Express in set notation the region containing (i) x elements, (ii) y elements, (iii) z elements.



(b) Given that $n(\varepsilon) = 150$, $n(A) = 42$, $n(B) = 50$, $n(A \cap B) = 12$, $n(B \cap C) = 18$, $n(A \cap C) = 13$, $n(A \cup C) = 82$, $n(A \cup B \cup C)' = 40$, evaluate x, y and z. (N85/P1/29)

(a) (i) $A \cap B \cap C$
(ii) $A \cap B' \cap C$
(iii) $A' \cap B \cap C$

(b) $z = n(B \cap C) - n(A \cap B \cap C)$
$= 18 - x$
$n(A' \cap B' \cap C) = n(A \cup C) - n(A) - z$
$= 82 - 42 - (18 - x)$
$= 40 - 18 + x$
$= 22 + x$

$n(A) + n(A' \cap B' \cap C) + n(A' \cap B) = n(A \cup B \cup C)$
$42 + (22 + x) + n(B) - n(A \cap B) = n(\varepsilon) - n(A \cup B \cup C)'$
$42 + 22 + x + 50 - 12 = 150 - 40$
$102 + x = 110$
$x = 8$

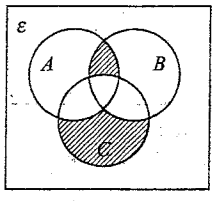
$y = n(A \cap C) - n(A \cap B \cap C)$
$= 13 - 8$
$= 5$

$z = n(B \cap C) - n(A \cap B \cap C)$
$= 18 - 8$
$= 10$

$\therefore x = 8, y = 5, z = 10$

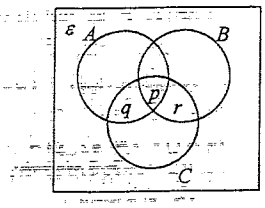
PAST EXAMINATION QUESTIONS

1. Use set notation to define the shaded regions.



2. ε = the set of real numbers. R = the set of rational numbers. I = the set of irrational numbers. N = the set of negative numbers. Draw the Venn diagram that illustrates the relationship of ε, R, I and N. What is the set $(R \cup I)'$?

3. Given that
$n(\varepsilon) = 20$, $n(A) = 10$,
$n(B) = 11$, $n(A \cap B) = 4$,
$n(B \cap C) = 7$, $n(A \cap C) = 5$,
$n(A \cup C) = 16$, $n(A \cup B \cup C)' = 1$.
Evaluate p, q and r.



4. Given that $\varepsilon = \{x: x \text{ is an integer} > 2\}$, draw a Venn diagram showing ε and the subsets
$D = \{x: x \text{ is odd}\}$
$P = \{x: x \text{ is a prime number}\}$
$C = \{x: x \text{ is a perfect cube}\}$
Place each of the numbers 27, 28, 29, 30 in the correct parts of the Venn diagrams.

5. There are 41 students in a school. They follow courses in English, Chinese and Malay. All students take one, two or all three of these subjects. A survey of course numbers gives the following data.

Malay	26
English	27
Both Malay and English	16
Both Chinese and Malay	15
English but not Chinese	6
English only	4

Use a Venn diagram to determine the number of students
(i) taking Chinese,
(ii) studying one subject only.

6. The set $P(n)$ is the set of prime factors of n, where n is an integer greater than or equal to 2.
Example: $P(24) = \{2, 3\}$.
List the numbers of (i) $P(240)$, (ii) $P(44) \cup P(30)$, (iii) $P(15) \cap P(21)$.

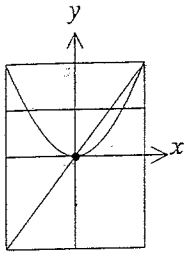
7. Each number of a sports club with 100 members plays at least one of the games: basketball, football, table tennis. The sets of members who play basketball, football, table tennis are denoted by B, F, T respectively. It is found that
$n(B' \cap F \cap T') = 31$, $n(B \cap F' \cap T') = 25$,
$n(B \cap F' \cap T) = 8$, $n(B' \cap F \cap T) = 5$,
$n(F) = 56$, $n(T) = 30$.
Evaluate $n((B \cup F) \cap T')$.

