Topic 2. Exercises on Complex Numbers II
Level 1

1. (a) Find $|z|$ and $\arg z$ when $z = i$.

$|z| = 1; \arg z = \dfrac{\pi}{2}$

(b) Find $|z|$ and $\arg z$ when $z = 2 + 2i$.

$|z| = 2\sqrt{2}; \arg z = \dfrac{\pi}{4}$

(c) Find $|z|$ and $\arg z$ when $z = -3 + 2i$.

$|z| = \sqrt{13}; \arg z = \pi - \tan^{-1}(2/3)$

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2. (a) Write down the moduli and arguments of $-\sqrt{3}+i$ and $4+4i$. Hence express in modulus/argument form $(-\sqrt{3}+i)(4+4i)$.

$$\boxed{8\sqrt{2}\text{cis}\left(-\frac{11\pi}{12}\right)}$$

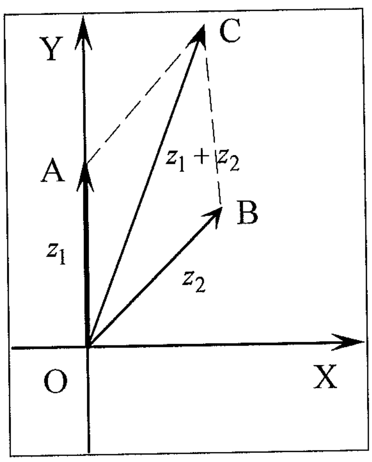
(b) Write $\left(1+\sqrt{3}i\right)^{-1}$ in modulus/argument form.

$$\boxed{\frac{1}{2}\text{cis}\left(-\frac{\pi}{3}\right)}$$

3. State the modulus and argument of $-1+i$. Hence write $(-1+i)^{18}$ in the form $a+ib$.

$\boxed{\sqrt{2}; \frac{3\pi}{4}; z^{18} = -512i}$.

4. On an Argand diagram the points A and B represent the complex numbers $z_1 = i$ and $z_2 = \frac{1}{\sqrt{2}}(1+i)$ respectively. Show that $\arg(z_1 + z_2) = \frac{3\pi}{8}$.



5. Use the vector representation of z_1 and z_2 on an Argand diagram to show that if $|z_1| = |z_2|$, then $\frac{z_1 + z_2}{z_1 - z_2}$ is imaginary.

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6. On an Argand diagram the points A and B represent z_1 and z_2 respectively. OAB is an equilateral triangle. Show that $z_1^2 + z_2^2 = z_1 z_2$.
(Hint: Express z_2 in terms of z_1).