

Topic 3. Exercises on Complex Numbers III
Level 1

1. Indicate on an Argand diagram the locus of the point P representing z when

(a) $\text{Im } z = 1$;

(b) $|z - 2 - i| = 2$;

(c) $\arg(z + i) = \frac{3\pi}{4}$.

(Answers on the last page of this workbook).

2. (a) The point P represent the complex number z on an Argand diagram. Describe the locus of P when $|z| = |z - 2|$.

- (b) $|z + i| \leq 2$ and $0 \leq \arg(z + 1) \leq \frac{\pi}{4}$. Sketch the region in an Argand diagram which contains the point P representing z .

(Answers on the last page of this workbook).

3. $\text{Arg}(z+3) = \frac{\pi}{3}$. Sketch the locus of the point P representing z on an Argand diagram.

Find the modulus and argument of z when $|z|$ takes its least value. Hence find in the form $a+ib$, the value of z for which $|z|$ is a minimum.

(Answers on the last page of this workbook).

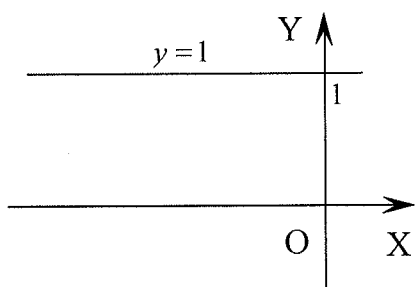
4. If $\arg(z - 2) = \arg(z + 2) + \frac{\pi}{3}$, show that the locus of the point P representing z on an Argand diagram is an arc of a circle and find the center and radius of this circle.

Centre $\left(0, \frac{2}{\sqrt{3}}\right)$; radius $\frac{4\sqrt{3}}{3}$
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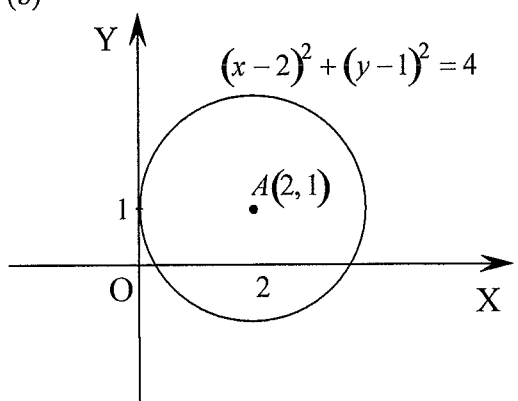
5. The complex number is given by $z = t + \frac{1}{t}$, where $t = r(\cos\theta + i\sin\theta)$. Show that the equation of the locus of the point P which represents z on an Argand diagram when $r = 2$ and θ varies is $\frac{4x^2}{25} + \frac{4y^2}{9} = 1$.

SOLUTIONS:

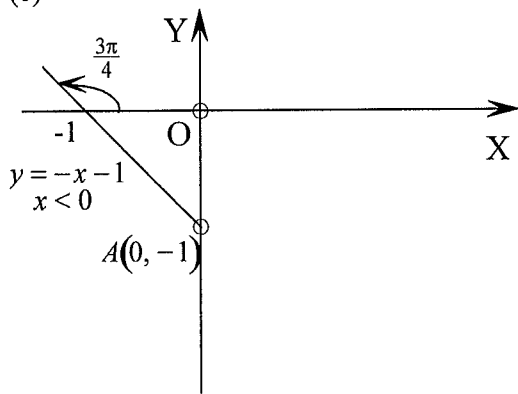
(1) (a)

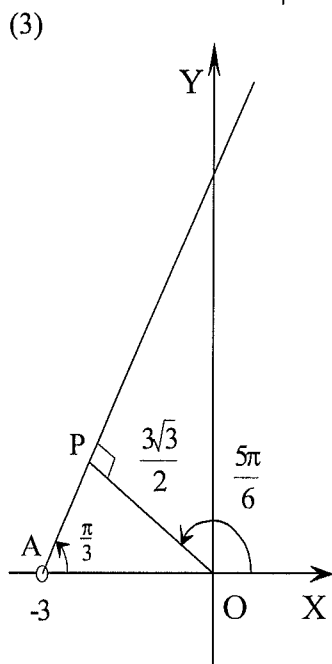
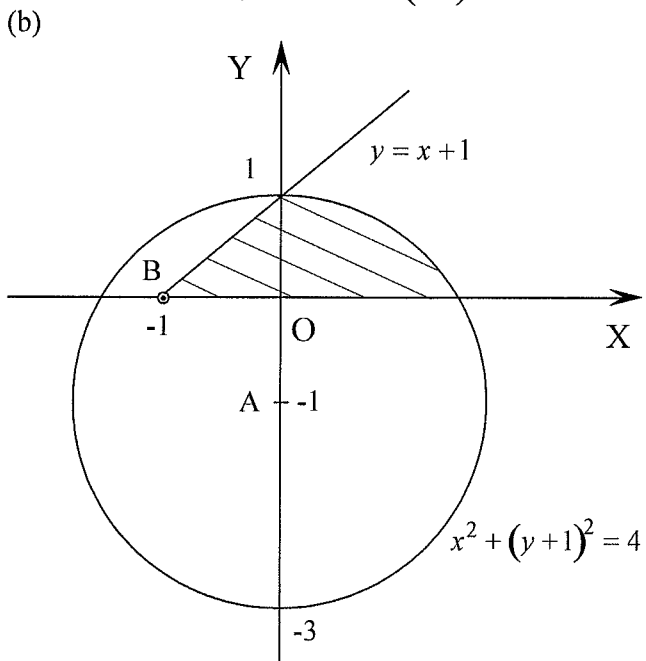
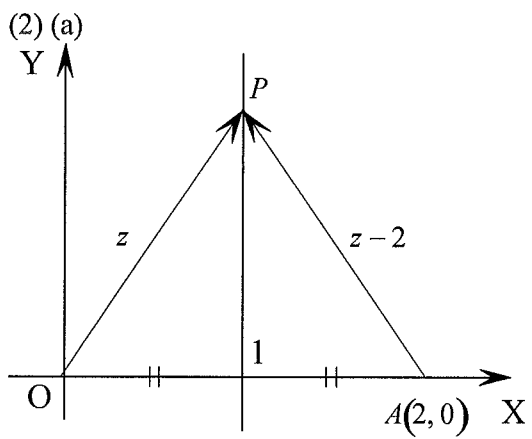


(b)



(c)





$$\frac{3}{4}(-3 + i\sqrt{3})$$