8. In a class of 12 pupils, T is the set of pupils who play the trumpet and E is the set of pupils who play the electric organ. Given $n(T) = 8$ and $n(E) = 6$, state the maximum and minimum values of (i) $n(T \cap E)$, (ii) $n(T \cup E)$.

9. A box contains 16 red, 12 blue and 32 green spheres. There are 24 glass spheres, and the remainder are made of plastic. The number of red plastic spheres is equal to the number of blue glass spheres. How many plastic spheres are green?

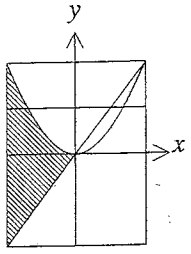
10. In a survey a researcher classified bicycles in three sets X, Y and Z. Given that $n(X) = 16$, $n(Y) = 24$, $n(Z) = 40$, $n(X \cap Y) = 14$, $n(Y \cap Z) = 12$, $n(X \cap Z) = 8$, evaluate $n(X \cap Y \cap Z')$.

11. A universal set ε is defined by $\varepsilon = \{(x, y) : -2 \le x \le 2$ and $-4 \le y \le 4\}$. The figure shows the boundaries of ε and of three subsets A, B and C defined by $A = \{(x, y) : y \le 2\}$, $B = \{(x, y) : y \ge 2x\}$, $C = \{(x, y) : y \le x^2\}$.

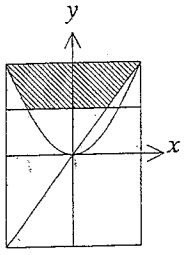


Draw the figure twice and, in each case, shade the region representing the following (i) ABC, (ii) $A'C'$. In each of the three diagrams below, express the set represented by the shaded area in terms of the symbols, A, B, C, A', B', C', $\cap$, $\cup$

(iii)



(iv)



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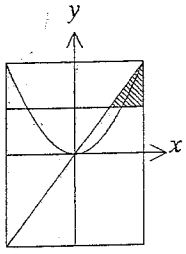
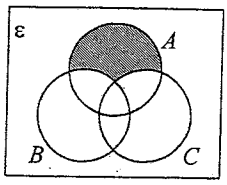


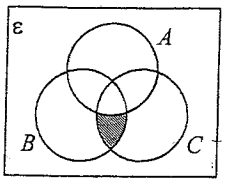
(J83/P1/29)

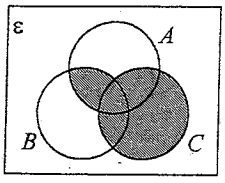
12. Three sets A, B, C are subsets of a universal set ε such that $n(A \cap B \cap C) = 15$, $n(A \cap B' \cap C') = 8$, $n(A' \cap B \cap C) = 3$. Draw a Venn diagram to illustrate the given information. Given also that $n(\varepsilon) = 90$, $n(A) = 54$, $n(B) = 48$, $n(C) = 38$, $n(A \cap B \cap C) = x$, express in terms of x.
(i) $n(A' \cap B' \cap C)$,
(ii) $n(A \cap B \cap C')$,
(iii) $n(A' \cap B \cap C)$.
Find
(iv) $n(A' \cap B' \cap C')$,
(v) the maximum value of x,
(vi) the minimum value of x. (J83/P2/29)

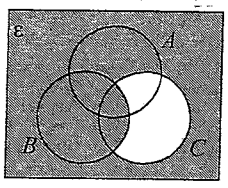
13. The set $P(n)$ is the set of prime factors of n, where n is an integar, $(n \ge 2)$,
e.g. $P(18) = \{2, 3\}$.
List the members of
(i) $P(180)$,
(ii) $P(12) \cap P(15)$,
(iii) $P(30) \cup P(42)$.
State whether each of the following statements is true or false, giving a counter example in those cases where you believe the statement to be false.
(iv) $P(a) = P(b) \Rightarrow a = b$.
(v) $P(a) \subset P(ab)$.
(vi) $P(a) \subset P(b)$ $a < b$.
(vii) $P(2^k) = P(2^{k+1})$ where k is a positive integer.
(viii) $a = b^2 \Rightarrow P(a) = P(b)$. (N83/P1/29)

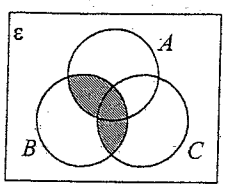
14. Each of the Venn diagrams below represents a universal set e and three subsets A, B and C. In each case express the set represent by the shaded region in terms of some or all of A, B, C, $\cap$, $\cup$, the complement sign ' and brackets.

(i) 

(ii) 

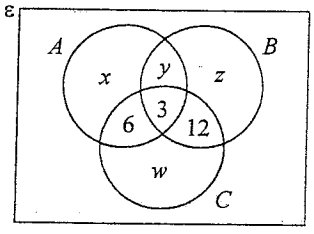
(iii) 

(iv) 

(v) 

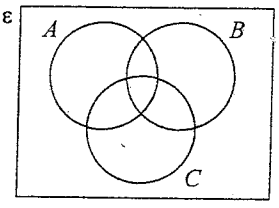
(N83/P2/29)

15. (a) In the given diagram $n(A) = 31$, $n(E) = 33$, $n(A \cup B) = 56$, $n(B \cup C) = 50$. Evaluate x, y, z and w. Given that $n(\varepsilon) = 70$, find $n(A \cup B \cup C)'$.



(b) Given that none of the sets A, B, C, D, E, F is ε or ϕ state, in set language, the deduction that can be made from each of the following:
(i) $A \cap B = A$,
(ii) $C \cup D = D$,
(iii) $E \cap F = E \cup F$. (J84/P1/29)

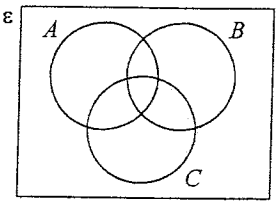
16. (a) The figure shows a universal set ε and three subsets A, B and C. Draw the figure three times and, by shading, illustrate regions.



(i) $(A \cap B')'$,
(ii) $A' \cap B' \cap C$
(iii) $(A \cup B) \cap C$.

(b) Given that C is the set of cars, T is the set of trains and P is the set of vehicles which take passengers,
(i) express in set language the statement " not all trains carry passengers",
(ii) interpret $(T \cup C)' \cap P \neq \phi$. (J84/P2/29)

17. (a) The figure shows a universal set ε and three subsets A, B and C. Draw the figure three times and, by shading, illustrate the regions:



(i) $(A \cup B) \cap C'$
(ii) $A' \cap B' \cap C'$.
(iii) $(A \cap B) \cup (B \cap C)$

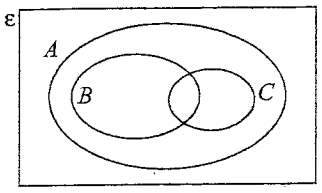
(b) In a class of 30 pupils, P is the set of pupils who play the piano and V is the set of pupils who play violin. Given that $n(P) = 20$ and $n(V) = 15$, state the maximum and minimum values of (i) $n(P \cap V)$, (ii) $n(P \cup V)$. (N84/P1/29)

18. A total of 90 copies of either the *Times*, the *Mail* or the *Express* was delivered to the 60 houses in a certain street. Each house took at least one paper. Five houses each took a copy of each paper. No house took only the *Mail*. Seven houses took both the *Times* and the *Mail*, but not the *Express*. Nine houses took both the *Mail* and the *Express*, but not the *Times*. Draw a

Venn diagram to illustrate this information and find (i) the number of houses which took both the *Times* and the *Express*, but not the *Mail*, (ii) the least and the greatest number of copies of the *Times* that might be required. (J85/P1/29)

19. (a) Given that $\varepsilon = \{x : x \text{ is an integer} > 2\}$, draw a Venn diagram showing ε and the subsets $O = \{x : x \text{ is odd}\}$, $P = \{x : x \text{ is a prime number}\}$, $S = \{x : x \text{ is a perfect square}\}$. Place each of the numbers 64, 65, 66, 67 in the correct parts of the Venn diagram.

(b) The figure shows a universal set ε and three subsets A, B and C. Determine whether or not each of the following sets is necessarily empty:
(i) $A' \cap B$, (ii) $A \cap C$, (iii) $A \cap (B \cup C)$.
Reduce to a simpler form: (iv) $A \cup B \cup C$,
(v) $A' \cap B' \cap C$, (vi) $A \cup B' \cup C$. (J85/P2/29)



20. There are 82 students in a college science department, and they follow courses in mathematics, physics and chemistry. All students take one, two or all of these subjects. An investigation of course numbers gives the following information:

	Students
Chemistry	52
Mathematics	54
Both mathematics and chemistry	32
Both physics and chemistry	30
Mathematics but not physics	12
Mathematics only	8

Use a Venn diagram to determine the number of students (i) studying physics, (ii) taking one subject only.

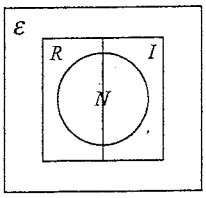
Eleven new students, each of whom studies exactly two subjects, join the department. The result is that the total number of students studying each of the three subjects are now equal. Determine the number now studying (iii) each subject, (iv) both physics and chemistry.
(N85/P2/29)

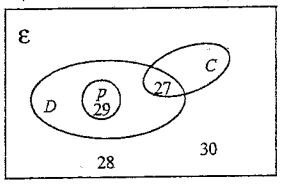
21. Given that C is the set of cars, T is the set of trains and P is the set of vehicles which take passengers,
(i) express in set language the statement "not all trains carry passengers",
(ii) make a statement explaining the meaning of $(T \cup C)' \cap P \neq \phi$. (sp/P1/2)

22. Sets H, M and P are defined by
$H = \{\text{students studying history}\}$,
$M = \{\text{students studying mathematics}\}$,
$P = \{\text{students studying physics}\}$.
Express the following statements in set notation.
(i) No student studies both history and physics.
(ii) All physics students also study mathematics.
Describe in words which students belong to the set
(iii) $H' \cap M \cap P'$,
(iv) $(H \cup M) \cap P'$.
(J2002/P2/3)

ANSWERS

TOPIC 1 SETS

1. $(A \cap B \cap C) \cup (A' \cap B' \cap C)$
2. 

$(R \cup I)' = \varnothing$
3. 3, 2, 4
4. 

5. (i) 26 students
   (ii) 17 students
6. (i) {2, 3}
   (ii) {2, 3, 5, 11}
   (iii) {2, 3, 5}
7. 70
8. (i) $2 \leq n(T \cap E) \leq 6$
   (ii) $8 \leq n(T \cup E) \leq 12$
9. 24 plastic spheres
10. 12
11. (iii) $B \cap C$
    (iv) $A' \cap B \cap C'$
    (v) $A' \cap B'$
12. (iv) 14
    (v) 20
    (vi) 1
13. (i) (2, 3, 5)
    (ii) (3)
    (iii) (2, 3, 5, 7)
    (iv) F
    (v) T
    (vi) F
    (vii) T
    (viii) T
14. $A \cap B' \cap C'$, $A' \cap B \cap C$, $(A \cap B) \cup C'$, $B \cup C'$, $(A \cup C) \cup B$
15. (a) 17, 5, 13, 11; 3;
    (b) (i) $A \subseteq B$
    (ii) $C \subseteq D$
    (iii) $E = F$
16. (b) (i) $T \cap P' \neq \phi$
    (ii) Not all vehicles which take passengers are cars or trains.
17. (b) (i) 15, 5
    (ii) 30, 20
18. (i) 4
    (ii) 16, 51
19. (b) (i) Yes
    (ii) No
    (iii) Yes
    (iv) A
    (v) A'
    (vi) ε
20. (i) 52
    (ii) 34
    (iii) 60
    (iv) 35
21. (i) $T \cap P' \neq \phi$
    (ii) Not all passengers travel on trains or in cars.
22. (i) $H \cap P = \varnothing$
    (ii) $P \subset M$
    (iii) Students who study mathematics but not history and not physics.
    (iv) Students who study history or mathematics but not physics.

TOPIC 2 IRRATIONAL ROOTS (SURDS)

1. (a) 25
   (b) 3
   (c) 2
   (d) 6